TAMKANG JOURNAL OF MATHEMATICS
Volume 45, Number 3, 275-284, September 2014
doi:10.5556/j.tkjm.45.2014.1072

Available online at http://journals.math.tku.edu.tw/

FRACTIONAL DIFFERENTIAL SUPERORDINATION

RABHA W. IBRAHIM

Abstract. The notion of differential superordination was introduced by S.S. Miller and P.T. Mocanu as a dual concept of differential subordination. Recently, in Tamkang J. Math. [7], the author have introduced the notion of fractional differential subordination. In this work, we consider the dual problem of determining properties of analytic functions that satisfy the fractional differential superordination. By employing some types of admissible functions, involving differential operator of fractional order, we illustrate geometric properties such as starlikeness and convexity for a class of analytic functions in the unit disk. Moreover, applications are posed in the sequel.

1. Introduction

Recently, the theory of fractional calculus has found interesting applications in the theory of analytic functions. The classical definitions of fractional operators and their generalizations have fruitfully been applied in obtaining, for example, the characterization properties, coefficient estimates [1], distortion inequalities [2] and convolution structures for various subclasses of analytic functions and the works in the research monographs. In [3], Srivastava and Owa, gave definitions for fractional operators (derivative and integral) in the complex z-plane $\mathbb{C}$ as follows:

Definition 1.1. The fractional derivative of order α is defined, for a function $f(z)$ by

$$D_z^\alpha f(z) := \frac{1}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z-\zeta)^\alpha} d\zeta; \ 0 \le \alpha < 1,$$

where the function $f(z)$ is analytic in simply-connected region of the complex z-plane $\mathbb{C}$ containing the origin and the multiplicity of $(z-\zeta)^{-\alpha}$ is removed by requiring $log(z-\zeta)$ to be real when$(z-\zeta) > 0$.

Received December 15, 2011, accepted November 14, 2012.
2010 *Mathematics Subject Classification*. 30C45.
Key words and phrases. Fractional calculus, analytic function, admissible function, fractional differential equation, subordination, superordination, fractional differential subordination, unit disk; bounded turning function, univalent function.

Definition 1.2. The fractional integral of order α is defined, for a function $f(z)$, by

$$I_z^\alpha f(z) := \frac{1}{\Gamma(\alpha)} \int_0^z f(\zeta)(z-\zeta)^{\alpha-1} d\zeta; \ \ \alpha > 0,$$

where the function $f(z)$ is analytic in simply-connected region of the complex z-plane ($\mathbb{C}$) containing the origin and the multiplicity of $(z-\zeta)^{\alpha-1}$ is removed by requiring $log(z-\zeta)$ to be real when $(z-\zeta) > 0$.

Remark 1.1. From Definitions 1.1 and 1.2, we have

$$D_z^\alpha \{z^\mu\} = \frac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)} \{z^{\mu-\alpha}\}, \qquad \mu > -1;\ 0 \le \alpha < 1$$

and

$$I_z^\alpha \{z^\mu\} = \frac{\Gamma(\mu+1)}{\Gamma(\mu+\alpha+1)} \{z^{\mu+\alpha}\}, \qquad \mu > -1;\ \alpha > 0.$$

Further properties of these operators with applications can be found in [2]–[5].

2. Preliminaries

Let $\mathcal{H}$ be the class of functions analytic in the unit disk $U = \{z : |z| < 1\}$ and for $a \in \mathbb{C}$ (set of complex numbers) and $n \in \mathbb{N}$ (set of natural numbers), let $\mathcal{H}[a,n]$ be the subclass of $\mathcal{H}$ consisting of functions of the form $f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \cdots$. Let $\mathcal{A}$ be the class of functions f, analytic in U and normalized by the conditions $f(0) = f'(0) - 1 = 0$. A function $f \in \mathcal{A}$ is called starlike of order μ if it satisfies the following inequality

$$\Re\{\frac{zf'(z)}{f(z)}\} > \mu, \ (z \in U)$$

for some $0 \le \mu < 1$. We denoted this class $\mathcal{S}(\mu)$. A function $f \in \mathcal{A}$ is called convex of order μ if it satisfies the following inequality

