ON CERTAIN INTEGRAL FORMULAS INVOLVING THE PRODUCT OF BESSEL FUNCTION AND JACOBI POLYNOMIAL

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Abstract. In the present paper, we establish some interesting integrals involving the product of Bessel function of the first kind with Jacobi polynomial, which are expressed in terms of Kampé de Fériet and Srivastava and Daoust functions. Some other integrals involving the product of Bessel (sine and cosine) function with ultraspherical polynomial, Gegenbauer polynomial, Tchebicheff polynomial, and Legendre polynomial are also established as special cases of our main results. Further, we derive an interesting connection between Kampé de Fériet and Srivastava and Daoust functions.

1. Introduction

Some very interesting integrals associated with a variety of special functions have been established by many authors (see, [9], [10], [13], [14]). Very recently, Choi and Agarwal [11] gave some interesting unified integrals involving the Bessel function of the first kind, which are expressed in terms of generalized (Wright) hypergeometric functions.

Motivated by the above-mentioned works, in the present paper, we establish a new class of integral formulas involving the product of Bessel function \( J_\nu(z) \) with Jacobi polynomial \( P_n^{(\alpha, \beta)}(z) \), which are expressed in terms of Kampé de Fériet and Srivastava and Daoust functions. Some other integrals involving the product of Bessel (sine and cosine) function with ultraspherical polynomial, Gegenbauer polynomial, Tchebicheff polynomial, and Legendre polynomial are also established as special cases of our main results. Next, we establish a very interesting connection between Kampé de Fériet function and Srivastava and Daoust function.

The Bessel function \( J_\nu(z) \) of the first kind and order \( \nu \) is defined by (see [4], [7])

\[
J_\nu(z) = \sum_{m=0}^{\infty} \frac{(-1)^m (\tfrac{z}{2})^{\nu+2m}}{m! \Gamma(\nu+m+1)}. \tag{1.1}
\]
It is well known that
\[ J_{\pm}\frac{1}{2}(z) = \sqrt{\frac{2}{\pi z}} \sin z \] (1.2)
and
\[ J_{-\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi z}} \cos z. \] (1.3)

The Jacobi polynomial \( P^{(\alpha, \beta)}_n(z) \) is defined by (see [4], [7])

\[
P^{(\alpha, \beta)}_n(z) = \frac{(1 + \alpha)_n}{n!} \binom{-n, 1 + \alpha + \beta + n; 1 - z/2}{1 + \alpha;},
\]
(1.4)
or equivalently,

\[
P^{(\alpha, \beta)}_n(z) = \sum_{k=0}^{n} \frac{(1 + \alpha)_n (1 + \alpha + \beta)_{n+k}}{k! (n-k)! (1 + \alpha)_k (1 + \alpha + \beta)_n} \left( \frac{z-1}{2} \right)^k.
\]
(1.5)

From (1.4) and (1.5) it follows that \( P^{(\alpha, \beta)}_n(z) \) is a polynomial of degree \( n \) and that

\[ P^{(\alpha, \beta)}_n(1) = \frac{(1 + \alpha)_n}{n!}. \] (1.6)

For \( \beta = \alpha \), the polynomial \( P^{(\alpha, \alpha)}_n(z) \) is called the ultraspherical polynomial and

\[ P^{(\mu, -\frac{1}{2}, -\frac{1}{2})}_n(z) = \frac{(\mu + \frac{1}{2})_n}{(2\mu)_n} C^\mu_n(z), \] (1.7)

where \( C^\mu_n(z) \) is the Gegenbauer polynomial (see [4], [7]),

\[ P^{(-\frac{1}{2}, -\frac{1}{2})}_n(z) = \frac{\frac{1}{2}_n}{n!} T_n(z) \] (1.8)
and

\[ P^{(\frac{1}{2}, \frac{1}{2})}_n(z) = \frac{\frac{3}{2}_n}{(n+1)!} U_n(z), \] (1.9)

where \( T_n(z) \) and \( U_n(z) \) are the Tchebicheff polynomials of the first and second kind, (see [4], [7]) and

\[ P^{(0,0)}_n(z) = P_n(z), \] (1.10)

where \( P_n(z) \) is the Legendre polynomial (see [4], [7]).

