INTUITIONISTIC FUZZY SEMI-GENERALIZED IRRESOLUTE MAPPINGS

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Abstract. The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy semi-generalized continuous mappings and intuitionistic fuzzy semi-generalized irresolute mappings in intuitionistic fuzzy topological space.

1. Introduction


Continuing the work done in the paper [12], we define the notion of intuitionistic fuzzy semi-generalized continuous mappings and intuitionistic fuzzy semi-generalized irresolute mappings. We discuss characterizations of intuitionistic fuzzy semi-generalized continuous mappings and irresolute mappings. We also established their properties and relationships with other classes of early defined forms of intuitionistic continuous mappings.

2. Preliminaries

Definition 2.1 ([1]). An intuitionistic fuzzy set (IFS, for short) $A$ in $X$ is an object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\}$$

where the functions $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of the membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set $A$, respectively, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

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Definition 2.2 ([11]). Let $A$ and $B$ be IFS's of the forms $A = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\}$ and $B = \{(x, \mu_B(x), \gamma_B(x)) \mid x \in X\}$. Then,

(a) $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \leq B$ and $B \leq A$,
(c) $\tilde{A} = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\}$,
(d) $A \cap B = \{(x, \mu_A(x) \land \mu_B(x), \gamma_A(x) \lor \gamma_B(x)) \mid x \in X\}$,
(e) $A \cup B = \{(x, \mu_A(x) \lor \mu_B(x), \gamma_A(x) \land \gamma_B(x)) \mid x \in X\}$,
(f) $0_- = \{(x, 0, 1) \mid x \in X\}$ and $1_- = \{(x, 1, 0) \mid x \in X\}$,
(g) $\overline{A} = A$, $\overline{0_-} = 1_-$. 

Definition 2.3 ([11]). Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of $X$ given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta), & \text{if } x = p, \\ (0, 1), & \text{otherwise.} \end{cases}$$

Definition 2.4 ([4]). An intuitionistic fuzzy topology (IFT for short) on $X$ is a family $\tau$ of IFS's in $X$ satisfying the following axioms:

(i) $0_- , 1_- \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFOS for short) in $X$. The complement $\tilde{A}$ of an IFS $A$ in IFTS($X, \tau$) is called an intuitionistic fuzzy closed set (IFCS for short) in $X$.

Definition 2.5 ([11]). Let $f$ be a mapping from a set $X$ to a set $Y$. If

$$B = \{(y, \mu_B(y), \gamma_B(y)) \mid y \in Y\}$$

is an IFS in $Y$, then the preimage of $B$ under $f$, denoted by $f^{-1}(B)$, is the IFS in $X$ defined by

$$f^{-1}(B) = \{(x, f^{-1}(\mu_B(x)), f^{-1}(\gamma_B(x))) \mid x \in X\}.$$
\[ \text{cl}(A) = \cap \{ K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K \}. \]

Note that, for any IFS \( A \) in \((X, \tau)\), we have

\[ \text{cl}(\bar{A}) = \overline{\text{int}(A)} \quad \text{and} \quad \text{int}(\bar{A}) = \overline{\text{cl}(A)}. \]

**Definition 2.7.** An IFS \( A = \{ (x, \mu_A(x), \gamma_A(x)) \mid x \in X \} \) in an IFTS \((X, \tau)\) is called

(i) intuitionistic fuzzy semi open set (IFSOS) if \( A \subseteq \text{cl}(\text{int}(A)). \) [6]

(ii) intuitionistic fuzzy \( \alpha \)-open set (IF\( \alpha \)OS) if \( A \subseteq \text{int}(\text{cl}(\text{int}(A))). \) [6]

(iii) intuitionistic fuzzy preopen set (IFPOS) if \( A \subseteq \text{int}(\text{cl}(A)). \) [6]

(iv) intuitionistic fuzzy regular open set (IFROS) if \( \text{int}(\text{cl}(A)) = A. \) [6]

(v) intuitionistic fuzzy semi-pre open set (IFSPOS) if there exists \( B \in \text{IFPO}(X) \) such that \( B \subseteq A \subseteq \text{cl}(B). \) [13]

An IFS \( A \) is called intuitionistic fuzzy semi closed set, intuitionistic fuzzy \( \alpha \)-closed set, intuitionistic fuzzy preclosed set, intuitionistic fuzzy regular closed set and intuitionistic fuzzy semi-preclosed set, respectively (IFSCS, IF\( \alpha \)CS, IFPCS, IFRCS and IFSPCS resp), if the complement \( \bar{A} \) is an IFSOS, IF\( \alpha \)OS, IFPOS, IFROS and IFSPOS respectively.

The family of all intuitionistic fuzzy semi open (resp. intuitionistic fuzzy \( \alpha \)-open, intuitionistic fuzzy preopen, intuitionistic fuzzy regular open and intuitionistic fuzzy semi-preopen) sets of an IFTS \((X, \tau)\) is denoted by IFSO\((X)\) (resp IF\( \alpha \)(\(X\)), IFPO\((X\)), IFRO\((X\)) and IFSPO\((X\)).

**Definition 2.8** ([12]). An intuitionistic fuzzy set \( A \) of an intuitionistic fuzzy topological space \((X, \tau)\) is called an intuitionistic fuzzy semi-generalized closed set (IFSGCS) if \( \text{scl}(A) \subseteq U, \) whenever \( A \subseteq U \) and \( U \) is intuitionistic fuzzy semi-open set.

The complement \( \bar{A} \) of intuitionistic fuzzy semi-generalized closed set \( A \) is called intuitionistic fuzzy semi-generalized open set (IFSGOS).

**Definition 2.9** ([12]). An intuitionistic fuzzy topological space \((X, \tau)\) is said to be intuitionistic fuzzy semi-T\(_{1/2}\) space, if every intuitionistic fuzzy sg-closed set in \( X \) is intuitionistic fuzzy semi-closed in \( X \).