$$\Re\{\frac{zf''(z)}{f'(z)} + 1\} > \mu, \ (z \in U)$$

for some $0 \le \mu < 1$. We denoted this class $\mathcal{C}(\mu)$. We note that $f \in \mathcal{C}(\mu)$ if and only if $zf' \in \mathcal{S}(\mu)$. Furthermore, Let $\mathcal{P}$ be the subclass of analytic functions in the unit disk and take the formula

$$p(z) = 1 + \sum_{n=1}^{\infty} a_n z^n, \quad \Re(p(z)) > 0, p(0) = 1.$$

Let f be analytic in U, g analytic and univalent in U and $f(0) = g(0)$. Then, by the symbol $f(z) \prec g(z)$ (f subordinate to g) in U, we shall mean $f(U) \subset g(U)$.

Let $\phi : \mathbb{C}^2 \to \mathbb{C}$ and let h be univalent in U. If p is analytic in U satisfying the differential subordination $\phi(p(z)), zp'(z)) \prec h(z)$ then p is called a solution of the differential subordination.

The univalent function q is called a dominant of the solutions of the differential subordination, $p \prec q$. If p and $\phi(p(z)), zp'(z))$ are univalent in U and satisfy the differential superordination $h(z) \prec \phi(p(z)), zp'(z))$ then p is called a solution of the differential superordination. An analytic function q is called subordinate of the solution of the differential superordination if $q \prec p$. For details (see [6]).

Analogs to this definition, Ibrahim [7] have imposed the concept of fractional differential subordination as follows

Definition 2.1. Let $\varphi : \mathbb{C}^2 \to \mathbb{C}$ and let h be univalent in U. If p is analytic in U satisfying the fractional differential subordination $\varphi\Big(p(z), z^\alpha D_z^\alpha p(z)\Big) \prec h(z)$, $0 \le \alpha < 1$, then p is called a solution of the fractional differential subordination. The univalent function q is called a dominant of the solutions of the fractional differential subordination, $p \prec q$. If p and $\varphi\Big(p(z), z^\alpha D_z^\alpha p(z)\Big)$ are univalent in U and satisfy the fractional differential superordination $h(z) \prec \varphi\Big(p(z), z^\alpha D_z^\alpha p(z)\Big)$ then p is called a solution of the fractional differential superordination. An analytic function q is called subordinate of the solution of the fractional differential superordination if $q \prec p$.

It is clear that when $\alpha = 0$, we have the differential subordination and the differential superordination of the first order. In the following sequel, we will assume that h is an analytic convex function in U with $h(0) = 1$. For $0 \le \alpha < 1$, consider the fractional differential equation

$$p(z) + \mu z^\alpha D_z^\alpha p(z) = g(z), \quad g(z) \prec h(z).$$

Definition 2.2 ([6])**.** We denote by Q the set of all functions $f(z)$ that are analytic and univalent on $\overline{U} - E(f)$ where $E(f) := \{\zeta \in \partial U : lim_{z \to \zeta} f(z) = \infty\}$ and are such that $f'(\zeta) \ne 0$ for $\zeta \in \partial U \setminus E(f)$. The subclass of Q for which $f(0) = a$ is denoted by $Q(a)$.

In this article we consider the dual problem of determining properties of analytic functions that satisfy the fractional differential superordination

$$h(z) \prec \varphi\Big(p(z), z^\alpha D_z^\alpha p(z)\Big).$$

Definition 2.3. Let Ω be a set in $\mathbb{C}$ and $q \in \mathcal{H}[a, n]$ be univalent in U. The class of admissible functions $\Phi_n[\Omega, q]$, consists of those functions $\varphi : \mathbb{C}^2 \to \mathbb{C}$, that satisfy the

$$\varphi\Big(q(z), z^\alpha D_z^\alpha q(z)\Big) \subset \Omega$$

and φ is univalent in U.

3. Fractional differential superordination

In this section, we establish some results which are related to the fractional superordination. In addition, we show that the problem of finding best subordinant of fractional superordination reduces to finding univalent solutions of fractional differential equations.