In 1921, the four Appell functions were unified and generalized by Kampé de Fériet, who defined a general hypergeometric function of two variables. The notation introduced by Kampé de Fériet for his double hypergeometric function of superior order was subsequently
abbreviated by Burchann and Chaundy. We recall here the definition of a more general double hypergeometric function in a slightly modified notation (see [7]):

\[
F_p^{m; n; k} [(a_p) : (b_q) ; (c_k) ; (\alpha_l) : (\beta_m) ; (\gamma_n) ; x, y] = \sum_{r,s=0}^{\infty} \frac{p}{p} \prod_{j=1}^{p} (a_j)^{r_j} \prod_{j=1}^{q} (b_j)^{r_j} \prod_{j=1}^{k} (c_j)^{s_j} x^r y^s \quad (1.11)
\]

where, for convergence,

(i) \( p + q < l + m + 1, p + k < l + n + 1, |x| < \infty, |y| < \infty \), or

(ii) \( p + q = l + m + 1, p + k = l + n + 1, \) and

\[
\left\{ \begin{array}{ll}
|x|^{1/(p-l)} + |y|^{1/(p-l)} < 1, & \text{if } p > l, \\
\max(|x|, |y|) < 1, & \text{if } p \leq l.
\end{array} \right.
\]

Srivastava and Daoust [7] multivariable hypergeometric function is given as follows:

\[
P_p^{m_1; \ldots; m_r; z_q_1; \ldots; z_q_r} [(a_j : \alpha_j^{(r)} \ldots \alpha_j^{(l)} , \ldots ; (c_j^{(r)} , r_j^{(r)} , \ldots ; (d_j^{(r)} , \delta_j^{(r)} , \ldots ; x_1 , x_2 , \ldots , x_r) \\
= \sum_{n_1, n_2, \ldots, n_r = 0}^{\infty} \frac{p}{p} \prod_{j=1}^{p} (a_j)^{n_1 \delta_j^{(r)}} \prod_{j=1}^{q_1} (c_j^{(r)} , r_j^{(r)} , \ldots ; (d_j^{(r)} , \delta_j^{(r)} , \ldots ; x_1 , x_2 , \ldots , x_r) \\
(1.12)
\]

where the multiple hypergeometric series converges absolutely under the parametric variable constraints, and \((\lambda)_\nu\) denotes the well known Pochhammer symbol.

In our present investigation, we also need to recall the following Obhettinger's integral formula [5]:

\[
\int_0^\infty x^{\mu-1} (x + a + \sqrt{x^2 + 2ax})^{-\lambda} dx = 2\lambda a^{-\lambda} \left( \frac{a}{2} \right)^\mu \Gamma(2\mu) \Gamma(\lambda - \mu) \Gamma(1 + \lambda + \mu),
\]

provided \( 0 < \Re(\mu) < \Re(\lambda) \).

2. Main results

In this section, we establish two interesting integrals involving the product of Bessel function with Jacobi polynomial, which are expressed in terms of Kampé de Fériet and Srivastava and Daoust functions.

**Theorem 2.1.** The following integral formula (in terms of Kampé de Fériet) holds true: For \( \Re(\nu) > -1, 0 < \Re(\mu) < \Re(\lambda + \nu) \) and \( x > 0 \),

