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NEW f-DIVERGNCE AND JENSEN-OSTROWSKI'S TYPE INEQUALITIES

RAM NARESH SARASWAT AND AJAY TAK

Abstract. In this paper we derive new information inequalities of Jensen-Ostrowski type, by considering two Jensen-Ostrowski type inequalities, new f-divergence and Chi-divergences. The special cases of these information inequalities are established as applications of new f-divergence and its particular instances.

1. Introduction

In this paper we apply inequalities of [3] to obtain information inequalities for new f-divergence measure. Let suppose that a set Ω and the σ-finite measure μ are given. Take the set of all probability densities on μ to be

$$P = \left(p \mid p : \Omega \to \Re,\ p(t) \geq 0,\ \int_{\Omega} p(t) d\mu(t) = 1 \right). \tag{1.1}$$

In this text we use the following definitions of divergence measures which are the particular instances of new f-divergence:

- Kullback-Leibler divergence measure [12]:

$$S_{KL}(\mu_1, \mu_2) = \int_{\Omega} p(t) \ln\left[\frac{p(t)}{q(t)}\right] d\mu(t), \tag{1.2}$$

$$S_{KL}(\mu_2, \mu_1) = \int_{\Omega} q(t) \ln\left[\frac{q(t)}{p(t)}\right] d\mu(t). \tag{1.3}$$

- Relative Jensen-Shannon divergence measure [13]:

$$S_F(\mu_2, \mu_1) = \int_{\Omega} q(t) \log\left(\frac{2q(t)}{p(t) + q(t)}\right) d\mu(t). \tag{1.4}$$

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Corresponding author: Ram Naresh Saraswat.

- Relative arithmetic-geometric divergence measure [14]:

$$S_G(\mu_2,\mu_1)=\int_\Omega\left(\frac{p(t)+q(t)}{2}\right)\log\left(\frac{p(t)+q(t)}{2q(t)}\right)d\mu(t). \tag{1.5}$$

- Triangular discrimination [6]:

$$S_f(\mu_1,\mu_2)=\int_\Omega\frac{(p(t)-q(t))^2}{2(p(t)+q(t)}d\mu(t)=\frac{1}{2}S_\Delta(\mu_1,\mu_2). \tag{1.6}$$

- Chi-divergences [13] and [14]:

$$S_f(\mu_1,\mu_2)=\frac{1}{4}\int_\Omega\left(\frac{p(t)-q(t)}{q(t)}\right)^2q(t)d\mu(t)=\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2), \tag{1.7}$$

$$S_f(\mu_1,\mu_2)=\frac{1}{2^k}\int_\Omega\left(\frac{p(t)-q(t)(2\lambda-1)}{q(t)}\right)^kq(t)d\mu(t)=\frac{1}{2^k}S_{\chi^2,2\lambda-1}(\mu_1,\mu_2). \tag{1.8}$$

The New f-divergence of the probability distributions μ_1 and μ_2 is defined as follows

$$S_f(\mu_1,\mu_2)=\int q(t)f\left(\frac{p(t)+q(t)}{2q(t)}\right)d\mu(t). \tag{1.9}$$

Where denote the density (Radon-Nikodym-derivative) of μ_i $(i=1,2)$ with respect to μ by $p(t)=\frac{d\mu_1(t)}{d\mu(t)}$ and $q(t)=\frac{d\mu_2(t)}{d\mu(t)}$. Define the convex functions $f:(0,\infty)\to\Re_+$, $f(1)=0$, appropriately for obtaining various divergences. The basic properties of New f-divergence are available in [10] and [11].

2. Generalized Jensen-Ostrowski type inequalities

The following Jensen-Ostrowski inequality is considered from [3] for functions with bounded second derivatives.

Theorem 2.1. *Let $f:I\to C$ be a differentiable function on $\dot{I}$, $f':[a,b]\subset\dot{I}\to C$ is absolutely continuous on $[a,b]$ and $\zeta\in[a,b]$ For some $\gamma,\Gamma\in C$, $\gamma\neq\Gamma$, assume that $f''\in\bar{U}_{[a,b]}(\gamma,\Gamma)=\bar{\Delta}_{[a,b]}(\gamma,\Gamma)$. If $g:\Omega\to[a,b]$ is Lebesgue μ-measurable on Ω such that fog, g, $(g-\zeta)^2\in L(\Omega,\mu)$, with $\int_\Omega d\mu=1$ then*

$$\left|\int_\Omega(fog)d\mu-f(\zeta)-\left(\int_\Omega gd\mu-\zeta\right)-f'(\zeta)-\frac{\gamma+\Gamma}{4}\int_\Omega(g-\zeta)^2d\mu\right|$$
$$\leq\frac{1}{4}|\Gamma-\gamma|\left[\sigma^2(g)+\left(\int_\Omega gd\mu-\zeta\right)^2\right]. \tag{2.1}$$

