



## NEW $f$ -DIVERGENCE AND JENSEN-OSTROWSKI'S TYPE INEQUALITIES

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**Abstract.** In this paper we derive new information inequalities of Jensen-Ostrowski type, by considering two Jensen-Ostrowski type inequalities, new  $f$ -divergence and Chi-divergences. The special cases of these information inequalities are established as applications of new  $f$ -divergence and its particular instances.

### 1. Introduction

In this paper we apply inequalities of [3] to obtain information inequalities for new  $f$ -divergence measure. Let suppose that a set  $\Omega$  and the  $\sigma$ -finite measure  $\mu$  are given. Take the set of all probability densities on  $\mu$  to be

$$P = \left( p \mid p : \Omega \rightarrow \mathfrak{R}, p(t) \geq 0, \int_{\Omega} p(t) d\mu(t) = 1 \right). \quad (1.1)$$

In this text we use the following definitions of divergence measures which are the particular instances of new  $f$ -divergence:

- Kullback-Leibler divergence measure [12]:

$$S_{KL}(\mu_1, \mu_2) = \int_{\Omega} p(t) \ln \left[ \frac{p(t)}{q(t)} \right] d\mu(t), \quad (1.2)$$

$$S_{KL}(\mu_2, \mu_1) = \int_{\Omega} q(t) \ln \left[ \frac{q(t)}{p(t)} \right] d\mu(t). \quad (1.3)$$

- Relative Jensen-Shannon divergence measure [13]:

$$S_F(\mu_2, \mu_1) = \int_{\Omega} q(t) \log \left( \frac{2q(t)}{p(t) + q(t)} \right) d\mu(t). \quad (1.4)$$

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- Relative arithmetic-geometric divergence measure [14]:

$$S_G(\mu_2, \mu_1) = \int_{\Omega} \left( \frac{p(t) + q(t)}{2} \right) \log \left( \frac{p(t) + q(t)}{2q(t)} \right) d\mu(t). \quad (1.5)$$

- Triangular discrimination [6]:

$$S_f(\mu_1, \mu_2) = \int_{\Omega} \frac{(p(t) - q(t))^2}{2(p(t) + q(t))} d\mu(t) = \frac{1}{2} S_{\Delta}(\mu_1, \mu_2). \quad (1.6)$$

- Chi-divergences [13] and [14]:

$$S_f(\mu_1, \mu_2) = \frac{1}{4} \int_{\Omega} \left( \frac{p(t) - q(t)}{q(t)} \right)^2 q(t) d\mu(t) = \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2), \quad (1.7)$$

$$S_f(\mu_1, \mu_2) = \frac{1}{2^k} \int_{\Omega} \left( \frac{p(t) - q(t)(2\lambda - 1)}{q(t)} \right)^k q(t) d\mu(t) = \frac{1}{2^k} S_{\chi^2, 2\lambda - 1}(\mu_1, \mu_2). \quad (1.8)$$

The New  $f$ -divergence of the probability distributions  $\mu_1$  and  $\mu_2$  is defined as follows

$$S_f(\mu_1, \mu_2) = \int q(t) f \left( \frac{p(t) + q(t)}{2q(t)} \right) d\mu(t). \quad (1.9)$$

Where denote the density (Radon-Nikodym-derivative) of  $\mu_i$  ( $i = 1, 2$ ) with respect to  $\mu$  by  $p(t) = \frac{d\mu_1(t)}{d\mu(t)}$  and  $q(t) = \frac{d\mu_2(t)}{d\mu(t)}$ . Define the convex functions  $f : (0, \infty) \rightarrow \mathfrak{R}_+$ ,  $f(1) = 0$ , appropriately for obtaining various divergences. The basic properties of New  $f$ -divergence are available in [10] and [11].

## 2. Generalized Jensen-Ostrowski type inequalities

The following Jensen-Ostrowski inequality is considered from [3] for functions with bounded second derivatives.