\[
\int_0^\infty x^{\mu-1} (x + a + \sqrt{x^2 + 2ax})^{-\lambda} J_\nu \left( \frac{y}{x + a + \sqrt{x^2 + 2ax}} \right) P_n^{(a,b)} \left( 1 - \frac{by}{x + a + \sqrt{x^2 + 2ax}} \right) dx
\]
\[ I = y^{\nu} 2^{-\nu} a^{\mu - \lambda - \nu} (1 + \alpha)_{n} \Gamma(2\mu) \Gamma(\lambda + \nu + 1) \Gamma(\lambda + \nu - \mu) \]
\[ \times \left[ \frac{n! \Gamma(v + 1) \Gamma(\lambda + v + 1) \Gamma(1 + \lambda + v + \mu)}{n! \Gamma(v + 1) \Gamma(\lambda + v) \Gamma(1 + \lambda + v + \mu)} \right] \]
\[ \times \int_{0}^{\infty} x^{\mu-1} \left( x + a + \sqrt{x^2 + 2ax} \right)^{-\lambda - v + 2m + k} \, dx. \]
Proof. Finally after a little simplification, summing up the above series with the help of (1.11), we arrive at the right-hand side of (1.1). This completes the proof. \( \square \)

**Theorem 2.2.** The following integral formula (in terms of Srivastava and Daoust function) holds true: For \( \Re(v) > -1, 0 < \Re(\mu) < \Re(\lambda + v) \) and \( x > 0 \),

\[
\int_{0}^{\infty} x^{\mu-1} (x + a + \sqrt{x^2 + 2ax})^{-\lambda} J_v \left( \frac{y}{x + a + \sqrt{x^2 + 2ax}} \right) P_{n}^{(a,\beta)} \left( 1 - \frac{by}{x + a + \sqrt{x^2 + 2ax}} \right) dx = y^\nu 2^{1-v-\mu} a^{\mu-\nu} \frac{\Gamma(2\mu)\Gamma(\lambda + v + 1)\Gamma(\lambda + v - \mu)}{\Gamma(\nu + 1)\Gamma(\lambda + v)\Gamma(1 + \lambda + v + \mu)}
\]

\[
\times P_{\frac{\alpha}{2} + 1}^{\frac{\nu}{2} + 1} \left( (\lambda + v + 1 : 2,3), (\lambda + v - \mu : 2,3), (1 + \alpha + \beta : 1,2),
\right)
\]

\[
(\lambda + v : 2,3), (1 + \lambda + v + \mu : 2,3), (v + 1 : 1,1), (1 + \alpha + \beta : 1,1),
\]

\[
(1 + \alpha : 1,1) ; \quad - \frac{y^2}{4a^2} ; \quad - \frac{by^3}{8a^3} ; \quad (1 : 1,1) ; \quad (1 + \alpha,1) ;
\]

(2.5)

**Proof.** In order to derive (2.5), we denote the left-hand side of (2.5) by \( I' \), expanding \( J_v \) and \( P_{n}^{(a,\beta)} \) in their series form and then using the lemma (see [4]):

\[
\sum_{n=0}^{\infty} \sum_{k=0}^{n} B(k, n) = \sum_{n=0}^{\infty} \sum_{k=0}^{n} B(k, n + k),
\]

we get

\[
I' = y^\nu 2^{-v} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-1)^{n+k} (1 + \alpha)_{n+k} (1 + \alpha + \beta)_{n+2k} (-b)^{k} y^{2n+3k}}{(n+k)! \Gamma(\nu+n+k+1) (1+\alpha+\beta)_{n+k} n! k!} \times \int_{0}^{\infty} x^{\mu-1} (x + a + \sqrt{x^2 + 2ax})^{-\lambda} (\nu+2n+3k) dx.
\]

(2.6)

