TAMKANG JOURNAL OF MATHEMATICS
Volume 44, Number 2, 113-122, Summer 2013
doi:10.5556/j.tkjm.44.2013.1126
Available online at http://journals.math.tku.edu.tw/

COMMENT ON "EXACT BAYESIAN VARIABLE SAMPLING PLANS FOR THE EXPONENTIAL DISTRIBUTION BASED ON TYPE-I HYBRID CENSORED SAMPLES" AND "ITS CORRECTIONS"

TACHEN LIANG

Abstract. We compare the performances of two sampling plans, namely, the Lin-Liang-Huang (2002)'s Bayesian sampling plan (n^*, ξ^*) and the Lin-Huang-Balakrishnan (2008a, 2010a)'s exact Bayesian sampling plan (n_0, r_0, t_0, ξ_0). We also comment the accuracy of the values of the design parameters (n_0, r_0, t_0, ξ_0) provided in Lin-Huang-Balakrishnan (2010a). We conclude that among the class of sampling plans (n, r, t, ξ) of Lin et al. (2008a, 2010a), the exact Bayesian sampling plan does not exist.

1. Introduction

Recently, Lin, et al. (2008a, 2008b, 2010a, 2010b, 2011), Huang (2010) and Lin and Huang (2011) have studied the problem of acceptance sampling for exponential distributions. Sampling plans have been designed based on different censoring schemes. However, those papers are carelessly written. In the papers, incorrect approaches are applied. The so claimed exact Bayesian sampling plans are not the true Bayesian sampling plans and the provided tables contains too many serious computational errors. We will present papers to discuss the related problems of acceptance sampling and comment the accuracy of those sampling plans proposed in the above mentioned papers. In this paper, we shall comment the results presented in Lin et al. (2008a, 2010a). Our comments are based on the Bayesian concept and the Bayesian analysis method introduced in Berger (1985).

Recently, Lin et al. (2008a) have studied the problem of acceptance sampling for exponential distributions based on type-I hybrid censored sample. Their method is described as follows.

Suppose we are given a batch of lifetime components for acceptance sampling. A sample of size n items is put on life test at the outset. Failed items are not replaced. We let $X_1, \ldots, X_n$

2010 *Mathematics Subject Classification*. 62F15; 62C10.
Key words and phrases. Acceptance sampling, Bayes risk, decision function, loss function, hybrid type-I censored sample, sampling plan.

denote the lifetimes of these n components. It is assumed that $X_1,\ldots,X_n$ are mutually independent, follow an exponential distribution, having expected lifetime $\theta = 1/\lambda$. It is assumed that the parameter λ follows a gamma (α,β) prior distribution. Let C_s denote the cost per item inspected. Let d denote an action on this problem of acceptance sampling. When $d = 1$, it means that the batch is accepted; and when $d = 0$, it means to reject the batch. We let C_r be the loss of rejecting the batch and let $h(\lambda) = a_0 + a_1\lambda + a_2\lambda^2$ be the loss of accepting the batch, where $a_0 \geq 0$, $a_1 \geq 0$, and $a_2 > 0$. The following loss function is considered:

$$L(d,\lambda,n) = dh(\lambda) + (1-d)C_r + nC_s. \tag{1}$$

Lin et al. (2008a) proposed using type-I hybrid censoring scheme to collect data from the sample. Let $Y_1,\ldots,Y_n$ be the ordered statistics of $X_1,\ldots,X_n$. For a given time $t > 0$ and an integer r, $1 \leq r \leq n$, type-I hybrid censoring scheme (type-IHCS) terminates the life-testing experiment at a random time $\tau \equiv \tau(n,r,t) = \min(Y_r,t)$. Define $D \equiv D(n,r,t) = \max\{j \mid 1 \leq j \leq r$, and $Y_j \leq t\} = \max\{j \mid Y_j \leq \tau\}$ if $Y_1 \leq \tau$; and $D = 0$ if $Y_1 > \tau$. D is the number of failures at the end of the life testing experiment. We denote the observable variable by $(D,\underline{Y}(D))$, where $\underline{Y}(D) = (Y_1,\ldots,Y_D)$. Let $\widetilde{\theta}_{ML}(D)$ denote the MLE of θ based on type-I hybrid censored sample $(D,\underline{Y}(D))$. Lin et al. (2008a) studied a type of sampling plans based on $\widetilde{\theta}_{ML}(D)$, in which, the associated decision function δ^{LHB} is given by:

If $D > 0$, then, $\delta^{LHB}(D,\underline{Y}(D)) = 1$ if $\widetilde{\theta}_{ML}(D) \geq \xi$; and 0, otherwise.
If $D = 0$, then no decision is made.