A generalized version of the Ostrowski inequality [5] is considered form [3]. It is inequality as well as bounds for the discrepancy in Jensens integral inequality. The theorem is following:

Theorem 2.2. *Let $f : I \to C$ be a differentiable function on I, $f' : [a,b] \subset \dot{I} \to C$ is absolutely continuous on $[a,b]$ and $\zeta \in [a,b]$ If $g : \Omega \to [a,b]$ is Lebesgue μ-measurable on Ω such that $fog,\ g,\ (g-\zeta)^2 \in L(\Omega,\mu)$, with $\int_\Omega d\mu = 1$ then we have the following Ostrowski type inequality:*

$$\begin{aligned}\Big|\int_\Omega (fog)d\mu - f(\zeta) - \Big(\int_\Omega g d\mu - \zeta\Big) - f'(\zeta)\Big| \\ \leq \frac{1}{2}\|f''\|_{[a,b],\infty}\Big[\sigma^2(g) + \Big(\int_\Omega g d\mu - \zeta\Big)^2\Big].\end{aligned} \tag{2.2}$$

We also have the following Jensen type inequality:

$$\Big|\int_\Omega (fog)d\mu - f\Big(\int_\Omega g d\mu\Big)\Big| \leq \frac{1}{2}\|f''\|_{[a,b],\infty}\sigma^2(g) \tag{2.3}$$

which is the best inequality one can get from (2.2).

3. Main results

In this section, we present our main results on the Jensen-Ostrowski type inequalities by using chi- divergences (1.7)-(1.8) and new f-divergence (1.9).

Proposition 3.1. *Let $f : (0,\infty) \to \Re$ be a differentiable convex function with the property that $f(1) = 0$. Assume that $\mu_1, \mu_2 \in \Omega$ and there exists constants $0 < r < 1 < R < \infty$ such that*

$$r \leq \frac{p(t)+q(t)}{2q(t)} \leq R \tag{3.1}$$

for $\mu - a.e. t \in \Omega$. If $\zeta \in [r,R]$ and f' is absolutely continuous on $[r,R]$, then we have the inequalities

$$\big|S_f(\mu_1,\mu_2) - f(\zeta) - (1-\zeta)f'(\zeta)\big| \leq \frac{1}{2}\|f''\|_{[r,R],\infty}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + (\zeta-1)^2\right]. \tag{3.2}$$

In particular, by choosing $\zeta = (r+R)/2$, we have

$$\begin{aligned}\Big|S_f(\mu_1,\mu_2) - f\Big(\frac{r+R}{2}\Big) - \Big(1 - \frac{r+R}{2}\Big)f'\Big(\frac{r+R}{2}\Big)\Big| \\ \leq \frac{1}{2}\|f''\|_{[r,R],\infty}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + \Big(\frac{r+R}{2} - 1\Big)^2\right]\end{aligned} \tag{3.3}$$

and when $\zeta = 1$, we have

$$|S_f(\mu_1,\mu_2)| \leq \frac{1}{2}\|f''\|_{[r,R],\infty}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{3.4}$$

Proof. We choose $g(t) = \frac{p(t)+q(t)}{2q(t)}$, noting that $\int_\Omega g d\mu(t) = 1$, in inequality (2.2), we have

$$\Big|\int_\Omega f\Big(\frac{p(t)+q(t)}{2q(t)}\Big)q(t)d\mu(t) - f(\zeta) - \Big(\int_\Omega g(t)d\mu - \zeta\Big)f'(\zeta)\Big|$$