**Theorem 2.1.** *Let  $f : I \rightarrow C$  be a differentiable function on  $I$ ,  $f' : [a, b] \subset I \rightarrow C$  is absolutely continuous on  $[a, b]$  and  $\zeta \in [a, b]$  For some  $\gamma, \Gamma \in C$ ,  $\gamma \neq \Gamma$ , assume that  $f'' \in \bar{U}_{[a, b]}(\gamma, \Gamma) = \bar{\Delta}_{[a, b]}(\gamma, \Gamma)$ . If  $g : \Omega \rightarrow [a, b]$  is Lebesgue  $\mu$ -measurable on  $\Omega$  such that  $f \circ g$ ,  $g$ ,  $(g - \zeta)^2 \in L(\Omega, \mu)$ , with  $\int_{\Omega} d\mu = 1$  then*

$$\begin{aligned} & \left| \int_{\Omega} (f \circ g) d\mu - f(\zeta) - \left( \int_{\Omega} g d\mu - \zeta \right) - f'(\zeta) - \frac{\gamma + \Gamma}{4} \int_{\Omega} (g - \zeta)^2 d\mu \right| \\ & \leq \frac{1}{4} |\Gamma - \gamma| \left[ \sigma^2(g) + \left( \int_{\Omega} g d\mu - \zeta \right)^2 \right]. \end{aligned} \quad (2.1)$$

A generalized version of the Ostrowski inequality [5] is considered form [3]. It is inequality as well as bounds for the discrepancy in Jensens integral inequality. The theorem is following:

**Theorem 2.2.** Let  $f : I \rightarrow C$  be a differentiable function on  $I$ ,  $f' : [a, b] \subset I \rightarrow C$  is absolutely continuous on  $[a, b]$  and  $\zeta \in [a, b]$ . If  $g : \Omega \rightarrow [a, b]$  is Lebesgue  $\mu$ -measurable on  $\Omega$  such that  $f \circ g, g, (g - \zeta)^2 \in L(\Omega, \mu)$ , with  $\int_{\Omega} d\mu = 1$  then we have the following Ostrowski type inequality:

$$\left| \int_{\Omega} (f \circ g) d\mu - f(\zeta) - \left( \int_{\Omega} g d\mu - \zeta \right) - f'(\zeta) \right| \leq \frac{1}{2} \|f''\|_{[a,b],\infty} \left[ \sigma^2(g) + \left( \int_{\Omega} g d\mu - \zeta \right)^2 \right]. \quad (2.2)$$

We also have the following Jensen type inequality:

$$\left| \int_{\Omega} (f \circ g) d\mu - f \left( \int_{\Omega} g d\mu \right) \right| \leq \frac{1}{2} \|f''\|_{[a,b],\infty} \sigma^2(g) \quad (2.3)$$

which is the best inequality one can get from (2.2).

### 3. Main results

In this section, we present our main results on the Jensen-Ostrowski type inequalities by using chi- divergences (1.7)-(1.8) and new  $f$ -divergence (1.9).

**Proposition 3.1.** Let  $f : (0, \infty) \rightarrow \mathfrak{R}$  be a differentiable convex function with the property that  $f(1) = 0$ . Assume that  $\mu_1, \mu_2 \in \Omega$  and there exists constants  $0 < r < 1 < R < \infty$  such that

$$r \leq \frac{p(t) + q(t)}{2q(t)} \leq R \quad (3.1)$$

for  $\mu$ -a.e.  $t \in \Omega$ . If  $\zeta \in [r, R]$  and  $f'$  is absolutely continuous on  $[r, R]$ , then we have the inequalities

$$\left| S_f(\mu_1, \mu_2) - f(\zeta) - (1 - \zeta) f'(\zeta) \right| \leq \frac{1}{2} \|f''\|_{[r,R],\infty} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right]. \quad (3.2)$$

In particular, by choosing  $\zeta = (r + R)/2$ , we have

$$\left| S_f(\mu_1, \mu_2) - f\left(\frac{r+R}{2}\right) - \left(1 - \frac{r+R}{2}\right) f'\left(\frac{r+R}{2}\right) \right| \leq \frac{1}{2} \|f''\|_{[r,R],\infty} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + \left(\frac{r+R}{2} - 1\right)^2 \right] \quad (3.3)$$

and when  $\zeta = 1$ , we have

$$|S_f(\mu_1, \mu_2)| \leq \frac{1}{2} \|f''\|_{[r,R],\infty} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \quad (3.4)$$

**Proof.** We choose  $g(t) = \frac{p(t)+q(t)}{2q(t)}$ , noting that  $\int_{\Omega} g d\mu(t) = 1$ , in inequality (2.2), we have

$$\left| \int_{\Omega} f\left(\frac{p(t)+q(t)}{2q(t)}\right) q(t) d\mu(t) - f(\zeta) - \left( \int_{\Omega} g(t) d\mu - \zeta \right) f'(\zeta) \right|$$