(2)

A sampling plan is a determination of the values of the parameters (n,r,t,ξ) or (n,r,t,δ^{LHB}). Using the loss function $L(d,\lambda,n)$ of (1), Lin et al.(2008a) attempted to find the best sampling plan, say $(n_0,r_0,t_0,\delta_0^{LHB})$ or (n_0,r_0,t_0,ξ_0), which will minimize the Bayes risks $R(n,r,t,\delta^{LHB})$ among all sampling plans of the types (n,r,t,ξ) described in (2).

Lin et al. (2008a) have provided tables for the values of the design parameters (n_0,r_0,t_0,ξ_0) and the associated Bayes risk $R_0 = R(n_0,r_0,t_0,\xi_0)$. However, they made serious computational errors. Most of the provided values of the design parameters (n_0,r_0,t_0,ξ_0) of their so claimed the exact Bayesian sampling plans are unreasonable and incorrect. Later, Lin et al. (2010a) have made “corrections" on those errors, and provided new “corrected" values for (n_0,r_0,t_0,ξ_0) and R_0.

However, we have observed that the type of sampling plans (n,r,t,ξ) possesses certain defects. (a) The type of decision functions δ^{LHB} of (2) is not derived through a Bayesian analysis, and is not a Bayes decision function. Thus, $(n_0,r_0,t_0,\delta_0^{LHB})$ is not a Bayesian sampling plan; (b) Since the cost of experimental time used for life test experiment is not included in the loss, for n being fixed, the best choice of (r,t) should be: $r = n$ and $t = \infty$, so that we are able to observe the complete lifetime data of the n components and obtain the most information about the expected lifetime $\theta = 1/\lambda$. Based on the complete lifetime data, we can make a suitable decision to reduce the cost of making a wrong decision. In this note, we shall consider a competitor, the Bayesian sampling plan of Lin et al. (2002). In Section 2, we shall compare the performances of the two sampling plans. In Section 3, we comment the accuracy of the values of (n_0, r_0, t_0, ξ_0) provided in Lin et al.(2010a). A concluding remark is given in Section 4. We conclude that among the class of sampling plans (n, r, t, ξ) of Lin et al. (2008a, 2010a), the Bayesian sampling plan does not exist.

2. Comparing the performances of two sampling plans

Lin et al.(2002) have studied the problem of deriving the Bayesian sampling plans (n^*, δ^*) for the problem of acceptance sampling using the loss function $L(d, \lambda, n)$ of (1) based on complete observations. Let $\widetilde{\theta}_{ML}$ denote the MLE of the expected lifetime θ based on the complete observations. Let

$$\xi^* = \max\Big(\frac{a_1(n^*+\alpha) + \sqrt{a_1^2(n^*+\alpha)^2 + 4(C_r - a_0)a_2(n^*+\alpha)(n^*+\alpha+1)}}{2(C_r - a_0)} - \beta, 0\Big)/n^*.$$

The Lin et al.(2002)'s Bayesian decision function δ^* can be presented as:

$$\delta^*(\widetilde{\theta}_{ML}) = 1 \text{ if } \widetilde{\theta}_{ML} \geq \xi^*\text{; and 0, otherwise.} \tag{3}$$

Due to (3), we may also denote the Bayesian sampling plan (n^*, δ^*) by (n^*, ξ^*) and its corresponding Bayes risk by $R^* = R(n^*, \xi^*)$.