$$
\begin{aligned}
&= |S_f(\mu_1,\mu_2) - f(\zeta) - (1-\zeta)f'(\zeta)| \\
&= \frac{1}{2}\|f''\|_{[r,R],\infty}\left[\int_\Omega\left(\frac{p(t)+q(t)}{2q(t)} - \int_\Omega g(t)d\mu\right)^2 q(t)d\mu(t) + \left(\int_\Omega g(t)d\mu - \zeta\right)^2\right] \\
&= \frac{1}{2}\|f''\|_{[r,R],\infty}\left[\int_\Omega\left(\frac{p(t)+q(t)}{2q(t)} - 1\right)^2 q(t)d\mu(t) + (\zeta-1)^2\right] \\
&= \frac{1}{2}\|f''\|_{[r,R],\infty}\left[\int_\Omega\left(\frac{p(t)-q(t)}{2q(t)}\right)^2 q(t)d\mu(t) + (\zeta-1)^2\right] \\
&= \frac{1}{2}\|f''\|_{[r,R],\infty}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + (\zeta-1)^2\right]
\end{aligned}
$$

and this completes the proof. □

Proposition 3.2. *Under the assumptions of Proposition 3.1, if f' is convex or $f''_\pm$ exists, then we have*

$$
\begin{aligned}
&\left|S_f(\mu_1,\mu_2) - f(\zeta) - (1-\zeta)f'(\zeta) + \frac{f''_+(r)+f''_-(R)}{4}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + (\zeta-1)^2\right]\right| \\
&\qquad \le \frac{1}{4}|f''_-(R) - f''_+(r)|\left[\frac{1}{4}S_{\chi^2,2\zeta-1}(\mu_1,\mu_2)\right] \qquad (3.5)
\end{aligned}
$$

for $\zeta \in [r,R]$. Some particular cases of interest are obtained by setting $\zeta = (r+R)/2$ and $\zeta = 1$.

Proof. When f' is convex, we set $\gamma = f''_+(r)$ and $\Gamma = f''_-(R)$ (cf. Remark 2 in [3]). For the case where $f''_\pm$ exists, we set γ and Γ appropriately to the values of $f''_+(r)$ and $f''_-(R)$, with $\gamma \le \Gamma$. Utilizing (2.1) for $g(t) = \frac{p(t)+q(t)}{2q(t)}$, and the measure $\int_\Omega q(t)d\mu = 1$, we have

$$
\begin{aligned}
&\left|\int_\Omega f\left(\frac{p(t)+q(t)}{2q(t)}\right)q(t)d\mu - f(\zeta) - \left(\int_\Omega q(t)d\mu - \zeta\right)f'(\zeta)\right. \\
&\qquad \left. + \frac{f''_+(r)+f''_-(R)}{4}\int_\Omega\left(\frac{p(t)+q(t)}{2q(t)} - \zeta\right)^2 q(t)d\mu\right| \\
&\qquad \left|S_f(\mu_1,\mu_2) - f(\zeta) - (1-\zeta)f'(\zeta) + \frac{f''_+(r)+f''_-(R)}{4}\left[\frac{1}{4}S_{\chi^2,2\zeta-1}(\mu_1,\mu_2)\right]\right| \\
&\quad \le \frac{1}{4}|f''_-(R) - f''_+(r)|\left[\int_\Omega\left(\frac{p(t)+q(t)}{2q(t)} - 1\right)^2 q(t)d\mu + (\zeta-1)^2\right] \\
&\quad = \frac{1}{4}|f''_-(R) - f''_+(r)|\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + (\zeta-1)^2\right]
\end{aligned}
$$

and this completes the proof. □

4. Special cases:

Example 4.1. If we consider the convex function $f : (0,\infty) \to \Re$, $f(t) = t\log(t)$, then we get (1.5).

We have $f'(t) = \log(t) + 1$, and $f''(t) = 1/t$. By Proposition 3.1, we have the following inequalities:

$$|S_G(\mu_2,\mu_1) - \zeta\log(\zeta) - (1-\zeta)(\log(\zeta)+1) = |S_G(\mu_2,\mu_1) - 1 + \zeta - \log(\zeta)|$$
$$\le \frac{1}{2}\Big[\sup_{t\in[r,R]} \frac{1}{t}\Big]\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + (\zeta-1)^2\right]$$
$$= \frac{1}{2r}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + (\zeta-1)^2\right]$$

for all $\zeta \in [r,R]$, and when $\zeta = 1$,

$$0 \le S_G(\mu_2,\mu_1) \le \frac{1}{2r}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{4.1}$$

Furthermore, by Proposition 3.2, we have the inequalities:

$$\left|S_G(\mu_2,\mu_1) - \log(\zeta) - 1 + \zeta + \frac{r+R}{4rR}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + (\zeta-1)^2\right]\right| \le \frac{R-r}{4rR}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)\right]$$

for $\zeta \in [r,R]$, and when $\zeta = 1$,

$$\left|S_G(\mu_2,\mu_1) + \frac{R+r}{4rR}S_{\chi^2}(\mu_1,\mu_2)\right| \le \frac{R-r}{4rR}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{4.2}$$

Example 4.2. If we consider the convex function $f:(0,\infty)\to\mathfrak{R}$, $f(t) = -\log(t)$, then we get (1.4).