**Definition 2.10** ([8]). Let \( p(\alpha, \beta) \) be an IFP of an IFTS\((X, \tau)\). An IFS \( A \) of \( X \) is called an intuitionistic fuzzy neighbourhood (IFN) of \( p(\alpha, \beta) \), if there exists an IFOS \( B \) in \( X \) such that \( p(\alpha, \beta) \in B \subseteq A. \)

**Definition 2.11.** Let \( f : X \rightarrow Y \) be a mapping from an IFTS \( X \) into an IFTS \( Y \). The mapping \( f \) is called

(i) **intuitionistic fuzzy continuous**, if \( f^{-1}(B) \) is an IFOS in \( X \), for each IFOS \( B \) in \( Y \). [6]
(ii) intuitionistic fuzzy semi-continuous, if $f^{-1}(B)$ is an IFOS in $X$, for each IFOS $B$ in $Y$.\cite{[6]}

(iii) intuitionistic fuzzy pre-continuous, if $f^{-1}(B)$ is an IFPOS in $X$, for each IFOS $B$ in $Y$.\cite{[6]}

(iv) intuitionistic fuzzy $\alpha$-continuous, if $f^{-1}(B)$ is an IF$\alpha$OS in $X$, for each IFOS $B$ in $Y$.\cite{[6]}

(v) intuitionistic fuzzy semi-pre continuous, if $f^{-1}(B)$ is an IF$\alpha$OS in $X$, for each IFOS $B$ in $Y$.\cite{[9]}

(vi) intuitionistic fuzzy completely continuous, if $f^{-1}(B)$ is an IFROS in $X$, for each IFOS $B$ in $Y$.\cite{[15]}

Lemma 2.12 (\cite{[15]}). Let $g : X \to X \times Y$ be the graph of a function $f : X \to Y$. If $A$ is an IFS of $X$ and $B$ is an IFS of $Y$, then $g^{-1}(A \times B)(x) = (A \cap f^{-1}(B))(x)$.

3. Intuitionistic fuzzy semi-generalized continuous mappings

In this section we introduce intuitionistic fuzzy semi-generalized continuous mappings and studied some of the properties regarding it.

Definition 3.1. Let $A$ be an IFS in an IFTS $(X, \tau)$. Then the intuitionistic fuzzy semi-generalized interior and intuitionistic fuzzy semi-generalized closure of $A$ are defined as follows.

$$
\text{sgint}(A) = \bigcup \{ G \mid G \text{ is an IFSGOS in } X \text{ and } G \subseteq A \},
$$

$$
\text{sgcl}(A) = \bigcap \{ K \mid K \text{ is an IFSGCS in } X \text{ and } A \subseteq K \}.
$$

Example 3.2. Let $X = \{a, b\}$.

Let $A = \left\{ x, \left( \begin{array}{cc} a & b \\ 0.2 & 0.3 \end{array} \right), \left( \begin{array}{cc} a & b \\ 0.7 & 0.7 \end{array} \right) \right\}$

Let $B = \left\{ x, \left( \begin{array}{cc} a & b \\ 0.4 & 0.7 \end{array} \right), \left( \begin{array}{cc} a & b \\ 0.6 & 0.1 \end{array} \right) \right\}$.

Then $\tau = \{0_-, 1_-, A, B\}$ is an IFTS on $X$.

Then $\text{sgint}(B) = A \cup 0_- = A$ and $\text{sgcl}(B) = 1_-$.\cite{[9]}

Proposition 3.3. If $A$ be an IFS in $X$, then $A \leq \text{sgcl}(A) \leq \text{scl}(A) \leq \text{cl}(A)$.

Proof. The result follows from Definition.\cite{[9]}

Theorem 3.4. If $A$ is an IFSGCS in $X$, then $\text{sgcl}(A) = A$.

Proof. Since $A$ is an IFSGCS, $\text{sgcl}(A)$ is the smallest IFSGCS which contains $A$, which is nothing but $A$. Hence $\text{sgcl}(A) = A$.\cite{[9]}

Theorem 3.5. If $A$ is an IFSGOS in $X$, then $\text{sgint}(A) = A$.

Proof. Similar to the above theorem. \qed

Definition 3.6. Let $(X, \tau)$ and $(Y, \kappa)$ be IFTs. A mapping $f : X \to Y$ is called intuitionistic fuzzy semi-generalized continuous (intuitionistic fuzzy sg-continuous), if $f^{-1}(B)$ is an IFSGCS in $X$ for every IFCS $B$ in $Y$.

Theorem 3.7. Every intuitionistic fuzzy continuous mapping is an intuitionistic fuzzy sg-continuous mapping.

Proof. Let $B$ be an IFCS in $Y$. Then by our assumption, $f^{-1}(B)$ is an IFCS in $X$. In [12], it has been proved that every intuitionistic fuzzy closed set is an intuitionistic fuzzy sg-closed set in $X$. Thus $f^{-1}(B)$ is an IFSGCS in $X$. Hence $f$ is an intuitionistic fuzzy sg-continuous mapping. \qed

The following example shows that the converse of above theorem is not true in general.

Example 3.8. Let $X = \{a, b\}$, $Y = \{c, d\}$.

$$A = \left< x, \left( \begin{array}{cc} a & b \\ 0.3 & 0.4 \\ 0.7 & 0.6 \end{array} \right) \right>$$

$$B = \left< x, \left( \begin{array}{cc} c & d \\ 0.7 & 0.8 \\ 0.3 & 0.2 \end{array} \right) \right>.$$

Then $\tau = \{0_, 1_, A\}$ and $\kappa = \{0_, 1_, B\}$ are IFTS on $X$ and $Y$ respectively. Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = c$, $f(b) = d$. Clearly $f$ is an intuitionistic fuzzy sg-continuous map.

Now we have $f^{-1}(B) = \left< x, \left( \begin{array}{cc} a & b \\ 0.7 & 0.8 \\ 0.3 & 0.2 \end{array} \right) \right>$. $f^{-1}(B) \notin \tau$, which shows that $f$ is not an intuitionistic fuzzy continuous map.