In the following, we provide numerical comparison on the performances of the two sampling plans (n_0, r_0, t_0, ξ_0) and (n^*, ξ^*). In Tables 1-6, we list the values of (n_0, r_0, t_0, ξ_0), R_0, (n^*, ξ^*) and R^*. The values of the entries (n_0, r_0, t_0, ξ_0) and R_0 are cited from Lin et al. (2010a). Since Lin et al. (2002) haven't provided the values of (n^*, ξ^*) and R^*, the IMSL, STAT/LIBRARY (1995) is employed to the Fortran program for searching the optimal design parameters (n^*, ξ^*) and computing the value of the Bayes risk R^*. The numerical results indicate that R^* is smaller than R_0 in all cases. In all cases studied, we observe that $n_0 = r_0$. Also, the Bayesian sampling plan (n^*, ξ^*) allows no sample situation, that is, n^* can be 0, (see Table 1, $(\alpha, \beta) = (2.5, 1.2)$ case, Table 4, $a = 0.5$ case, and Table 6, $C_r = 10$ and 100 cases), while the sampling plan (n_0, r_0, t_0, ξ_0) requests n_0 to be at least one.

3. On the accuracy of (n_0, r_0, t_0, ξ_0)

Lin et al. (2008a,2010a) claimed that their sampling plans (n_0, r_0, t_0, ξ_0) are the Bayesian sampling plans among all sampling plans having the type of decision function as defined in

Table 1: Numerical comparison on the performance of (n^*, ξ^*) and (n_0, r_0, t_0, ξ_0) with $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, $a_2 = 2$, and (α, β) varying.

		Lin et al.(2002)		Lin et al.(2010a)	
α	β	(n^*, ξ^*)	R^*	(n_0, r_0, t_0, ξ_0)	R_0
2.5	0.4	(1, 0.7930)	29.7506	(2, 2, 1.1207, 0.5603)	29.8119
2.5	0.6	(3, 0.4022)	27.7267	(3, 3, 0.8537, 0.4268)	27.8193
1.5	0.8	(3, 0.2333)	16.5825	(3, 3, 0.5262, 0.2631)	16.7533
2.0	0.8	(4, 0.2899)	21.1398	(4, 4, 0.6051, 0.3026)	21.2875
2.5	0.8	(4, 0.3281)	24.8419	(4, 4, 0.6808, 0.3404)	24.9893
3.0	0.8	(3, 0.3865)	27.5581	(3, 3, 0.8170, 0.4085)	27.6521
3.5	0.8	(2, 0.5032)	29.2789	(2, 2, 1.0037, 0.5019)	29.3642
2.5	1.0	(4, 0.2781)	21.7081	(4, 4, 0.5819, 0.2910)	21.8515
2.5	1.2	(0, 0.0)	18.3194	(3, 3, 0.4158, 0.2079)	18.7384
10.0	3.0	(2, 0.3970)	29.5166	(2, 2, 0.8194, 0.4097)	29.5959

Table 2: Numerical comparison on the performance of (n^*, ξ^*) and (n_0, r_0, t_0, ξ_0) with $(\alpha, \beta) = (2.5, 0.8)$, $C_s = 0.5$, $C_r = 30$, $a_1 = 2$, $a_2 = 2$ and a_0 varying.

	Lin et al.(2002)		Lin et al.(2010a)	
a_0	(n^*, ξ^*)	R^*	(n_0, r_0, t_0, ξ_0)	R_0
0.1	(4, 0.3091)	23.8394	(4, 4, 0.6539, 0.3269)	23.9966
0.5	(4, 0.3129)	24.0549	(4, 4, 0.6539, 0.3269)	24.2101
1.5	(4, 0.3229)	24.5833	(4, 4, 0.6808, 0.3404)	24.7318
2	(4, 0.3281)	24.8419	(4, 4, 0.6808, 0.3404)	24.9893
3	(4, 0.3391)	25.3474	(3, 3, 0.7346, 0.3673)	25.4818
5	(3, 0.3751)	26.2801	(3, 3, 0.7884, 0.3942)	26.3831
10	(3, 0.4619)	28.2637	(3, 3, 0.9768, 0.4884)	28.3359

Table 3: Numerical comparison on the performance of (n^*, ξ^*) and (n_0, r_0, t_0, ξ_0) with $(\alpha, \beta) = (2.5, 0.8)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_2 = 2$ and a_1 varying.