$$\begin{aligned}
&= |S_f(\mu_1, \mu_2) - f(\zeta) - (1 - \zeta)f'(\zeta)| \\
&= \frac{1}{2} \|f''\|_{[r, R], \infty} \left[ \int_{\Omega} \left( \frac{p(t) + q(t)}{2q(t)} - \int_{\Omega} g(t) d\mu \right)^2 q(t) d\mu + \left( \int_{\Omega} g(t) d\mu - \zeta \right)^2 \right] \\
&= \frac{1}{2} \|f''\|_{[r, R], \infty} \left[ \int_{\Omega} \left( \frac{p(t) + q(t)}{2q(t)} - 1 \right)^2 q(t) d\mu + (\zeta - 1)^2 \right] \\
&= \frac{1}{2} \|f''\|_{[r, R], \infty} \left[ \int_{\Omega} \left( \frac{p(t) - q(t)}{2q(t)} \right)^2 q(t) d\mu + (\zeta - 1)^2 \right] \\
&= \frac{1}{2} \|f''\|_{[r, R], \infty} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right]
\end{aligned}$$

and this completes the proof.  $\square$

**Proposition 3.2.** *Under the assumptions of Proposition 3.1, if  $f'$  is convex or  $f''_{\pm}$  exists, then we have*

$$\begin{aligned}
&\left| S_f(\mu_1, \mu_2) - f(\zeta) - (1 - \zeta)f'(\zeta) + \frac{f''_+(r) + f''_-(R)}{4} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right] \right| \\
&\leq \frac{1}{4} |f''_-(R) - f''_+(r)| \left[ \frac{1}{4} S_{\chi^2, 2\zeta-1}(\mu_1, \mu_2) \right]
\end{aligned} \tag{3.5}$$

for  $\zeta \in [r, R]$ . Some particular cases of interest are obtained by setting  $\zeta = (r + R)/2$  and  $\zeta = 1$ .

**Proof.** When  $f'$  is convex, we set  $\gamma = f''_+(r)$  and  $\Gamma = f''_-(R)$  (cf. Remark 2 in [3]). For the case where  $f''_{\pm}$  exists, we set  $\gamma$  and  $\Gamma$  appropriately to the values of  $f''_+(r)$  and  $f''_-(R)$ , with  $\gamma \leq \Gamma$ . Utilizing (2.1) for  $g(t) = \frac{p(t)+q(t)}{2q(t)}$ , and the measure  $\int_{\Omega} q(t) d\mu = 1$ , we have

$$\begin{aligned}
&\left| \int_{\Omega} f\left(\frac{p(t) + q(t)}{2q(t)}\right) q(t) d\mu - f(\zeta) - \left( \int_{\Omega} q(t) d\mu - \zeta \right) f'(\zeta) \right. \\
&\quad \left. + \frac{f''_+(r) + f''_-(R)}{4} \int_{\Omega} \left( \frac{p(t) + q(t)}{2q(t)} - \zeta \right)^2 q(t) d\mu \right| \\
&\quad \left| S_f(\mu_1, \mu_2) - f(\zeta) - (1 - \zeta)f'(\zeta) + \frac{f''_+(r) + f''_-(R)}{4} \left[ \frac{1}{4} S_{\chi^2, 2\zeta-1}(\mu_1, \mu_2) \right] \right| \\
&\leq \frac{1}{4} |f''_-(R) - f''_+(r)| \left[ \int_{\Omega} \left( \frac{p(t) + q(t)}{2q(t)} - 1 \right)^2 q(t) d\mu + (\zeta - 1)^2 \right] \\
&= \frac{1}{4} |f''_-(R) - f''_+(r)| \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right]
\end{aligned}$$

and this completes the proof.  $\square$

#### 4. Special cases:

**Example 4.1.** If we consider the convex function  $f : (0, \infty) \rightarrow \mathfrak{R}$ ,  $f(t) = t \log(t)$ , then we get (1.5).

We have  $f'(t) = \log(t) + 1$ , and  $f''(t) = 1/t$ . By Proposition 3.1, we have the following inequalities:

$$\begin{aligned} |S_G(\mu_2, \mu_1) - \zeta \log(\zeta) - (1 - \zeta)(\log(\zeta) + 1)| &= |S_G(\mu_2, \mu_1) - 1 + \zeta - \log(\zeta)| \\ &\leq \frac{1}{2} \left[ \sup_{t \in [r, R]} \frac{1}{t} \right] \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right] \\ &= \frac{1}{2r} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right] \end{aligned}$$

for all  $\zeta \in [r, R]$ , and when  $\zeta = 1$ ,

$$0 \leq S_G(\mu_2, \mu_1) \leq \frac{1}{2r} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \quad (4.1)$$

Furthermore, by Proposition 3.2, we have the inequalities:

$$\left| S_G(\mu_2, \mu_1) - \log(\zeta) - 1 + \zeta + \frac{r+R}{4rR} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right] \right| \leq \frac{R-r}{4rR} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) \right]$$

for  $\zeta \in [r, R]$ , and when  $\zeta = 1$ ,

$$\left| S_G(\mu_2, \mu_1) + \frac{R+r}{4rR} S_{\chi^2}(\mu_1, \mu_2) \right| \leq \frac{R-r}{4rR} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \quad (4.2)$$

**Example 4.2.** If we consider the convex function  $f : (0, \infty) \rightarrow \mathfrak{R}$ ,  $f(t) = -\log(t)$ , then we get (1.4).