Theorem 3.9. Every intuitionistic fuzzy $\alpha$-continuous mapping is an intuitionistic fuzzy sg-continuous mapping.

Proof. Let $B$ be an IFCS in $Y$. Since $f$ is an intuitionistic fuzzy $\alpha$-continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy $\alpha$-closed set in $X$. In [12], it has been proved that every IF$\alpha$CS is an intuitionistic fuzzy sg-closed set in $X$. Thus $f^{-1}(B)$ is an IFSGCS in $X$. Hence $f$ is an intuitionistic fuzzy sg-continuous mapping. \qed

The following example shows that the converse of the above theorem is not true in general.
Example 3.10. Let $X = \{a, b\}$, $Y = \{u, v\}$.

Let $A = \left\langle x, \begin{pmatrix} a & b \\ 0.7 & 0.5 \end{pmatrix}, \begin{pmatrix} a & b \\ 0.3 & 0.5 \end{pmatrix} \right\rangle$

$B = \left\langle y, \begin{pmatrix} u & v \\ 0.25 & 0.3 \end{pmatrix}, \begin{pmatrix} u & v \\ 0.2 & 0.2 \end{pmatrix} \right\rangle$.

Then $\tau = \{0_-, 1_-, A\}$ and $\kappa = \{0_-, 1_-, B\}$ are IFTS on $X$ and $Y$ respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \kappa)$ by $f(a) = u$, $f(b) = v$. Clearly $f$ is intuitionistic fuzzy $sg$-continuous map.

Now we have $f^{-1}(B) = \left\langle x, \begin{pmatrix} a & b \\ 0.25 & 0.3 \end{pmatrix}, \begin{pmatrix} a & b \\ 0.2 & 0.2 \end{pmatrix} \right\rangle$.

$\text{cl}(f^{-1}(B)) = 1_-$, $\text{int}(\text{cl}(f^{-1}(B))) = \text{int}(1_-) = 1_-$

$\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) = \text{cl}(1_-) = 1_-$. Thus $\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \not\subseteq f^{-1}(B)$, which shows that $f$ is not an intuitionistic fuzzy $\alpha$-continuous map.

Thus the class of intuitionistic fuzzy $\alpha$-continuous maps is properly contained in the class of intuitionistic fuzzy $sg$-continuous maps.

Forthcoming theorem and example shows that the class of intuitionistic fuzzy semi-continuous maps is properly contained in the class of intuitionistic fuzzy $sg$-continuous maps.

Theorem 3.11. Every intuitionistic fuzzy semi-continuous mapping is intuitionistic fuzzy $sg$-continuous mapping.

Proof. Let $f : X \rightarrow Y$ be any function from IFTS $X$ in to $Y$ such that $f$ is intuitionistic fuzzy semi-continuous. By definition of intuitionistic fuzzy semi-continuous, $f^{-1}(A)$ is IFSCS in $X$ for every IFCS $A$ in $Y$. In [12], it has been proved that every intuitionistic fuzzy semi-closed set is an intuitionistic fuzzy $sg$-closed set in $X$. Thus $f^{-1}(B)$ IFSGCS in $X$. Hence $f$ is an intuitionistic fuzzy $sg$-continuous mapping.

The converse of the above theorem is not true as seen from the following example. \hfill $\square$

Example 3.12. Let $X = \{a, b\}$, $Y = \{u, v\}$.

Let $A = \left\langle x, \begin{pmatrix} a & b \\ 0.2 & 0.4 \end{pmatrix}, \begin{pmatrix} a & b \\ 0.6 & 0.25 \end{pmatrix} \right\rangle$

$B = \left\langle y, \begin{pmatrix} u & v \\ 0.3 & 0.5 \end{pmatrix}, \begin{pmatrix} u & v \\ 0.4 & 0.5 \end{pmatrix} \right\rangle$.

Then $\tau = \{0_-, 1_-, A\}$ and $\kappa = \{0_-, 1_-, B\}$ are IFTS on $X$ and $Y$ respectively. Define a mapping $f : X \rightarrow Y$ by $f(a) = u$, $f(b) = v$. Clearly $f$ is intuitionistic fuzzy $sg$-continuous map.
Now we have $f^{-1}(B) = \left< x, \left( \frac{a}{0.3}, \frac{b}{0.5} \right), \left( \frac{a}{0.4}, \frac{b}{0.5} \right) \right>$.

$\text{cl}(f^{-1}(B)) = 1_\sim \quad \text{int}[\text{cl}(f^{-1}(B))] = \text{int}(1_\sim) = 1_\sim \quad \text{Thus int}[\text{cl}(f^{-1}(B))] \not\subseteq f^{-1}(B)$, which shows that $f$ is not intuitionistic fuzzy semi-continuous mapping.

**Theorem 3.13.** Every intuitionistic fuzzy sg-continuous mapping is intuitionistic fuzzy semi-pre continuous mapping.

**Proof.** Let $B$ be an IFCS in $Y$. Since $f$ is intuitionistic fuzzy sg-continuous map, $f^{-1}(B)$ is an intuitionistic fuzzy sg-closed set in $X$. In paper [12], it has been proved that, every intuitionistic fuzzy sg-closed set is an intuitionistic fuzzy semi-pre closed set. Therefore $f^{-1}(B)$ is an IFSPCS in $X$. Hence $f$ is an intuitionistic fuzzy semi-pre continuous mapping. $\square$

The converse of the above theorem is not true as seen from the following example.

**Example 3.14.** Let $X = \{a, b\}, Y = \{u, v\}$.

Let $A = \left< x, \left( \frac{a}{0.4}, \frac{b}{0.5} \right), \left( \frac{a}{0.1}, \frac{b}{0.3} \right) \right>$

$B = \left< y, \left( \frac{u}{0.15}, \frac{v}{0.3} \right), \left( \frac{u}{0.5}, \frac{v}{0.7} \right) \right>$.