On using (1.13) in the above expression and after a little simplifications, we arrive at

\[
I' = y^\nu 2^{1-v-\mu} a^{\mu-\nu} \frac{\Gamma(2\mu)\Gamma(\lambda + v + 1)\Gamma(\lambda + v - \mu)}{\Gamma(\nu + 1)\Gamma(\lambda + v)\Gamma(1 + \lambda + v + \mu)}
\]

\[
\times P_{\frac{\alpha}{2} + 1}^{\frac{\nu}{2} + 1} \left( (\lambda + v + 1 : 2,3), (\lambda + v - \mu : 2,3), (1 + \alpha + \beta : 1,2),
\right)
\]

\[
(\lambda + v : 2,3), (1 + \lambda + v + \mu : 2,3), (v + 1 : 1,1), (1 + \alpha + \beta : 1,1),
\]

\[
(1 + \alpha : 1,1) ; \quad - \frac{y^2}{4a^2} ; \quad - \frac{by^3}{8a^3} ; \quad (1 : 1,1) ; \quad (1 + \alpha,1) ;
\]
where \( P_{2n+3k}^{(n,0)}(1) \) is the ultraspherical polynomial (see [4], [7]).

This corollary can be established with the help of Theorem 2.1 by putting \( \beta = \alpha \).
Corollary 3.2. The following integral formula holds true under the same condition of Theorem 2.1:

\[
\int_0^\infty x^{\mu-1} (x + a + \sqrt{x^2 + 2ax})^{-\lambda} J_\nu \left( \frac{y}{x + a + \sqrt{x^2 + 2ax}} \right) C_n^l \left( 1 - \frac{by}{x + a + \sqrt{x^2 + 2ax}} \right) dx \\
= y^\nu 2^{1-\nu-\mu} a^{\mu-\lambda-\nu} \frac{(l)_n \Gamma(2\mu) \Gamma(\lambda + v + 1) \Gamma(\lambda + v - \mu)}{n! \Gamma(v + 1) \Gamma(\lambda + v) \Gamma(1 + \lambda + v + \mu)} \\
\times \frac{\Delta(2; \lambda + v + 1), \Delta(2; \lambda + v - \mu)}{\Delta(2; -n), \Delta(2; 2l + n)} \\
+ \frac{by}{2a} \frac{n(2l + n)(\lambda + v + 1)(\lambda + v - \mu)}{(\lambda + v)(1 + \lambda + v + \mu)(l + \frac{1}{2})} \\
\times \int_F^{4:0;4}_{4:1;3} \left[ \Delta(2; \lambda + v + 1), \Delta(2; \lambda + v - \mu + 1) \right] \\
\left[ \Delta(2; -n + 1), \Delta(2; 1 + 2l + n) \right] \\
\left[ \Delta(2; \lambda + v + 1), \Delta(2; \lambda + v + \mu) : v + 1 \right] \\
\left[ \Delta(2; l + \frac{1}{2}), \frac{3}{2} \right]
\right]
\]

where \( C_n^l(z) \) is the Gegenbauer polynomial (see [4], [7]).

The above corollary can be established with the help of Theorem 2.1 by putting \( \beta = \alpha = l - \frac{1}{2} \) and then using (1.7).

Corollary 3.3. The following integral formula holds true under the same condition of Theorem 2.1:

\[
\int_0^\infty x^{\mu-1} (x + a + \sqrt{x^2 + 2ax})^{-\lambda} J_\nu \left( \frac{y}{x + a + \sqrt{x^2 + 2ax}} \right) T_n \left( 1 - \frac{by}{x + a + \sqrt{x^2 + 2ax}} \right) dx \\
= y^\nu 2^{1-\nu-\mu} a^{\mu-\lambda-\nu} \frac{\Gamma(2\mu) \Gamma(\lambda + v + 1) \Gamma(\lambda + v - \mu)}{\Gamma(v + 1) \Gamma(\lambda + v) \Gamma(1 + \lambda + v + \mu)} \\
\times \frac{\Delta(2; \lambda + v + 1), \Delta(2; \lambda + v - \mu)}{\Delta(2; -n), \Delta(2; 2l + n)} \\
+ \frac{by}{a} \frac{n^2(\lambda + v + 1)(\lambda + v - \mu)}{(\lambda + v)(1 + \lambda + v + \mu)} \\
\times \int_F^{4:0;4}_{4:1;3} \left[ \Delta(2; \lambda + v + 1), \Delta(2; \lambda + v - \mu + 1) \right] \\
\left[ \Delta(2; -n + 1), \Delta(2; n + 1) \right] \\
\left[ \Delta(2; \lambda + v + 1), \Delta(2; 2 + \lambda + v + \mu) : v + 1 \right] \\
\left[ \Delta(2; \frac{3}{2}), \frac{3}{2} \right]
\right]
\]

where \( T_n(z) \) is the Tchebicheff polynomial of the first kind (see [4], [7]).
The above corollary can be established with the help of Theorem 2.1 by putting \( \beta = \alpha = -\frac{1}{2} \)
and then using (1.8).