	Lin et al.(2002)		Lin et al.(2010a)	
a_1	(n^*, ξ^*)	R^*	(n_0, r_0, t_0, ξ_0)	R_0
0.1	(4, 0.2694)	22.6418	(4, 4, 0.5732, 0.2866)	22.8050
0.5	(4, 0.2813)	23.1538	(4, 4, 0.6001, 0.3000)	23.3141
1.5	(4, 0.3121)	24.3181	(4, 4, 0.6539, 0.3269)	24.4684
2	(4, 0.3281)	24.8419	(4, 4, 0.6808, 0.3404)	24.9893
3	(4, 0.3616)	25.7826	(3, 3, 0.7884, 0.3942)	25.8993
5	(3, 0.4543)	27.2215	(3, 3, 0.9768, 0.4884)	27.3145
10	(2, 0.7786)	29.2149	(2, 2, 1.5956,0.7978)	29.3100

Table 4: Numerical comparison on the performance of (n^*, ξ^*) and (n_0, r_0, t_0, ξ_0) with $(\alpha, \beta) = (2.5, 0.8)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$ and a_2 varying.

	Lin et al.(2002)		Lin et al.(2010a)	
a_2	(n^*, ξ^*)	R^*	(n_0, r_0, t_0, ξ_0)	R_0
0.5	(0, 0)	15.0859	(2, 2, 0.0081, 0.0041)	16.1299
1	(3, 0.1811)	20.8052	(3, 3, 0.3848, 0.1924)	20.9471
1.5	(4, 0.2662)	23.2670	(4, 4, 0.5463, 0.2731)	23.4151
2	(4, 0.3281)	24.8419	(4, 4, 0.7077, 0.3539)	24.9893
3	(3, 0.4545)	26.7561	(3, 3, 0.9768, 0.4884)	26.8623
5	(3, 0.4536)	28.5115	(3, 3, 1.3804, 0.6902)	28.5944
10	(1, 1.7000)	29.8049	(2, 2, 2.3759, 1.1879)	29.8832

Table 5: Numerical comparison on the performance of (n^*, ξ^*) and (n_0, r_0, t_0, ξ_0) with $(\alpha, \beta) = (2.5, 0.8)$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, $a_2 = 2$, and C_s varying.

	Lin et al.(2002)		Lin et al.(2010a)	
C_s	(n^*, ξ^*)	R^*	(n_0, r_0, t_0, ξ_0)	R_0
0.1	(10, 0.3165)	22.5339	(11, 11, 0.6270, 0.3135)	22.6644
0.3	(5, 0.3237)	23.9619	(5, 5, 0.6808, 0.3404)	24.1174
0.4	(4, 0.3281)	24.4419	(4, 4, 0.6808, 0.3404)	24.5893
0.5	(4, 0.3281)	24.8419	(4, 4, 0.6808, 0.3404)	24.9893
0.6	(3, 0.3355)	25.1847	(3, 3, 0.7077, 0.3539)	25.3067
1.0	(2, 0.3500)	26.2298	(3, 3, 0.7077, 0.3539)	26.5067
2.0	(1, 0.3930)	27.7605	(2, 2, 0.7077, 0.3539)	28.5744

Table 6: Numerical comparison on the performance of (n^*, ξ^*) and (n_0, r_0, t_0, ξ_0) with $(\alpha, \beta) = (2.5, 0.8)$, $C_s = 0.5$, $a_0 = 2$, $a_1 = 2$, $a_2 = 2$, and C_r varying.

	Lin et al.(2002)		Lin et al.(2010a)	
C_r	(n^*, ξ^*)	R^*	(n_0, r_0, t_0, ξ_0)	R_0
10	(0 ,)	10.0000	(1, 1, 2.6988, 1.3494)	10.4252
15	(1, 1.0490)	14.8625	(2, 2, 1.5687, 0.7844)	15.0499
20	(2, 0.5635)	18.8571	(2, 2, 1.1382, 0.5691)	18.9763
30	(4, 0.3281)	24.8419	(4, 4, 0.6808, 0.3404)	24.9893
40	(5, 0.2479)	28.9908	(5, 5, 0.5194, 0.2597)	29.2025
50	(5, 0.1987)	31.8898	(5, 5, 0.4117, 0.2059)	32.1352
100	(0, 0.0)	35.5938	(2, 2, 0.0081, 0.0041)	36.8491

(2) and achieve the minimum Bayes risk which is denoted by $R(n_0, r_0, t_0, \xi_0)$. However, such a claim is not true. We are able to find many such type of sampling plans which are better than the sampling plans (n_0, r_0, t_0, ξ_0). In the following, for each given (n_0, r_0) and given