We have  $f'(t) = 1/t$ , and  $f''(t) = -1/t^2$ , and we note that

$$\frac{1}{4} \int_{\Omega} q(t) \left( \left( \frac{p(t)}{q(t)} \right)^2 - 1 \right) d\mu(t) = \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2).$$

By Proposition 3.1, we have the following inequalities:

$$\begin{aligned} |S_F(\mu_2, \mu_1) + \log(\zeta) + \frac{1}{\zeta} - 1| &\leq \frac{1}{2} \left[ \sup_{t \in [r, R]} \frac{1}{t^2} \right] \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right] \\ &= \frac{1}{2r^2} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right] \end{aligned}$$

for all  $\zeta \in [r, R]$ , and when  $\zeta = 1$ ,

$$0 \leq S_F(\mu_2, \mu_1) \leq \frac{1}{2r^2} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \quad (4.3)$$

Furthermore, by Proposition 3.2, we have the inequalities:

$$\left| S_F(\mu_2, \mu_1) + \log(\zeta) + \frac{1}{\zeta} - 1 + \frac{r^2 + R^2}{4r^2 R^2} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right] \right| \leq \frac{R^2 - r^2}{4r^2 R^2} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) \right]$$

for  $\zeta \in [r, R]$ , and when  $\zeta = 1$ ,

$$\left| S_F(\mu_2, \mu_1) + \frac{R^2 + r^2}{4r^2 R^2} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) \right| \leq \frac{R^2 - r^2}{4rR} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \quad (4.4)$$

**Example 4.3.** If we consider the convex function  $f : (0, \infty) \rightarrow \mathfrak{R}$ ,  $f(t) = -\frac{(t-1)^2}{t}$ , then we get (1.6).

We have  $f'(t) = 1 - 1/t^2$ , and  $f''(t) = 2/t^2$ , and  $f'''(t) = 2/t^3$ . By Proposition 3.1, we have the following inequalities:

$$\begin{aligned} \left| \frac{1}{2} S_{\Delta}(\mu_1, \mu_2) - \frac{(\zeta-1)^2}{\zeta} - (1-\zeta) \left(1 - \frac{1}{\zeta^2}\right) \right| &= \left| \frac{1}{2} S_{\Delta}(\mu_1, \mu_2) - \frac{(\zeta-1)^2}{\zeta} - (1-\zeta) \frac{(\zeta^2-1)}{\zeta^2} \right| \\ &= \frac{1}{2} \left[ \sup_{t \in [r, R]} \frac{2}{t^3} \right] \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta-1)^2 \right] \\ &= \frac{1}{r^3} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta-1)^2 \right] \end{aligned}$$

for all  $\zeta \in [r, R]$ , and when  $\zeta = 1$ ,

$$0 \leq \frac{1}{2} S_{\Delta}(\mu_1, \mu_2) \leq \frac{1}{r^3} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \quad (4.5)$$

Furthermore, by Proposition 3.2, we have the inequalities:

$$\begin{aligned} \left| \frac{1}{2} S_{\Delta}(\mu_1, \mu_2) - \frac{(\zeta-1)^2}{\zeta} - (1-\zeta) \left(1 - \frac{1}{\zeta^2}\right) + \frac{r^3 + R^3}{4r^3 R^3} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta-1)^2 \right] \right| \\ \leq \frac{r^3 - R^3}{2r^3 R^3} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) \right] \end{aligned}$$

for  $\zeta \in [r, R]$ , and when  $\zeta = 1$ ,

$$\left| \frac{1}{2} S_{\Delta}(\mu_1, \mu_2) + \frac{R^3 + r^3}{2r^3 R^3} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) \right| \leq \frac{r^3 - R^3}{2r^3 R^3} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \quad (4.6)$$