We have $f'(t) = 1/t$, and $f''(t) = -1/t^2$, and we note that

$$\frac{1}{4}\int_\Omega q(t)\left(\left(\frac{p(t)}{q(t)}\right)^2 - 1\right)d\mu(t) = \frac{1}{4}S_{\chi^2}(\mu_1,\mu_2).$$

By Proposition 3.1, we have the following inequalities:

$$|S_F(\mu_2,\mu_1) + \log(\zeta) + \frac{1}{\zeta} - 1| \le \frac{1}{2}\Big[\sup_{t\in[r,R]} \frac{1}{t^2}\Big]\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + (\zeta-1)^2\right]$$
$$= \frac{1}{2r^2}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + (\zeta-1)^2\right]$$

for all $\zeta \in [r,R]$, and when $\zeta = 1$,

$$0 \le S_F(\mu_2,\mu_1) \le \frac{1}{2r^2}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{4.3}$$

Furthermore, by Proposition 3.2, we have the inequalities:

$$\left|S_F(\mu_2,\mu_1) + \log(\zeta) + \frac{1}{\zeta} - 1 + \frac{r^2+R^2}{4r^2R^2}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2) + (\zeta-1)^2\right]\right| \le \frac{R^2-r^2}{4r^2R^2}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)\right]$$

for $\zeta \in [r,R]$, and when $\zeta = 1$,

$$\left|S_F(\mu_2,\mu_1) + \frac{R^2+r^2}{4r^2R^2}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)\right| \le \frac{R^2-r^2}{4rR}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{4.4}$$

Example 4.3. If we consider the convex function $f:(0,\infty)\to\Re$, $f(t)=-\frac{(t-1)^2}{t}$, then we get (1.6).

We have $f'(t)=1-1/t^2$, and $f''(t)=2/t^2$, and $f''(t)=2/t^3$. By Proposition 3.1, we have the following inequalities:

$$\left|\frac{1}{2}S_\Delta(\mu_1,\mu_2)-\frac{(\zeta-1)^2}{\zeta}-(1-\zeta)\left(1-\frac{1}{\zeta^2}\right)\right| = \left|\frac{1}{2}S_\Delta(\mu_1,\mu_2)-\frac{(\zeta-1)^2}{\zeta}-(1-\zeta)\frac{(\zeta^2-1)}{\zeta^2}\right|$$
$$=\frac{1}{2}\left[\sup_{t\in[r,R]}\frac{2}{t^3}\right]\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)+(\zeta-1)^2\right]$$
$$=\frac{1}{r^3}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)+(\zeta-1)^2\right]$$

for all $\zeta\in[r,R]$, and when $\zeta=1$,

$$0\le\frac{1}{2}S_\Delta(\mu_1,\mu_2)\le\frac{1}{r^3}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{4.5}$$

Furthermore, by Proposition 3.2, we have the inequalities:

$$\left|\frac{1}{2}S_\Delta(\mu_1,\mu_2)-\frac{(\zeta-1)^2}{\zeta}-(1-\zeta)\left(1-\frac{1}{\zeta^2}\right)+\frac{r^3+R^3}{4r^3R^3}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)+(\zeta-1)^2\right]\right|$$
$$\le\frac{r^3-R^3}{2r^3R^3}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)\right]$$

for $\zeta\in[r,R]$, and when $\zeta=1$,

$$\left|\frac{1}{2}S_\Delta(\mu_1,\mu_2)+\frac{R^3+r^3}{2r^3R^3}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)\right|\le\frac{r^3-R^3}{2r^3R^3}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{4.6}$$