Then $\tau = \{0_\sim, 1_\sim, A\}$ and $\kappa = \{0_\sim, 1_\sim, B\}$ are IFTS on $X$ and $Y$ respectively. Define a mapping $g: (X, \tau) \rightarrow (Y, \kappa)$ by $g(a) = u, g(b) = v$. Clearly $g$ is intuitionistic fuzzy semi-pre continuous map. Infact we have

$g^{-1}(B) = \left< x, \left( \frac{a}{0.15}, \frac{b}{0.3} \right), \left( \frac{a}{0.5}, \frac{b}{0.7} \right) \right>$

$scl(g^{-1}(B)) = 1_\sim \not\subseteq A$. Hence $g$ is not intuitionistic fuzzy semi-generalized continuous mapping.

**Remark 3.15.** Intuitionistic fuzzy pre-continuity is independent from intuitionistic fuzzy sg-continuity.

The proof follows from the following examples.

**Example 3.16.** Let $X = \{a, b\}, Y = \{u, v\}$.

Let $A = \left< x, \left( \frac{a}{0.3}, \frac{b}{0.4} \right), \left( \frac{a}{0.7}, \frac{b}{0.6} \right) \right>$

$B = \left< y, \left( \frac{u}{0.6}, \frac{v}{0.5} \right), \left( \frac{u}{0.4}, \frac{v}{0.5} \right) \right>$. 


Then $\tau = \{0, 1\}$ and $\kappa = \{0, 1\}$ are IFTS on $X$ and $Y$ respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \kappa)$ by $f(a) = u$, $f(b) = v$. Clearly $f$ is intuitionistic fuzzy sg-continuous map. In fact we have

$$f^{-1}(B) = \langle x, \left(\frac{a}{0.15}, \frac{b}{0.7}\right), \left(\frac{a}{0.15}, \frac{b}{0.7}\right)\rangle$$

$$\text{cl}(f^{-1}(B)) = 1. \cap \bar{A} = \bar{A}. \int\text{cl}(f^{-1}(B)) = \int\bar{A} = 0. \cup A = A. \text{ Hence } f^{-1}(B) \not\subseteq A = \int\text{cl}(f^{-1}(B))$$

which shows that $f$ is not an intuitionistic fuzzy pre-continuous map.

**Example 3.17.** Let $X = \{a, b\}$, $Y = \{u, v\}$.

Let $A = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}\right)\rangle$,

$B = \langle y, \left(\frac{u}{0.2}, \frac{v}{0.3}\right), \left(\frac{u}{0.4}, \frac{v}{0.7}\right)\rangle$.

Then $\tau = \{0, 1\}$ and $\kappa = \{0, 1\}$ are IFTS on $X$ and $Y$ respectively. Define a mapping $h : (X, \tau) \rightarrow (Y, \kappa)$ by $h(a) = u$, $h(b) = v$. Clearly $h$ is intuitionistic fuzzy pre-continuous map. In fact we have

$$h^{-1}(B) = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.2}, \frac{b}{0.3}\right)\rangle$$

$$\text{scl}(h^{-1}(B)) = 1. \cup A, \text{ but } \text{scl}(h^{-1}(B)) \not\subseteq A, \text{ which shows that } f \text{ is not an intuitionistic fuzzy sg-continuous map.}$$

The above diagram shows the relationships between intuitionistic fuzzy sg-continuous mappings and some other mappings. The reverse implications are not true in the above diagram.
Theorem 3.18. Let $f : X \rightarrow Y$ be a mapping from a IFTS $X$ into an IFTS $Y$. Then the following statements are equivalent.

(i) $f$ is intuitionistic fuzzy sg-continuous mapping.
(ii) $f^{-1}(B)$ is an IFSGOS in $X$, for every IFOS $B$ in $X$.

Proof. (i)⇒(ii) Let $B$ be an IFOS in $Y$, then $\overline{B}$ is an IFCS in $Y$. Since $f$ is intuitionistic fuzzy sg-continuous mapping $f^{-1}(\overline{B})$ is an IFSGCS in $X$. Then $f^{-1}(\overline{B}) = f^{-1}(B)$, implies $f^{-1}(B)$ is an IFSGOS in $Y$.

(ii)⇒(i) Let $B$ be an IFCS in $Y$. By our assumption $f^{-1}(\overline{B})$ is an IFSGOS in $X$ for every IFOS $\overline{B}$ in $Y$. But $f^{-1}(\overline{B}) = f^{-1}(B)$, which in turn implies $f^{-1}(B)$ is an IFSGCS in $X$. Hence $f$ is intuitionistic fuzzy sg-continuous mapping. □

Theorem 3.19. Let $f : X \rightarrow Y$ be an intuitionistic fuzzy sg-continuous mapping. Then the following statements hold.

(i) $f(\text{sgc}1(A)) \leq \text{c}1(f(A))$, for every intuitionistic fuzzy set $A$ in $X$.
(ii) $\text{sgc}1(f^{-1}(B)) \leq f^{-1}(\text{c}1(B))$ for every intuitionistic fuzzy set $B$ in $Y$.

Proof. (i) Let $A \subseteq X$. Then $\text{c}1(f(A))$ is an intuitionistic fuzzy closed set in $Y$. Since $f$ is intuitionistic fuzzy sg-continuous, $f^{-1}[\text{c}1(f(A))]$ is intuitionistic fuzzy sg-closed in $X$. Since $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}[\text{c}1(f(A))]$ and $f^{-1}[\text{c}1(f(A))]$ is intuitionistic fuzzy sg-closed, implies $\text{sgc}1(A) \subseteq f^{-1}[\text{c}1(f(A))]$. Hence $f(\text{sgc}1(A)) \leq \text{c}1(f(A))$.

(ii) Replacing $A$ by $f^{-1}(B)$ in (i), we get

\[
\begin{align*}
\text{c}1(f^{-1}(B)) & \leq \text{c}1(f(B)) \\
\text{sgc}1(f^{-1}(B)) & \leq f^{-1}(\text{c}1(B))
\end{align*}
\]

Hence $\text{sgc}1(\text{f}^{-1}(B)) \leq f^{-1}[\text{c}1(B)]$. □

Theorem 3.20. Let $f : X \rightarrow Y$ be a function and $g : X \rightarrow X \times Y$ the graph of the function $f$. Then $f$ is intuitionistic fuzzy sg-continuous if $g$ is so.