**Corollary 3.4.** The following integral formula holds true under the same condition of Theorem 2.1:

\[
\int_0^\infty x^{\mu-1} (x + a + \sqrt{x^2 + 2ax})^{-\lambda} J_\nu \left( \frac{y}{x + a + \sqrt{x^2 + 2ax}} \right) U_n \left( 1 - \frac{by}{x + a + \sqrt{x^2 + 2ax}} \right) dx = y^\nu 2^{1 - \nu - \mu} a^{\mu - \lambda - \nu} \frac{(n + 1) \Gamma(2\mu) \Gamma(\lambda + \nu + 1) \Gamma(\lambda + \nu - \mu)}{\Gamma(\nu + 1) \Gamma(\lambda + \nu) \Gamma(1 + \lambda + \nu + \mu)} \\
\times F_4^{4; 0; 4}_{4; 1; 3} \left[ \begin{array}{c}
\Delta(2; \lambda + \nu + 1), \ D(2; \lambda + \nu - \mu) : - \frac{y^2}{4a^2}, \ \frac{b^2y^2}{4a^2} \\
\Delta(2; \lambda + \nu), \ D(2; 1 + \lambda + \nu + \mu) : v + 1, \ \Delta(2; \frac{3}{2}), \ \frac{1}{2}
\end{array} \right] \\
+ \frac{by}{3a} \frac{n(2 + n)(\lambda + \nu + 1)(\lambda + \nu - \mu)}{(\lambda + v)(1 + \lambda + \nu + \mu)} \\
\times F_4^{4; 0; 4}_{4; 1; 3} \left[ \begin{array}{c}
\Delta(2; \lambda + \nu + 2), \ D(2; \lambda + \nu - \mu + 1) : - \frac{y^2}{4a^2}, \ \frac{b^2y^2}{4a^2} \\
\Delta(2; \lambda + \nu + 1), \ D(2; 2 + \lambda + \nu + \mu) : v + 1, \ \Delta(2; \frac{3}{2})
\end{array} \right],
\]

(3.4)

where \( U_n(z) \) is the Tchebichef polynomial of the second kind (see [4], [7]).

The above corollary can be established with the help of Theorem 2.1 by putting \( \beta = \alpha = \frac{1}{2} \)
and then using (1.9).

**Corollary 3.5.** The following integral formula holds true under the same condition of Theorem 2.1:

\[
\int_0^\infty x^{\mu-1} (x + a + \sqrt{x^2 + 2ax})^{-\lambda} J_\nu \left( \frac{y}{x + a + \sqrt{x^2 + 2ax}} \right) P_n \left( 1 - \frac{by}{x + a + \sqrt{x^2 + 2ax}} \right) dx = y^\nu 2^{1 - \nu - \mu} a^{\mu - \lambda - \nu} \frac{\Gamma(2\mu) \Gamma(\lambda + \nu + 1) \Gamma(\lambda + \nu - \mu)}{\Gamma(\nu + 1) \Gamma(\lambda + \nu) \Gamma(1 + \lambda + \nu + \mu)} \\
\times F_4^{4; 0; 4}_{4; 1; 3} \left[ \begin{array}{c}
\Delta(2; \lambda + \nu + 1), \ D(2; \lambda + \nu - \mu) : - \frac{y^2}{4a^2}, \ \frac{b^2y^2}{4a^2} \\
\Delta(2; \lambda + \nu), \ D(2; 1 + \lambda + \nu + \mu) : v + 1, \ \Delta(2; 1), \ \frac{1}{2}
\end{array} \right] \\
+ \frac{by}{2a} \frac{n(1 + n)(\lambda + \nu + 1)(\lambda + \nu - \mu)}{(\lambda + \nu)(1 + \lambda + \nu + \mu)} \\
\times F_4^{4; 0; 4}_{4; 1; 3} \left[ \begin{array}{c}
\Delta(2; \lambda + \nu + 2), \ D(2; \lambda + \nu - \mu + 1) : - \frac{y^2}{4a^2}, \ \frac{b^2y^2}{4a^2} \\
\Delta(2; \lambda + \nu + 1), \ D(2; 2 + \lambda + \nu + \mu) : v + 1, \ \Delta(2; 2), \ \frac{3}{2}
\end{array} \right],
\]