Example 4.4. If we consider the convex function $f:(0.5,\infty)\to\Re$, $f(t)=-\log(2t-1)$, then we get (1.3). We have $f'(t)=\frac{-2}{2t-1}$, and $f''(t)=\frac{4}{(2t-1)^2}$, By Proposition 3.1, we have the following inequalities:

$$\left|S_{KL}(\mu_2,\mu_1)-\log(2\zeta-1)-(1-\zeta)\frac{2}{(2t-1)}\right|\le\frac{1}{2}\left[\sup_{t\in[r,R]}\frac{4}{(2t-1)^2}\right]\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)+(\zeta-1)^2\right]$$
$$=\frac{2}{(2r-1)^2}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)+(\zeta-1)^2\right]$$

for all $\zeta\in[r,R]$, and when $\zeta=1$,

$$0\le S_{KL}(\mu_2,\mu_1)\le\frac{2}{(2r-1)^2}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{4.7}$$

Furthermore, by Proposition 3.2, we have the inequalities:

$$\left|S_F(\mu_2,\mu_1)+\log(2\zeta-1)+\frac{2(1-\zeta)}{(2\zeta-1)}+\frac{4(r^2+R^2)-4(r+R)+2}{(2r-1)^2(2R-1)^2}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)+(\zeta-1)^2\right]\right|$$

$$\le \frac{4(R^2-r^2)-4(R-r)}{(2r-1)^2(2R-1)^2}\left[\frac{1}{4}S_{\chi^2,2\zeta-1}(\mu_1,\mu_2)\right]$$

for $\zeta \in [r,R]$, and when $\zeta = 1$,

$$\left|S_F(\mu_1,\mu_2)+\frac{4(r^2+R^2)-4(r+R)+2}{(2r-1)^2(2R-1)^2}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)\right| \le \frac{4(R^2-r^2)-4(R-r)}{(2r-1)^2(2R-1)^2}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{4.8}$$

Example 4.5. If we consider the convex function $f:(0.5,\infty)\to\Re$, $f(t)=(2t-1)\log(2t-1)$, then we get (1.2). We have $f'(t)=2\log(2t-1)+2$ and $f''(t)=\frac{4}{(2t-1)}$, By Proposition 3.1, we have the following inequalities:

$$\begin{aligned}&\left|S_{KL}(\mu_1,\mu_2)-(2\zeta-1)\log(2\zeta-1)-2(1-\zeta)(\log(2\zeta-1)+1)\right|\\ &=\left|S_{KL}(\mu_1,\mu_2)-\log(2\zeta-1)+2(\zeta-1)\right|\\ &\le \frac{1}{2}\left[\sup_{t\in[r,R]}\frac{4}{(2t-1)}\right]\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)+(\zeta-1)^2\right]\\ &=\frac{2}{(2r-1)}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)+(\zeta-1)^2\right]\end{aligned}$$

for all $\zeta\in[r,R]$, and when $\zeta=1$,

$$0\le S_{KL}(\mu_1,\mu_2)\le\frac{2}{(2r-1)}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{4.9}$$

Furthermore, by Proposition 3.2, we have the inequalities:

$$\begin{aligned}&\left|S_{KL}(\mu_1,\mu_2)-\log(2\zeta-1)+(2\zeta-1)+\frac{2(r+R-1)}{(2r-1)(2R-1)}\left[\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)+(\zeta-1)^2\right]\right|\\ &\le\frac{2(R-r)}{(2r-1)(2R-1)}\left[\frac{1}{4}S_{\chi^2,2\zeta-1}(\mu_1,\mu_2)\right]\end{aligned}$$

for $\zeta\in[r,R]$, and when $\zeta=1$,

$$\left|S_{KL}(\mu_1,\mu_2)+\frac{2(r+R-1)}{(2r-1)(2R-1)}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2)\right| \le\frac{2(R-r)}{(2r-1)(2R-1)}\frac{1}{4}S_{\chi^2}(\mu_1,\mu_2). \tag{4.10}$$

5. Conclusions

In this paper, we have been derived new information inequalities of Jensen-Ostrowski type, by considering two above inequalities of Section 2, new f-divergence and Chi-divergences. Particular cases of these inequalities have been also established in terms of divergences like as Kullback-Leibler divergence, Relative Jensen-Shannon divergence, Relative arithmetic-geometric divergence and Triangular discrimination.

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Department of Mathematics and Statistics, School of Basic Science, Manipal University, Jaipur-303007, India.

E-mail: sarswatrn@gmail.com

Department of Mathematics and Statistics, School of Basic Science, Manipal University, Jaipur-303007, India.

E-mail: ajaytak86@gmail.com