Proof. Let $B$ be an IFOS in $Y$. Then by Lemma 2.11, $f^{-1}(B) = f^{-1}(1_\times \times B) = 1_\times \cap f^{-1}(B) = g^{-1}(1_\times \times B)$. Since $B$ is an IFOS in $Y$, $1_\times \times B$ is an IFOS in $X \times Y$. Also since $g$ is intuitionistic fuzzy sg-continuous implies that $g^{-1}(1_\times \times B)$ is an IFSGOS in $X$. Therefore $f^{-1}(B)$ is an IFSGOS in $X$. Hence $f$ is intuitionistic fuzzy sg-continuous mapping. □

Theorem 3.21. Let $f : X \rightarrow Y$ is a mapping from an IFTS $X$ into an IFTS $Y$. If any union of IFSGCS is IFSGCS, then the following statements are equivalent.
(i) \( f \) is intuitionistic fuzzy sg-continuous mapping.

(ii) For each IFP \( p_{(a,b)} \in X \) and every IFN \( A \) of \( f(p_{(a,b)}) \), there exists an IFSGCS \( B \) such that \( p_{(a,b)} \in B \leq f^{-1}(A) \).

(iii) For each IFP \( p_{(a,b)} \in X \) and every IFN \( A \) of \( f(p_{(a,b)}) \), there exists an IFSGCS \( B \) such that \( p_{(a,b)} \in B \) and \( f(B) \leq A \).

**Proof.** (i)\(\Rightarrow\) (ii): Assume that \( f \) is intuitionistic fuzzy sg-continuous. Let \( p_{(a,b)} \) be an IFP in \( X \) and \( A \) be an IFN of \( f(p_{(a,b)}) \). Then by Definition of IFN, there exists an IFCS \( C \) in \( Y \), such that \( f(p_{(a,b)}) \in C \leq A \). Taking \( B = f^{-1}(C) \in X \), since \( f \) is intuitionistic fuzzy sg-continuous, \( f^{-1}(C) \) is IFSGCS and

\[
p_{(a,b)} \in B \leq f^{-1}[f(p_{(a,b)})] \leq f^{-1}(C) = B \leq f^{-1}(A).
\]

Hence \( p_{(a,b)} \in B \leq f^{-1}(A) \).

(ii)\(\Rightarrow\) (iii): Let \( p_{(a,b)} \) be an IFP in \( X \) and \( A \) be an IFN of \( f(p_{(a,b)}) \), such that there exists an IFSGCS \( B \) with \( p_{(a,b)} \in B \leq f^{-1}(A) \). From this we have \( p_{(a,b)} \in B \) and \( B \leq f^{-1}(A) \). This implies \( f(B) \leq f(f^{-1}(A)) = A \). Hence (iii) holds.

(iii)\(\Rightarrow\) (i): Assume that (iii) holds. Let \( B \) be an IFCS in \( Y \) and take \( p_{(a,b)} \in f^{-1}(B) \). Then \( f(p_{(a,b)}) \in f(f^{-1}(B)) \leq B \). Since \( B \) is IFCS in \( Y \), it follows that \( B \) is an IFN of \( f(p_{(a,b)}) \). Then from (iii), there exists an IFSGCS \( A \) such that \( p_{(a,b)} \in A \) and \( f(A) \leq B \). This shows that \( p_{(a,b)} \in A \leq f^{-1}(f(A)) \leq f^{-1}(B) \). (i.e) \( p_{(a,b)} \in A \leq f^{-1}(B) \). Since \( p_{(a,b)} \) is an arbitrary IFP and \( f^{-1}(B) \) is union of all IFP contained in \( f^{-1}(B) \), by assumption \( f^{-1}(B) \) is an IFSGCS. Hence \( f \) is intuitionistic fuzzy sg-continuous mapping.

**Theorem 3.22.** Let \( f : X \rightarrow Y \) is a mapping from an IFTS \( X \) into an IFTS \( Y \). Then the following statements are equivalent.

(i) \( f \) is intuitionistic fuzzy sg-continuous mapping.

(ii) \( f^{-1}(B) \) is an IFSGOS in \( X \), for every IFOS \( B \) in \( Y \).

(iii) \( f(sgc1(A)) \leq c1(f(A)) \), for every fuzzy set \( A \) in \( X \).

(iv) \( sgc1(f^{-1}(B)) \leq f^{-1}(c1(B)) \) for every fuzzy set \( B \) in \( Y \).

(v) For each IFP \( p_{(a,b)} \in X \) and every IFN \( A \) of \( f(p_{(a,b)}) \), there exists an IFSGCS \( B \) such that \( p_{(a,b)} \in B \leq f^{-1}(A) \).

(vi) For each IFP \( p_{(a,b)} \in X \) and every IFN \( A \) of \( f(p_{(a,b)}) \), there exists an IFSGCS \( B \) such that \( p_{(a,b)} \in B \) and \( f(B) \leq A \).

**Proof.** Follows from the Theorems 3.18, 3.19 and 3.22.

**Theorem 3.23.** If \( f : X \rightarrow Y \) is intuitionistic fuzzy sg-continuous and \( g : Y \rightarrow Z \) is intuitionistic fuzzy completely continuous, then \( g \circ f : X \rightarrow Z \) is intuitionistic fuzzy sg-continuous.
**Proof.** Let $B$ be any IFCS in $Z$. Since $g$ is intuitionistic fuzzy completely continuous, $g^{-1}(B)$ is an IFRCS in $Y$. In [6], it has been proved that every IFRCS is an IFCS. Therefore $g^{-1}(B)$ is an IFCS in $Y$. Also since $f$ is intuitionistic fuzzy $sg$-continuous mapping $f^{-1}[g^{-1}(B)]$ is an IFSGCS in $X$.