Theorem 3.1 ([7]). *Let $p(z) = a + a_n z^n + \cdots$ be analytic in U with $p(z) \neq a$ and $n \geq 1$, and let $q \in Q$ with $q(0) = a$. If p is not subordination to q, then there exist points $z_0 = r_0 e^{i\theta_0} \in U$ and $\zeta_0 \in \partial U \setminus E(q)$ for which $p(U_{r_0}) \subset q(U)$ and a real number m such that*

(i) $p(z_0) = q(\zeta_0)$

(ii) $z_0^\alpha D_{z_0}^\alpha p(z_0) = m\zeta_0 \sum_{k=0}^{\infty} \binom{\alpha}{k} q^{(k)}(\zeta_0)$,

where m is real.

Theorem 3.2. *Let $\Omega \subset \mathbb{C}$, $q \in \mathscr{H}[a,n]$, $\varphi : \mathbb{C}^2 \to \mathbb{C}$ and suppose that*

$$\varphi\Big(q(z), z^\alpha D_z^\alpha q(z)\Big) \in \Omega, \quad z \in U.$$

If $p \in Q(a)$ and $\varphi\Big(p(z), z^\alpha D_z^\alpha p(z)\Big)$ is univalent then

$$\Omega \subset \{\varphi\Big(p(z), z^\alpha D_z^\alpha p(z)\Big), z \in U\} \tag{1}$$

implies

$$q(z) \prec p(z), \quad z \in U.$$

Proof. Assume q not subordinate to p. By Theorem 3.1, there exist points $z_0 = r_0 e^{i\theta_0} \in U$ and $\zeta_0 \in \partial U \setminus E(q)$ for which $p(U_{r_0}) \subset q(U)$ and a real number m such that $p(z_0) = q(\zeta_0)$ and

$$z_0^\alpha D_{z_0}^\alpha p(z_0) = m\zeta_0 \sum_{k=0}^{\infty} \binom{\alpha}{k} q^{(k)}(\zeta_0),$$

where m is real, we obtain

$$\varphi\Big(p(\zeta_0), \zeta_0^\alpha D_{\zeta_0}^\alpha p(\zeta_0)\Big)$$

Since ζ_0 is a boundary point we deduce a contradiction with the assumption (1), and we have $q(z) \prec p(z)$.

We next consider the special case, when h is analytic on U and $h(U) = \Omega \neq \mathbb{C}$. In this case, we have the class $\phi_n[h,q]$ and the following result is an immediate consequence of Theorem 3.2.

Theorem 3.3. *Let h be analytic in U, $q \in \mathcal{H}[a,n]$, $\varphi : \mathbb{C}^2 \to \mathbb{C}$ and suppose that*

$$\varphi\Big(q(z), z^{\alpha} D_z^{\alpha} q(z)\Big) \in h(U), \quad z \in U.$$

If $p \in Q(a)$ and $\varphi(p(z), z^{\alpha} D_z^{\alpha} p(z))$ is univalent then

$$h(z) \prec \varphi\Big(p(z), z^{\alpha} D_z^{\alpha} p(z)\Big) \tag{2}$$

implies

$$q(z) \prec p(z), \quad z \in U.$$

Next we consider another fractional differential superordination takes the form

$$h(z) \prec A(z)p(z) + B(z)z^{\alpha} D_z^{\alpha} p(z) \tag{3}$$

where h is analytic in U, and $A(z)p(z) + B(z)z^{\alpha} D_z^{\alpha} p(z)$ is univalent in U.

Remark 3.1. If $A(z) = B(z) = 1$ and $\alpha = 1$, then (3) becomes

$$h(z) \prec p(z) + zp'(z)$$

a differential superordination studied by S.S. Miller and P.T. Mocanu in [8].

Theorem 3.4. *Let h be analytic in U, $h(0) = 0$, $q \in \mathcal{H}[0,n]$ be convex in U, $\varphi : \mathbb{C}^2 \to \mathbb{C}$ and suppose that*

$$\varphi\Big(q(z), z^{\alpha} D_z^{\alpha} q(z)\Big) \in h(U), \quad z \in U.$$

If $p \in \mathcal{H}[0,1] \cap Q$ and $z^{\alpha} D_z^{\alpha} p(z)B(z)$ is univalent in U then

$$h(z) \prec z^{\alpha} D_z^{\alpha} p(z)B(z) \tag{4}$$

implies

$$q(z) \prec p(z), \quad z \in U,$$

where

$$q(z) = I_z^{\alpha} \frac{h(z)}{z^{\alpha}}. \tag{5}$$

Proof. Operating (5) by D_z^{α} we obtain

$$z^{\alpha} D_z^{\alpha} = h(z), \quad z \in U.$$

Let $\varphi\Big(p(z), z^{\alpha} D_z^{\alpha} p(z)\Big) := z^{\alpha} D_z^{\alpha} p(z)B(z)$, then (4) becomes

$$h(z) \prec \varphi\Big(p(z), z^{\alpha} D_z^{\alpha} p(z)\Big).$$

In view of Theorem 3.3, we have the desired result.