**Example 4.4.** If we consider the convex function  $f : (0.5, \infty) \rightarrow \mathfrak{R}$ ,  $f(t) = -\log(2t-1)$ , then we get (1.3). We have  $f'(t) = \frac{-2}{2t-1}$ , and  $f''(t) = \frac{4}{(2t-1)^2}$ . By Proposition 3.1, we have the following inequalities:

$$\begin{aligned} \left| S_{KL}(\mu_2, \mu_1) - \log(2\zeta-1) - (1-\zeta) \frac{2}{(2t-1)} \right| &\leq \frac{1}{2} \left[ \sup_{t \in [r, R]} \frac{4}{(2t-1)^2} \right] \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta-1)^2 \right] \\ &= \frac{2}{(2r-1)^2} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta-1)^2 \right] \end{aligned}$$

for all  $\zeta \in [r, R]$ , and when  $\zeta = 1$ ,

$$0 \leq S_{KL}(\mu_2, \mu_1) \leq \frac{2}{(2r-1)^2} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \quad (4.7)$$

Furthermore, by Proposition 3.2, we have the inequalities:

$$\left| S_F(\mu_2, \mu_1) + \log(2\zeta-1) + \frac{2(1-\zeta)}{(2\zeta-1)} + \frac{4(r^2 + R^2) - 4(r+R) + 2}{(2r-1)^2(2R-1)^2} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta-1)^2 \right] \right|$$

$$\leq \frac{4(R^2 - r^2) - 4(R - r)}{(2r - 1)^2(2R - 1)^2} \left[ \frac{1}{4} S_{\chi^2, 2\zeta - 1}(\mu_1, \mu_2) \right]$$

for  $\zeta \in [r, R]$ , and when  $\zeta = 1$ ,

$$\begin{aligned} & \left| S_F(\mu_1, \mu_2) + \frac{4(r^2 + R^2) - 4(r + R) + 2}{(2r - 1)^2(2R - 1)^2} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) \right| \\ & \leq \frac{4(R^2 - r^2) - 4(R - r)}{(2r - 1)^2(2R - 1)^2} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \end{aligned} \quad (4.8)$$

**Example 4.5.** If we consider the convex function  $f : (0.5, \infty) \rightarrow \mathfrak{R}$ ,  $f(t) = (2t - 1) \log(2t - 1)$ , then we get (1.2). We have  $f'(t) = 2 \log(2t - 1) + 2$  and  $f''(t) = \frac{4}{(2t - 1)^2}$ . By Proposition 3.1, we have the following inequalities:

$$\begin{aligned} & |S_{KL}(\mu_1, \mu_2) - (2\zeta - 1) \log(2\zeta - 1) - 2(1 - \zeta)(\log(2\zeta - 1) + 1)| \\ & = |S_{KL}(\mu_1, \mu_2) - \log(2\zeta - 1) + 2(\zeta - 1)| \\ & \leq \frac{1}{2} \left[ \sup_{t \in [r, R]} \frac{4}{(2t - 1)} \right] \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right] \\ & = \frac{2}{(2r - 1)} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right] \end{aligned}$$

for all  $\zeta \in [r, R]$ , and when  $\zeta = 1$ ,

$$0 \leq S_{KL}(\mu_1, \mu_2) \leq \frac{2}{(2r - 1)} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \quad (4.9)$$

Furthermore, by Proposition 3.2, we have the inequalities:

$$\begin{aligned} & \left| S_{KL}(\mu_1, \mu_2) - \log(2\zeta - 1) + (2\zeta - 1) + \frac{2(r + R - 1)}{(2r - 1)(2R - 1)} \left[ \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) + (\zeta - 1)^2 \right] \right| \\ & \leq \frac{2(R - r)}{(2r - 1)(2R - 1)} \left[ \frac{1}{4} S_{\chi^2, 2\zeta - 1}(\mu_1, \mu_2) \right] \end{aligned}$$

for  $\zeta \in [r, R]$ , and when  $\zeta = 1$ ,

$$\begin{aligned} & \left| S_{KL}(\mu_1, \mu_2) + \frac{2(r + R - 1)}{(2r - 1)(2R - 1)} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2) \right| \\ & \leq \frac{2(R - r)}{(2r - 1)(2R - 1)} \frac{1}{4} S_{\chi^2}(\mu_1, \mu_2). \end{aligned} \quad (4.10)$$

## 5. Conclusions

In this paper, we have been derived new information inequalities of Jensen-Ostrowski type, by considering two above inequalities of Section 2, new  $f$ -divergence and Chi-divergences. Particular cases of these inequalities have been also established in terms of divergences like as Kullback-Leibler divergence, Relative Jensen-Shannon divergence, Relative arithmetic-geometric divergence and Triangular discrimination.

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