(3.5)

where \( P_n(z) \) is the Legendre polynomial (see [4], [7]).
The above corollary can be established with the help of Theorem 2.1 by putting $\beta = \alpha = 0$ and then using (1.10).

**Corollary 3.6.** The following integral formula holds true: For $0 < \Re(\mu) < \Re(\lambda + \frac{1}{2})$ and $x > 0$,

$$
\int_0^\infty x^{\mu-1}(x + a + \sqrt{x^2 + 2ax})^{-(\lambda - \frac{1}{2})} \sin \left( \frac{y}{x + a + \sqrt{x^2 + 2ax}} \right) P_n^{(\alpha, \beta)} \left( 1 - \frac{by}{x + a + \sqrt{x^2 + 2ax}} \right) dx
$$

$$
= 2^{1-\mu} a^{\mu-\lambda-\frac{1}{2}} (1 + \alpha)_n \Gamma(2\mu) \Gamma(\lambda + \frac{1}{2}) \left( \lambda - \mu + \frac{1}{2} \right)
$$

$$
\times F^4_{4; 0; 4} \left[ \Delta(2; \lambda + \frac{3}{2}), \Delta(2; \lambda - \mu + \frac{1}{2}); _{-} ; \Delta(2; -n), \Delta(2; 1 + \alpha + \beta + n); \right]
$$

$$
\frac{by}{2a} \left( \lambda + \frac{1}{2} \right) \left( \lambda - \mu + \frac{1}{2} \right) \left( \lambda - \mu - \frac{1}{2} \right)
$$

$$
\times F^4_{4; 4; 1; 3} \left[ \Delta(2; \lambda + \frac{3}{2}), \Delta(2; \lambda - \mu + \frac{1}{2}); _{-} ; \Delta(2; -n + 1), \Delta(2; 1 + \alpha + \beta + n); \right]
$$

$$
\Delta(2; \lambda + \frac{3}{2}), \Delta(2; \lambda + \mu + \frac{3}{2}); _{-} ; \Delta(2; 2 + \alpha), \frac{3}{2};
$$

$$
\Delta(2; \lambda + \frac{3}{2}), \Delta(2; \lambda + \mu + \frac{3}{2}); _{-} ; \Delta(2; 2 + \alpha), \frac{3}{2};
$$

(3.6)

The above corollary can be established with the help of Theorem 2.1 by putting $\nu = \frac{1}{2}$ and then using (1.2).

**Corollary 3.7.** The following integral formula holds true: For $0 < \Re(\mu) < \Re(\lambda - \frac{1}{2})$ and $x > 0$,

$$
\int_0^\infty x^{\mu-1}(x + a + \sqrt{x^2 + 2ax})^{-(\lambda - \frac{1}{2})} \cos \left( \frac{y}{x + a + \sqrt{x^2 + 2ax}} \right) P_n^{(\alpha, \beta)} \left( 1 - \frac{by}{x + a + \sqrt{x^2 + 2ax}} \right) dx
$$