We have $(g \circ f)^{-1}[B] = f^{-1}[g^{-1}(B)]$ is IFSGCS in $X$, for every IFCS $B$ in $Z$. Hence $g \circ f$ is intuitionistic fuzzy $sg$-continuous mapping. □

**Theorem 3.24.** If $f : X \to Y$ is intuitionistic fuzzy $sg$-continuous and $g : Y \to Z$ is intuitionistic fuzzy continuous, then $g \circ f : X \to Z$ is intuitionistic fuzzy $sg$-continuous.

**Proof.** Let $B$ be any intuitionistic fuzzy closed set in $Z$. Since $g$ is intuitionistic fuzzy continuous, $g^{-1}(B)$ is intuitionistic fuzzy closed set in $Y$. Since $f$ is intuitionistic fuzzy $sg$-continuous mapping $f^{-1}[g^{-1}(B)]$ is an intuitionistic fuzzy $sg$-closed set in $X$.

$$(g \circ f)^{-1}[B] = f^{-1}[g^{-1}(B)]$$ is intuitionistic fuzzy $sg$-closed set, for every intuitionistic fuzzy closed $B$ in $Z$.

Hence $g \circ f$ is intuitionistic fuzzy $sg$-continuous mapping. □

**Theorem 3.25.** Let $f : X \to Y$ is a mapping from an IFTS $X$ into an IFTS $Y$. If $X$ is intuitionistic fuzzy semi-$T_{1/2}$ space, then $f$ is intuitionistic fuzzy $sg$-continuous iff it is intuitionistic fuzzy semi-continuous.

**Proof.** Let $f$ be intuitionistic fuzzy $sg$-continuous mapping and let $A$ be an intuitionistic fuzzy closed set in $Y$. Then by definition of intuitionistic fuzzy semi-generalized continuous $f^{-1}(A)$ is intuitionistic fuzzy $sg$-closed in $X$. Since $X$ is intuitionistic fuzzy semi-$T_{1/2}$ space, $f^{-1}(A)$ is intuitionistic fuzzy semi-closed set.

Hence $f$ is intuitionistic fuzzy semi-continuous.

Conversely assume that $f$ is intuitionistic fuzzy semi-continuous. Then by **Theorem 3.11** $f$ is intuitionistic fuzzy $sg$-continuous mapping. □

**Theorem 3.26.** Let $X$, $X_1$, $X_2$ are IFTS’s and $p_i : X_1 \times X_2 \to X_i$ ($i = 1, 2$) are projections of $X_1 \times X_2$ onto $X_i$. If $f : X \to X_1 \times X_2$ is intuitionistic fuzzy $sg$-continuous, then $p_i \circ f$ ($i = 1, 2$) is intuitionistic fuzzy $sg$-continuous mapping.

**Proof.** It follows from the facts that projections are intuitionistic fuzzy continuous mappings. □
4. Intuitionistic fuzzy semi-generalized irresolute mappings

**Definition 4.1.** A mapping \( f : X \to Y \) from an IFTS \( X \) into an IFTS \( Y \) is said to be intuitionistic fuzzy semi-generalized irresolute (intuitionistic fuzzy sg-irresolute) if \( f^{-1}(B) \) is an IFSGCS in \( X \) for every IFSGCS \( B \) in \( Y \).

**Theorem 4.2.** Let \( f : X \to Y \) is a mapping from an IFTS \( X \) into an IFTS \( Y \). Then every intuitionistic fuzzy sg-irresolute mapping is intuitionistic fuzzy sg-continuous.

**Proof.** Assume that \( f : X \to Y \) is an intuitionistic fuzzy sg-irresolute mapping and let \( A \) be an IFCS in \( Y \). In [12], it has been proved that every intuitionistic fuzzy closed set is an intuitionistic fuzzy sg-closed. Therefore \( A \) is an IFSGCS in \( Y \). Since \( f \) is intuitionistic fuzzy sg-irresolute, by definition \( f^{-1}(A) \) is IFSGCS in \( X \). Hence \( f \) is intuitionistic fuzzy sg-continuous. \( \square \)

**Example 4.3.** Let \( X = \{a, b, c\}, Y = \{u, v, w\} \).

Let 
\[
A = \left\{ \begin{array}{l}
x, 
\left( \begin{array}{ccc}
0.8 & 0.4 & 0.4 \\
0.1 & 0.6 & 0.6
\end{array} \right) \\
\end{array} \right\}, \\
B = \left\{ \begin{array}{l}
y, 
\left( \begin{array}{ccc}
0 & 0.4 & 0.2 \\
0 & 0.6 & 0.6
\end{array} \right) \\
\end{array} \right\}.
\]

Then \( \tau = \{0, 1, A\} \) and \( \kappa = \{0, 1, B\} \) are IFTS on \( X \) and \( Y \) respectively. Define a mapping \( h : (X, \tau) \to (Y, \kappa) \) by \( h(a) = u, h(b) = v, h(c) = w \). Clearly \( h \) is intuitionistic fuzzy sg-continuous map. In fact we have
\[
C = \left\{ \begin{array}{l}
y, 
\left( \begin{array}{ccc}
0 & 0.4 & 0.2 \\
0 & 0.6 & 0.6
\end{array} \right) \\
\end{array} \right\}
\]
be an IFSGCS in \( Y \).

\[
h^{-1}(C) = \left\{ \begin{array}{l}
x, 
\left( \begin{array}{ccc}
0 & 0.4 & 0.2 \\
0 & 0.6 & 0.6
\end{array} \right) \\
\end{array} \right\}.
\]

\( \text{scl}(h^{-1}(C)) = 1 = 1 \). \( h^{-1}(C) \subset A \), but \( \text{scl}(h^{-1}(C)) \not\subset A \), which shows that \( h^{-1}(C) \) is not an IFSGCS in \( X \). Therefore \( f \) is not an intuitionistic fuzzy sg-irresolute map.

**Theorem 4.4.** Let \( f : X \to Y \) be a mapping from a IFTS \( X \) into an IFTS \( Y \). Then the following statements are equivalent.