Remark 3.2. For $B(z)=1, \alpha=1$, the result was obtained in [8, Theorem 9].

Example 3.1. Let $h(z)=z^{\alpha}$, from Theorem 3.4, we have $q(z)=\frac{z^{\alpha}}{\Gamma(\alpha+1)}$. If $p\in\mathscr{H}[0,1]$ and $z^{\alpha}D_z^{\alpha}p(z)B(z)$ is univalent in U then

$$z^{\alpha}\prec z^{\alpha}D_z^{\alpha}p(z)B(z) \tag{6}$$

implies

$$\frac{z^{\alpha}}{\Gamma(\alpha+1)}\prec p(z), \quad z\in U.$$

Theorem 3.5. *Let h be analytic in $U, h(0)=0, q\in\mathscr{H}[0,n]$ be convex in U, $\varphi:\mathbb{C}^2\to\mathbb{C}$ and suppose that*

$$\varphi\Big(q(z), z^{\alpha}D_z^{\alpha}q(z)\Big)\in h(U), \quad z\in U.$$

If $p\in\mathscr{H}[0,1]\cap Q$ and $p(z)+\frac{z^{\alpha}D_z^{\alpha}p(z)A(z)}{\gamma}$ is univalent in U then

$$h(z)\prec p(z)+\frac{z^{\alpha}D_z^{\alpha}p(z)A(z)}{\gamma} \tag{7}$$

implies

$$q(z)\prec p(z),$$

where $z\in U, \gamma\neq 0$ and

$$q(z)=\gamma I_z^{\alpha}[\frac{h(z)-q(z)}{z^{\alpha}}]. \tag{8}$$

Proof. Operating (7) by D_z^{α} we obtain

$$h(z)=q(z)+\frac{z^{\alpha}D_z^{\alpha}q(z)}{\gamma}, \quad z\in U.$$

Let

$$\varphi\Big(p(z), z^{\alpha}D_z^{\alpha}p(z)\Big):=p(z)+\frac{z^{\alpha}D_z^{\alpha}p(z)A(z)}{\gamma},$$

then (6) becomes

$$h(z)\prec\varphi\Big(p(z), z^{\alpha}D_z^{\alpha}p(z)\Big).$$

In virtue of Theorem 3.3, implies that $q(z)\prec p(z)$.

Remark 3.3. For $A(z)=1, \alpha=1$, the result was obtained in [8, Theorem 6].

Example 3.2. Let $\gamma=1, h(z)=(\frac{1}{\Gamma(2-\alpha)}+1)z$, from Theorem 3.5, we have $q(z)=z$. If $p\in\mathscr{H}[0,1]$ and $p(z)+z^{\alpha}D_z^{\alpha}p(z)A(z)$ is univalent in U then

$$(\frac{1}{\Gamma(2-\alpha)}+1)z\prec p(z)+z^{\alpha}D_z^{\alpha}p(z)A(z) \tag{9}$$

implies

$$z\prec p(z), \quad z\in U.$$

In general we have the following example:

Example 3.3. Assume that $\gamma = 1$, and

$$h(z) = (\frac{\Gamma(\mu+1)}{\Gamma(\mu+1-\alpha)} + 1)z^{\mu}$$

from Theorem 3.5, we have $q(z) = z^{\mu}$. If $p \in \mathcal{H}[0,1]$ and $p(z) + z^{\alpha} D_z^{\alpha} p(z) A(z)$ is univalent in U then

$$(\frac{\Gamma(\mu+1)}{\Gamma(\mu+1-\alpha)} + 1)z^{\mu} \prec p(z) + z^{\alpha} D_z^{\alpha} p(z) A(z) \tag{10}$$

implies

$$z^{\mu} \prec p(z),$$

where $z \in U$, $\mu > -1$.