Table 7: Numerical values of $\xi(n_0, r_0, t)$ and $R(n_0, r_0, t, \xi_1)$ for $(\alpha, \beta) = (2.5, 0.4)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, and $a_2 = 2$ case.

t	$\xi(n_0, r_0, t)$	$R(n_0, r_0, t, \xi_1)$
0.5000	0.4999	31.612509
1.0000	0.5418	29.830957
2.0000	0.5488	29.760890
3.0000	0.5497	29.753666
4.0000	0.5499	29.752197
5.0000	0.5500	29.751767
6.0000	0.5500	29.751609
8.0000	0.5500	29.751509
10.0000	0.5500	29.751482

$(n^*, \xi^*) = (1, 0.7930)$, $R^* = 29.750554$, $(n_0, r_0, t_0, \xi_0) = (2, 2, 1.1207, 0.5603)$, $R_0 = 29.8119$

Table 8: Numerical values of $\xi(n_0, r_0, t)$ and $R(n_0, r_0, t, \xi_1)$ for $(\alpha, \beta) = (2.5, 0.6)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, and $a_2 = 2$ case.

t	$\xi(n_0, r_0, t)$	$R(n_0, r_0, t, \xi_1)$
0.5000	0.3932	29.367186
1.0000	0.4008	27.750468
2.0000	0.4020	27.727979
3.0000	0.4021	27.726865
4.0000	0.4021	27.726705
5.0000	0.4021	27.726669
6.0000	0.4021	27.726658
8.0000	0.4021	27.726653
10.0000	0.4021	27.726651

$(n^*, \xi^*) = (3, 0.4022)$, $R^* = 27.726651$, $(n_0, r_0, t_0, \xi_0) = (3, 3, 0.8537, 0.4268)$, $R_0 = 27.8193$

$t > 0$, we let $\xi_1 = \xi_1(t) \equiv \xi(n_0, r_0, t)$ be the value of the best design parameter for the decision function of the type given in (2). We also denote the Bayes risk associated with the sampling plan (n_0, r_0, t, ξ_1) by $R(n_0, r_0, t, \xi_1)$. For cases considered in Table 1, certain numerical results of (n_0, r_0, t, ξ_1) and $R(n_0, r_0, t, \xi_1)$ are computed and given in Tables 7-16, respectively. In each case studied, the numerical output indicates that $R(n_0, r_0, t, \xi_1)$ decreases in t, and as $t > t_0$, $R(n_0, r_0, t, \xi_1) < R(n_0, r_0, t_0, \xi_0)$, which is a contradiction to the Lin et al. (2008a,2010a)'s claim that (n_0, r_0, t_0, ξ_0) is the exact Bayesian sampling plan. We have observed that: for cases in which $n_0 = n^*$, $R(n_0, r_0, t, \xi_1) \geq R^*$ for all $t > 0$, and $R(n_0, r_0, t, \xi_1)$ approximates R^* when t is sufficiently large; and for cases where $n_0 \neq n^*$, $R(n_0, r_0, t, \xi_1) > R^*$ for all $t > 0$, and $R(n_0, r_0, t, \xi_1)$ does not approximate R^* though t is very large, see Tables 7 and 15 .

Table 9: Numerical values of $\xi(n_0, r_0, t)$ and $R(n_0, r_0, t, \xi_1)$ for $(\alpha, \beta) = (1.5, 0.8)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, and $a_2 = 2$ case.

t	$\xi(n_0, r_0, t)$	$R(n_0, r_0, t, \xi_1)$
0.5000	0.2231	16.705377
1.0000	0.2314	16.602043
2.0000	0.2331	16.584421
3.0000	0.2333	16.582895
4.0000	0.2333	16.582607
5.0000	0.2333	16.582526
6.0000	0.2333	16.582497
8.0000	0.2333	16.582480
10.0000	0.2333	16.582475

$(n^*, \xi^*) = (3, 0.2333)$, $R^* = 16.582472$, $(n_0, r_0, t_0, \xi_0) = (3, 3, 0.5262, 0.2631)$, $R_0 = 16.7533$

Table 10: Numerical values of $\xi(n_0, r_0, t)$ and $R(n_0, r_0, t, \xi_1)$ for $(\alpha, \beta) = (2.5, 0.8)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, and $a_2 = 2$ case.