(i) \( f \) is intuitionistic fuzzy sg-irresolute mapping.

(ii) \( f^{-1}(B) \) is an IFSGOS in \( X \), for every IFSGOS \( B \) in \( X \).

**Proof.** Similar to Theorem 3.18. \( \square \)

**Theorem 4.5.** Let \( f : X \to Y \) be a mapping from an IFTS \( X \) into an IFTS \( Y \). Then the following statements are equivalent.
(i) \( f \) is an intuitionistic fuzzy semi-generalized irresolute mapping.

(ii) \( f^{-1}(B) \) is an IFSGOS in \( X \) for each IFSGOS \( B \) in \( Y \).

(iii) \( \text{sgcl}(f^{-1}(B)) \leq f^{-1}(\text{sgcl}(B)) \), for each IFS \( B \) of \( Y \).

(iv) \( f^{-1}(\text{sgint}(B)) \leq \text{sgint}(f^{-1}(B)) \), for each IFS \( B \) of \( Y \).

**Proof.** (i)\(\Rightarrow\)(ii) It can be proved by using the complement and **Definition 4.1**.

(ii)\(\Rightarrow\)(iii) Let \( B \) be an IFS in \( Y \). Since \( B \leq \text{sgcl}(B) \), \( f^{-1}(B) = f^{-1}(\text{sgcl}(B)) \). Since \( \text{sgcl}(B) \) is an IFSGCS in \( Y \), by our assumption, \( f^{-1}(\text{sgcl}(B)) \) is an IFSGCS in \( X \). Therefore \( \text{sgcl}(f^{-1}(B)) \leq f^{-1}(\text{sgcl}(B)) \).

(iii)\(\Rightarrow\)(iv) By taking complement we get the result.

(iv)\(\Rightarrow\)(i) Let \( B \) be any IFSGOS in \( Y \). Then \( \text{sgint}(B) = B \). By our assumption we have \( f^{-1}(B) = f^{-1}(\text{sgint}(B)) \leq \text{sgint}(f^{-1}(B)) \), so \( f^{-1}(B) \) is an IFSGCS in \( X \). Hence \( f \) is intuitionistic fuzzy sg-irresolute mapping.

**Theorem 4.6.** Let \( f : X \to Y \) be intuitionistic fuzzy sg-irresolute mapping. Then \( f \) is intuitionistic fuzzy irresolute mapping if \((X, \tau)\) is intuitionistic fuzzy semi-\( T_{1/2} \) space.

**Proof.** Let \( B \) be an IFSCS in \( Y \). Then \( B \) is an IFSGCS in \( Y \). Since \( f \) is intuitionistic fuzzy sg-irresolute, \( f^{-1}(B) \) is an IFSGCS in \( X \). But \((X, \tau)\) is intuitionistic fuzzy semi-\( T_{1/2} \) space implies \( f^{-1}(B) \) is an IFSCS in \( X \). Hence \( f \) is intuitionistic fuzzy irresolute.

**Theorem 4.7.** If a mapping \( f : X \to Y \) is intuitionistic fuzzy sg-irresolute mapping, then \( f(\text{sgcl}(B)) \leq \text{scl}(f(B)) \) for every IFS \( B \) of \( X \).

**Proof.** Let \( B \) be an IFS of \( X \). Since \( \text{scl}(f(B)) \) is an IFSGCS in \( Y \), by our assumption \( f^{-1}(\text{scl}(f(B))) \) is an IFSGCS in \( X \). Furthermore \( B \leq f^{-1}(f(B)) \leq f^{-1}(\text{scl}(f(B))) \) and hence \( \text{sgcl}(B) \leq f^{-1}(\text{scl}(f(B))) \) and consequently \( f[\text{sgcl}(B)] \leq f[f^{-1}(\text{scl}(f(B)))] \leq \text{scl}(f(B)) \).

**Theorem 4.8.** Let \((Y, \kappa)\) be an IFTS such that every IFSCS in \( Y \) is an IFCS. If \( f : (X, \tau) \to (Y, \kappa) \) is bijective and intuitionistic fuzzy sg-continuous then \( f \) is intuitionistic fuzzy sg-irresolute.

**Proof.** Let \( B \) be an IFSGCS in \( Y \) and let \( f^{-1}(B) \leq A \), where \( A \) is an IFSOS in \( X \). Then \( B \leq f(A) \). Since \( f(A) \) is an IFSO in \( Y \) and \( B \) is an IFSGCS in \( Y \), then \( \text{scl}(B) \leq f(A) \) and hence \( f^{-1}(\text{scl}(A)) \leq f^{-1}(f(A)) = A \). Since \( f \) is intuitionistic fuzzy sg-continuous and \( \text{scl}(B) \) is an IFCS in \( Y \), then \( f^{-1}(\text{scl}(B)) \) is an IFSGCS in \( X \). Therefore \( \text{scl}(f^{-1}(\text{scl}(B))) \leq A \) and so \( \text{scl}(f^{-1}(B)) \leq A \). Hence \( f^{-1}(B) \) is an IFSGCS in \( X \). Hence \( f \) is intuitionistic fuzzy sg-irresolute mapping.
Theorem 4.9. Let $f : X \rightarrow Y$ be an intuitionistic fuzzy sg-irresolute mappings. Then $f$ is intuitionistic fuzzy irresolute, if $(X, \tau)$ is an intuitionistic fuzzy semi-$T_{1/2}$ space.

Proof. Let $A$ be any IFSCS in $Y$. In [12], it has been proved that every IFSCS is an IFSGCS. Therefore $A$ is an IFSGCS in $Y$ and $f$ is an intuitionistic fuzzy sg-irresolute. Then by definition $f^{-1}(A)$ is IFSGCS in $X$. But $(X, \tau)$ is an intuitionistic fuzzy semi-$T_{1/2}$ space, so $f^{-1}(A)$ is an IFSCS. Hence $f$ is an intuitionistic fuzzy irresolute.