$$
= 2^{1-\mu} a^{\mu-\lambda-\frac{1}{2}} (1 + \alpha)_n \Gamma(2\mu) \Gamma(\lambda + \frac{1}{2}) \left( \lambda - \mu - \frac{1}{2} \right)
$$

$$
\times F^4_{4; 0; 4} \left[ \Delta(2; \lambda + \frac{1}{2}), \Delta(2; \lambda - \mu - \frac{1}{2}); _{-} ; \Delta(2; -n), \Delta(2; 1 + \alpha + \beta + n); \right]
$$

$$
\frac{by}{2a} \left( \lambda - \frac{1}{2} \right) \left( \lambda + \mu + \frac{1}{2} \right) \left( \lambda - \mu - \frac{1}{2} \right)
$$

$$
\times F^4_{4; 4; 1; 3} \left[ \Delta(2; \lambda + \frac{3}{2}), \Delta(2; \lambda - \mu - \frac{1}{2}); _{-} ; \Delta(2; -n + 1), \Delta(2; 2 + \alpha + \beta + n); \right]
$$

$$
\Delta(2; \lambda + \frac{3}{2}), \Delta(2; \lambda + \mu + \frac{3}{2}); _{-} ; \Delta(2; 2 + \alpha), \frac{3}{2};
$$

$$
\Delta(2; \lambda + \frac{3}{2}), \Delta(2; \lambda + \mu + \frac{3}{2}); _{-} ; \Delta(2; 2 + \alpha), \frac{3}{2};
$$

(3.7)

The above corollary can be established with the help of Theorem 2.1 by putting $\nu = -\frac{1}{2}$ and then using (1.3).
Remark 2. In a similar way, with the help of Theorem 2.1, we can find some other integral formulas involving the product of sine (cosine) function with ultraspherical polynomial, Gegenbauer polynomial, Tchebicheff polynomial and Legendre polynomial. Also, we can establish some other interesting special cases with the help of Theorem 2.2 by choosing some suitable values of $\alpha$, $\beta$ and $\nu$.

4. Connection between the Kampé de Fériet and Srivastava and Daoust functions

In this section, we give an interesting relation between Kampé de Fériet and Srivastava and Daoust functions as follow:

\[
\begin{align*}
F_4: 0; 4 &\quad 4: 1; 3 \\
\Delta(2; \lambda + v + 1), &\quad \Delta(2; \lambda + v - \mu) : \quad \Delta(2; -n), &\quad \Delta(2; 1 + \alpha + \beta + n); &\quad -\frac{y^2}{4a^2}, \quad \frac{b^2 y^2}{4a^2} \\
\Delta(2; \lambda + v), &\quad \Delta(2; 1 + \lambda + v + \mu) : \quad v + 1; &\quad \Delta(2; 1 + \alpha), \\
\end{align*}
\]

\[
+ \frac{b y}{2a} \frac{n(1 + \alpha + \beta + n)(\lambda + v + 1)(\lambda + v - \mu)}{\lambda + v + 1) (1 + \lambda + v + \mu)(1 + \alpha)} \Delta(2; \lambda + v + 2), \quad \Delta(2; \lambda + v - \mu + 1) : \quad \Delta(2; -n + 1), \quad \Delta(2; 2 + \alpha + \beta + n); \\
\Delta(2; 2 + \alpha), \quad \frac{3}{2}; \quad \frac{-y^2}{4a^2}, \quad \frac{b^2 y^2}{4a^2} = \frac{n!}{(1 + \alpha)^n}
\]

The above relation can be established by comparing (2.1) and (2.5).

5. Concluding remarks

In the present investigation, we have established some unified integral formulas, which are expressed in terms of Kampé de Fériet and Srivastava and Daoust functions. Further, we have derived a connection between Kampé de Fériet and Srivastava and Daoust functions from our main results. Also, it is noticed that, the Bessel function can be expressed in terms of the Fox $H$-function. Therefore, the results presented in this paper are easily converted in terms of the Fox $H$-function after some suitable parametric replacement.
References


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