t	$\xi(n_0, r_0, t)$	$R(n_0, r_0, t, \xi_1)$
0.5000	0.2868	21.580729
1.0000	0.2896	21.144029
2.0000	0.2899	21.139957
3.0000	0.2899	21.139797
4.0000	0.2899	21.139778
5.0000	0.2899	21.139774
6.0000	0.2899	21.139773
8.0000	0.2899	21.139773
10.0000	0.2899	21.139773

$(n^*, \xi^*) = (4, 0.2899)$, $R^* = 21.139773$, $(n_0, r_0, t_0, \xi_0) = (4, 4, 0.6051, 0.3026)$, $R_0 = 21.2875$

Table 11: Numerical values of $\xi(n_0, r_0, t)$ and $R(n_0, r_0, t, \xi_1)$ for $(\alpha, \beta) = (2.5, 0.8)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, and $a_2 = 2$ case.

t	$\xi(n_0, r_0, t)$	$R(n_0, r_0, t, \xi_1)$
0.5000	0.3253	25.895917
1.0000	0.3279	24.849182
2.0000	0.3281	24.842000
3.0000	0.3281	24.841873
4.0000	0.3281	24.841861
5.0000	0.3281	24.841858
6.0000	0.3281	24.841858
8.0000	0.3281	24.841858
10.0000	0.3281	24.841857

$(n^*, \xi^*) = (4, 0.3281)$, $R^* = 24.841857$, $(n_0, r_0, t_0, \xi_0) = (4, 4, 0.6808, 0.3404)$, $R_0 = 24.9893$

Table 12: Numerical values of $\xi(n_0, r_0, t)$ and $R(n_0, r_0, t, \xi_1)$ for $(\alpha, \beta) = (3.0, 0.8)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, and $a_2 = 2$ case.

t	$\xi(n_0, r_0, t)$	$R(n_0, r_0, t, \xi_1)$
0.5000	0.3780	28.829113
1.0000	0.3853	27.577942
2.0000	0.3864	27.559127
3.0000	0.3865	27.558238
4.0000	0.3865	27.558125
5.0000	0.3865	27.558101
6.0000	0.3865	27.558095
8.0000	0.3865	27.558092
10.0000	0.3865	27.558091

$(n^*, \xi^*) = (3, 0.3865)$, $R^* = 27.558091$, $(n_0, r_0, t_0, \xi_0) = (3, 3, 0.8170, 0.4085)$, $R_0 = 27.6521$

Table 13: Numerical values of $\xi(n_0, r_0, t)$ and $R(n_0, r_0, t, \xi_1)$ for $(\alpha, \beta) = (3.5, 0.8)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, and $a_2 = 2$ case.

t	$\xi(n_0, r_0, t)$	$R(n_0, r_0, t, \xi_1)$
0.5000	0.6382	30.254936
1.0000	0.4965	29.366270
2.0000	0.5025	29.287118
3.0000	0.5031	29.280417
4.0000	0.5032	29.279298
5.0000	0.5032	29.279021
6.0000	0.5032	29.278933
8.0000	0.5032	29.278886
10.0000	0.5032	29.278876

$(n^*, \xi^*) = (2, 0.5032)$, $R^* = 29.278871$, $(n_0, r_0, t_0, \xi_0) = (2, 2, 1.0037, 0.5019)$, $R_0 = 29.3642$

Table 14: Numerical values of $\xi(n_0, r_0, t)$ and $R(n_0, r_0, t, \xi_1)$ for $(\alpha, \beta) = (2.5, 1.0)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, and $a_2 = 2$ case.

t	$\xi(n_0, r_0, t)$	$R(n_0, r_0, t, \xi_1)$
0.5000	0.2753	22.027740
1.0000	0.2779	21.711446
2.0000	0.2781	21.708202
3.0000	0.2781	21.708092
4.0000	0.2781	21.708080
5.0000	0.2781	21.708078
6.0000	0.2781	21.708078
8.0000	0.2781	21.708078
10.0000	0.2781	21.708078

$(n^*, \xi^*) = (4, 0.2781)$, $R^* = 21.708077$, $(n_0, r_0, t_0, \xi_0) = (4, 4, 0.5819, 0.2910)$, $R_0 = 21.8515$