Theorem 4.10. If any union of IFSGCS is an IFSGCS, then a mapping $f : X \rightarrow Y$ from an IFTS $X$ into an IFTS $Y$ is intuitionistic fuzzy sg-irresolute if and only if for each IFP $p(a, \beta)$ in $X$ and IFSGCS $B$ in $Y$ such that $f(p(a, \beta)) \in B$, there exists an IFSGCS $A$ in $X$ such that $p(a, \beta) \in A$ and $f(A) \subseteq B$.

Proof. Let $f$ be any intuitionistic fuzzy sg-irresolute mapping, $p(a, \beta)$ an IFP in $X$ and $B$ be any IFSGCS in $Y$, such that $f(p(a, \beta)) \in B$. Then $p(a, \beta) \in f^{-1}(B) = \text{sgcl}(f^{-1}(B))$. We take $A = \text{sgcl}(f^{-1}(B))$. Then $A$ is an IFSGCS in $X$, containing IFP $p(a, \beta)$ and $f(A) = f[\text{sgcl}(f^{-1}(B))] \subseteq f[f^{-1}(B)] \subseteq B$.

Conversely assume that $B$ be any IFSGCS in $Y$ and IFP $p(a, \beta)$ in $X$, such that $p(a, \beta) \in f^{-1}(B)$. By assumption there exists IFSGCS $A$ in $X$ such that $p(a, \beta) \in A$ and $f(A) \subseteq B$. Therefore $p(a, \beta) \in A \subseteq f^{-1}(B)$ and $p(a, \beta) \in A = \text{sgcl}(A) \subseteq \text{sgcl}(f^{-1}(B))$. Since $p(a, \beta)$ is an arbitrary IFP and $f^{-1}(B)$ is union of all IFP contained in $f^{-1}(B)$, $f^{-1}(B)$ is an IFSGCS in $X$, so $f$ is an intuitionistic fuzzy semi-generalized irresolute mapping.

Corollary 4.11. A mapping $f : X \rightarrow Y$ from an IFTS $X$ into an IFTS $Y$ is intuitionistic fuzzy semi-generalized irresolute if and only if for each IFP $p(a, \beta)$ in $X$ and IFSGCS $B$ in $Y$ such that $f(p(a, \beta)) \in B$, there exists an IFSGCS $A$ in $X$ such that $p(a, \beta) \in A$ and $A \subseteq f^{-1}(B)$.

Proof. Follows from Theorem 4.10.

Theorem 4.12. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are intuitionistic fuzzy sg-irresolute mappings, where $X, Y, Z$ are IFTS. Then $g \circ f$ is an intuitionistic fuzzy sg-irresolute mapping.

Proof. Let $A$ be an intuitionistic fuzzy sg-closed set in $Z$. Since $g$ is an intuitionistic fuzzy semi-generalized irresolute mapping $g^{-1}(A)$ is an intuitionistic fuzzy sg-closed set in $Y$. Also since $f$ is intuitionistic fuzzy semi-generalized irresolute mapping, $f^{-1}[g^{-1}(A)]$ is an intuitionistic fuzzy sg-closed set in $X$.

$$(g \circ f)^{-1}(A) = f^{-1}[g^{-1}(A)]$$

for each $A$ in $Z$. Hence $(g \circ f)^{-1}(A)$ is an intuitionistic fuzzy sg-closed set in $X$. Therefore $g \circ f$ is an intuitionistic fuzzy semi-generalized irresolute mapping.
**Theorem 4.13.** Let $f : X \to Y$ and $g : Y \to Z$ are intuitionistic fuzzy semi-generalized irresolute and intuitionistic fuzzy continuous mappings respectively, where $X$, $Y$, $Z$ are IFTS. Then $g \circ f$ is an intuitionistic fuzzy semi-generalized continuous mapping.

**Proof.** Let $A$ be any IFCS in $Z$. Since $g$ is intuitionistic fuzzy semi-generalized continuous, $g^{-1}(A)$ is an IFSGCS in $Y$. Also, since $f$ is intuitionistic fuzzy semi-generalized irresolute, $f^{-1}[g^{-1}(A)]$ is an IFSGCS in $X$.

$$(g \circ f)^{-1}(A) = f^{-1}[g^{-1}(A)]$$ is an IFSGCS in $X$. Hence $g \circ f$ is intuitionistic fuzzy semi-generalized continuous. 

**Theorem 4.14.** Let $(X, \tau)$, $(Y, \kappa)$, $(Z, \delta)$ be any intuitionistic fuzzy topological spaces. Let $f : (X, \tau) \to (Y, \kappa)$ be intuitionistic fuzzy semi-generalized irresolute and $g : (Y, \kappa) \to (Z, \sigma)$ is intuitionistic fuzzy continuous, then $g \circ f$ is intuitionistic fuzzy semi-generalized continuous.

**Proof.** Let $B$ be any intuitionistic fuzzy closed set in $Z$. Since $g$ is intuitionistic fuzzy continuous, $g^{-1}(B)$ is IFCS in $Y$. In paper [12], it has been proved that every IFCS is an IFSGCS. Therefore $f^{-1}(g^{-1}(B))$ is an IFSGCS in $Y$. But since $f$ is an intuitionistic fuzzy sgirresolute mapping $f^{-1}(g^{-1}(B))$ is an IFSGCS in $X$.

$$(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$$ is IFSGCS in $X$ for every IFCS ‘$B$’ in $X$.

Hence $g \circ f$ is intuitionistic fuzzy sg-continuous.

**Theorem 4.15.** Let $X, X_1, X_2$ are IFTS’s and $p_i : X_1 \times X_2 \to X_i$ ($i = 1, 2$) are projections of $X_1 \times X_2$ onto $X_i$. If $f : X \to X_1 \times X_2$ is intuitionistic fuzzy semi-generalized irresolute, then $p_i f$ is intuitionistic fuzzy semi-generalized continuous mapping.

**Proof.** $p_i f : X \to X_i$ ($i = 1, 2$). It follows from the fact that $p_i$ ($i = 1, 2$) are intuitionistic fuzzy continuous mappings and by **Theorem 5.5**.

**References**


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