Example 3.4. Assume that $\gamma = 1$, and

$$h(z) = (\frac{1}{\Gamma(2-\alpha)} + 1)z + (\frac{2!}{\Gamma(3-\alpha)} + 1)z^2$$

from Theorem 3.5, we have $q(z) = z + z^2$. Since $\Re(2 - \frac{1}{2z+1}) > 0$ yields that q is convex in U. If $p \in \mathcal{H}[0,1]$ and $p(z) + z^{\alpha} D_z^{\alpha} p(z) A(z)$ is univalent in U then

$$(\frac{1}{\Gamma(2-\alpha)} + 1)z + (\frac{2!}{\Gamma(3-\alpha)} + 1)z^2 \prec p(z) + z^{\alpha} D_z^{\alpha} p(z) A(z) \tag{11}$$

implies

$$z + z^2 \prec p(z).$$

4. Applications

In this section, we deduce some applications of Theorems 3.2, 3.3 for special functions involving starlike, convex and bounded turning functions.

Let $\mathcal{A}$ be the subclass of $\mathcal{H}[a,n]$ consisting of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in U. \tag{12}$$

By letting $p(z) = \frac{f(z)}{z}$, $p(z) = \frac{zf'(z)}{f(z)}$, $p(z) = 1 + \frac{zf''(z)}{f'(z)}$, $p(z) = f'(z)$ in Theorem 3.3, we have the following results:

Theorem 4.6. *Let $\varphi \in \Phi[\Omega, q]$ with $q(0) = 1$. If $f \in \mathcal{A}$ satisfies*

$$\Omega \subset \Big\{\varphi\Big(\frac{f(z)}{z}, z^\alpha D_z^\alpha \frac{f(z)}{z}; z\Big), z \in U\Big\},$$

or

$$h(z) \prec \varphi\Big(\frac{f(z)}{z}, z^\alpha D_z^\alpha \frac{f(z)}{z}\Big), \quad z \in U,$$

where $h : U \to \Omega$ is a conformal mapping such that $\Omega = h(U)$, then $q(z) \prec \frac{f(z)}{z}$.

Theorem 4.7. *Let $\varphi \in \Phi[\Omega, q]$ with $q(0) = 1$. If $f \in \mathcal{A}$ satisfies*

$$\Omega \subset \Big\{\varphi\Big(\frac{zf'(z)}{f(z)}, z^\alpha D_z^\alpha \frac{zf'(z)}{f(z)}; z\Big)\Big\},$$

or

$$h(z) \prec \varphi\Big(\frac{zf'(z)}{f(z)}, z^\alpha D_z^\alpha \frac{zf'(z)}{f(z)}\Big), \quad z \in U,$$

where $h : U \to \Omega$ is a conformal mapping such that $\Omega = h(U)$, then $q(z) \prec \frac{zf'(z)}{f(z)}$.

Theorem 4.8. *Let $\varphi \in \Phi[\Omega, q]$ with $q(0) = 1$. If $f \in \mathcal{A}$ satisfies*

$$\Omega \subset \Big\{\varphi\Big(1 + \frac{zf''(z)}{f'(z)}, z^\alpha D_z^\alpha \Big(1 + \frac{zf''(z)}{f'(z)}\Big); z\Big)\Big\},$$

or

$$h(z) \prec \varphi\Big(1 + \frac{zf''(z)}{f'(z)}, z^\alpha D_z^\alpha \Big(1 + \frac{zf''(z)}{f'(z)}\Big)\Big), \quad z \in U,$$

where $h : U \to \Omega$ is a conformal mapping such that $\Omega = h(U)$, then $q(z) \prec 1 + \frac{zf''(z)}{f'(z)}$.

Theorem 4.9. *Let $\varphi \in \Phi[\Omega, q]$ with $q(0) = 1$. If $f \in \mathcal{A}$ satisfies*

$$\Omega \subset \{\varphi\Big(z^\alpha D_z^\alpha f'(z)\Big)\},$$

or

$$h(z) \prec \varphi\Big(f'(z), z^\alpha D_z^\alpha f'(z)\Big), \quad z \in U,$$

where $h : U \to \Omega$ is a conformal mapping such that $\Omega = h(U)$, then $q(z) \prec f'(z)$.