4. Concluding remark

Lin et al.(2008a, 2010a) have studied the problem of acceptance sampling for exponential distributions using the loss function $L(d, \lambda, n)$. The hybrid type-I censoring scheme is used to

Table 15: Numerical values of $\xi(n_0, r_0, t)$ and $R(n_0, r_0, t, \xi_1)$ for $(\alpha, \beta) = (2.5, 1.2)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, and $a_2 = 2$ case.

t	$\xi(n_0, r_0, t)$	$R(n_0, r_0, t, \xi_1)$
0.5000	0.1932	18.677392
1.0000	0.2008	18.605391
2.0000	0.2020	18.594293
3.0000	0.2021	18.593585
4.0000	0.2021	18.593481
5.0000	0.2021	18.593456
6.0000	0.2021	18.593449
8.0000	0.2021	18.593445
10.0000	0.2021	18.593444

$(n^*, \xi^*) = (0, 0.00)$, $R^* = 18.319444$, $(n_0, r_0, t_0, \xi_0) = (3, 3, 0.4158, 0.2079)$, $R_0 = 18.7384$

Table 16: Numerical values of $\xi(n_0, r_0, t)$ and $R(n_0, r_0, t, \xi_1)$ for $(\alpha, \beta) = (10.0, 3.0)$, $C_s = 0.5$, $C_r = 30$, $a_0 = 2$, $a_1 = 2$, and $a_2 = 2$ case.

t	$\xi(n_0, r_0, t)$	$R(n_0, r_0, t, \xi_1)$
0.5000	0.3732	29.856178
1.0000	0.3935	29.553686
2.0000	0.3969	29.517816
3.0000	0.3970	29.516645
4.0000	0.3970	29.516571
5.0000	0.3970	29.516563
6.0000	0.3970	29.516562
8.0000	0.3970	29.516562
10.0000	0.3970	29.516562

$(n^*, \xi^*) = (2, 0.3970)$, $R^* = 29.516562$, $(n_0, r_0, t_0, \xi_0) = (2, 2, 0.8194, 0.4097)$, $R_0 = 29.5959$

collect data. However, Lin et al. (2008a, 2010a) failed to carry out Bayesian analysis to derive the Bayesian decision functions. For the loss function $L(d, \lambda, n)$, the numerical results show that the hybrid type-I censoring scheme is not a suitable sampling scheme for collecting data from the sample. Thus, their proposed sampling plans $(n_0, r_0, t_0, \delta_0^{LHB})$ have certain defects and are not the Bayesian sampling plans. The Bayesian sampling plan (n^*, δ^*) is a challenging competitor to the sampling plan $(n_0, r_0, t_0, \delta_0^{LHB})$. The numerical results indicate that (n^*, δ^*) performs better than $(n_0, r_0, t_0, \delta_0^{LHB})$ in all cases studied. When $r_0 = n_0 \neq n^*$, $R(n_0, r_0, t, \xi_1) \geq R^*$ for all $t > 0$, and $R(n_0, r_0, t, \xi_1)$ decreases in t for $t > 0$, and approximates R^* when t is sufficiently large. However, when $r_0 = n_0 \neq n^*$, $R(n_0, r_0, t, \xi_1) > R^*$, for all $t > 0$, and there is some gap between $R(n_0, r_0, t, \xi_1)$ and R^* even for very large t. Also, $R(n_0, r_0, t, \xi_1)$ decreases in t for $t > 0$.

Lin et al. (2008a, 2010a) attempted to find the exact Bayesian sampling plans from among the class of sampling plans (n, r, t, ξ) of the type having decision functions given in (2). However, since $R(n_0, r_0, t, \xi_1)$ decreases in t for $t > 0$, thus, for any $t_1 > 0$, the sampling plan $(n_0, r_0, t_1, \xi_1(t_1))$ is always dominated by $(n_0, r_0, t_2, \xi_1(t_2))$ as $t_2 > t_1$. This fact implies that among the class of sampling plans (n, r, t, ξ), the exact Bayesian sampling plan does not exist. Thus, the Lin et al. (2008a, 2010a)'s goal to find the exact Bayesian sampling plans will never be successfully achieved.

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Department of Mathematics, Wayne State University, Detroit, MI 48202, USA.
E-mail: liang@math.wayne.edu