Remark 4.1. Consequently from Theorems 4.1–4.4 in [7], we have the following sandwich relations:

$$q_1(z) \prec \frac{f(z)}{z} \prec q_2(z)$$

$$q_1(z) \prec \frac{zf'(z)}{f(z)} \prec q_2(z)$$

$$q_1(z) \prec 1 + \frac{zf''(z)}{f'(z)} \prec q_2(z)$$

and

$$q_1(z) \prec f'(z) \prec q_2(z).$$

Finally, as a direct application of Theorem 3.2, we pose the following result

Theorem 4.10. *Let*

$$r < \frac{1}{\left[1 + \frac{1}{\Gamma(1-\alpha)} \int_0^1 \frac{1+rt}{(1-t)^\alpha (1-rt)^3} dt\right]^{1/(1-\alpha)}}, \quad 0 < \alpha < 1.$$

If $p \in \mathcal{H}[r,1] \cap Q$ *and* $p(z) + z^\alpha D_z^\alpha p(z)$ *are univalent in* U *then*

$$U \subset \{p(z) + z^\alpha D_z^\alpha p(z), \, z \in U\}$$

implies

$$U_r \subset p(U).$$

Proof. Since p is univalent in U then in view of [2, Theorem 3], we have

$$|D_z^\alpha p(z)| \leq \frac{r^{1-\alpha}}{\Gamma(1-\alpha)} \int_0^1 \frac{1+rt}{(1-t)^\alpha (1-rt)^3} dt, \quad 0 < \alpha < 1.$$

Assume

$$q(z) = r^{1-\alpha} z, \quad z \in U,$$

then q is univalent in U. We evaluate

$$\begin{aligned} \left|\varphi\Big(q(z), z^\alpha D_z^\alpha q(z)\Big)\right| &= \left|q(z) + z^\alpha D_z^\alpha q(z)\right| \\ &\leq r^{1-\alpha} + \frac{r^{1-\alpha}}{\Gamma(1-\alpha)} \int_0^1 \frac{1+rt}{(1-t)^\alpha (1-rt)^3} dt \\ &= r^{1-\alpha}\left[1 + \frac{1}{\Gamma(1-\alpha)} \int_0^1 \frac{1+rt}{(1-t)^\alpha (1-rt)^3} dt\right] \\ &< 1. \end{aligned}$$

Since $\varphi\Big(q(z), z^\alpha D_z^\alpha q(z)\Big) \in U$ and $U \subset \{p(z) + z^\alpha D_z^\alpha p(z), \, z \in U\}$ by applying Theorem 3.2, we obtain

$$q(z) \prec p(z), \quad \text{i.e.} \quad U_r \subset p(U).$$

References

[1] M. Darus and R. W. Ibrahim, *Radius estimates of a subclass of univalent functions*, Math. Vesnik, **63**(2011), No.1, 55-58.

[2] H. M. Srivastava , Y. Ling and G. Bao, *Some distortion inequalities assotiated with the fractional drivatives of analytic and univalent functions*, J. Ineq. Pure and Applied Math., **2**(2001), No.2, 1–6.

[3] H. M. Srivastava and S. Owa, Univalent Functions, Fractional Calculus, and Their Applications, Halsted Press, John Wiley and Sons, New York, Chichester, Brisban, and Toronto, 1989.

[4] R. W. Ibrahim, M. Darus, *Subordination and superordination for analytic functions involving fractional integral operator*, Complex Variables and Elliptic Equations, **53** (2008), 1021–1031.

[5] R. W. Ibrahim, M. Darus, *Subordination and superordination for univalent solutions for fractional differential equations*, J. Math. Anal. Appl., **345** (2008), 871–879.

[6] S. S. Miller and P. T. Mocanu, Differential Subordinantions: Theory and Applications. Pure and Applied Mathematics No.225 Dekker, New York, 2000.

[7] R. W. Ibrahim, *The concept of fractional differential subordination*, Tamkang J. Of Math., **44**(2013), 53–60.

[8] S. S. Miller and P. T. Mocanu, *Subordinants of differential superordinations, Complex Variables*, **48**(2003), 815–826.

Institute of Mathematical Sciences, University Malaya, 50603, Kuala Lumpur, Malaysia.

E-mail: rabhaibrahim@yahoo.com