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FEKETE-SZEGÖ PROBLEM FOR A CLASS OF ANALYTIC FUNCTIONS DEFINED BY CONVOLUTION

S. SIVAPRASAD KUMAR AND VIRENDRA KUMAR

Abstract. Let g and h be two fixed normalized analytic functions and ϕ be starlike with respect to 1, whose range is symmetric with respect to the real axis. Let $\mathcal{M}^{\alpha,\beta}_{g,h}(\phi)$ be the class of analytic functions $f(z) = z + a_2z^2 + a_3z^3 + \ldots$, satisfying the subordination

$$\left(\frac{(f*g)(z)}{z}\right)^{\alpha}\left(\frac{(f*h)(z)}{z}\right)^{\beta} \prec \phi(z),$$

where α and β are real numbers and are not zero simultaneously. In the present investigation, sharp upper bounds of the Fekete-Szegö functional $|a_3 - \mu a_2^2|$ for functions belonging to the class $\mathcal{M}^{\alpha,\beta}_{g,h}(\phi)$ are obtained and certain applications are also discussed.

1. Introduction

Let $\mathcal{A}$ denote the class of functions f analytic in $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ and normalized by the conditions $f(0) = 0$ and $f'(0) - 1 = 0$. Thus, if $f \in \mathcal{A}$, then

$$f(z) = z + a_2z^2 + a_3z^3 + \cdots. \tag{1.1}$$

We denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of univalent functions. For two functions f and g analytic in $\mathbb{D}$, we say that f is *subordinate* to g, denoted by $f \prec g$, if there is an analytic function w with $|w(z)| \le |z|$ such that $f(z) = g(w(z))$. If g is univalent, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(\mathbb{D}) \subseteq g(\mathbb{D})$.

Let ϕ be an analytic univalent function with positive real part in $\mathbb{D}$ and $\phi(\mathbb{D})$ be symmetric with respect to the real axis, starlike with respect to $\phi(0) = 1$ and $\phi'(0) > 0$. Let $\mathcal{P}(\phi)$ be the class of analytic functions p in $\mathbb{D}$ with $p(0) = 1$ and $p(\mathbb{D}) \subset \phi(\mathbb{D})$ or equivalently $p \prec \phi$. Let $\mathcal{P} = \mathcal{P}((1+z)/(1-z))$ is the class of analytic functions with positive real part in the unit disk $\mathbb{D}$. Let $\mathcal{S}^*(\phi)$ be the class of functions $f \in \mathcal{S}$ such that $zf'(z)/f(z) \in \mathcal{P}(\phi)$ and $\mathcal{C}(\phi)$ be the class of

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functions $f \in \mathcal{S}$ such that $1+zf''(z)/f'(z) \in \mathcal{P}(\phi)$. These classes were introduced and studied by Ma and Minda [6]. The classes $\mathcal{S}^*(\phi)$ and $\mathcal{C}(\phi)$ reduce to several well-known classes. For example, the class $\mathcal{S}^*((1+Az)/(1+Bz)) =: \mathcal{S}^*[A,B]$ $(-1 \le B < A \le 1)$ was introduced by Janowski [4]; $\mathcal{S}^*((1+z)/(1-z)) =: \mathcal{S}^*$ and $\mathcal{C}((1+z)/(1-z)) =: \mathcal{C}$ are the well-known classes of starlike and convex functions respectively.

Ali et al. [1] introduced the class $\mathcal{M}(\alpha,\phi)$ of α-convex function with respect to ϕ consisting of functions f in $\mathcal{A}$, satisfying

$$(1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1+\frac{zf''(z)}{f'(z)}\right) \prec \phi(z).$$

The class $\mathcal{M}(\alpha,\phi)$ includes several known classes namely $\mathcal{S}^*(\phi)$, $\mathcal{C}(\phi)$ and $\mathcal{M}(\alpha,(1+(1-2\alpha)z)/(1-z)) =: \mathcal{M}(\alpha)$. The class $\mathcal{M}(\alpha)$ is the class of α-convex functions, introduced and studied by Miller and Mocanu [8].

Bieberbach, in 1916, proved that if $f \in \mathcal{S}$, then $|a_2^2 - a_3| \le 1$. In 1933, Fekete and Szegö [3] proved that

$$|a_2^2 - \mu a_3| \le \begin{cases} 4\mu - 3 & (\mu \ge 1), \\ 1+\exp\left(-\frac{2\mu}{1-\mu}\right) & (0 \le \mu \le 1), \\ 3-4\mu & (\mu \le 0), \end{cases}$$

holds for the functions $f \in \mathcal{S}$ and the result is sharp. The problem of finding the sharp bounds for the non-linear functional $|a_3 - \mu a_2^2|$ of any compact family of functions is popularly known as the Fekete-Szegö problem. For related results, refer [1, 2, 10, 13, 14, 17] and the references cited therein.

Obradović [10] introduced the class of functions $f \in \mathcal{A}$ satisfying

$$\operatorname{Re}\left\{f'(z)\left(\frac{z}{f(z)}\right)^{\lambda+1}\right\} > 0 \quad (0 < \lambda < 1).$$

Tuneksi and Darus [17] obtained Fekete-Szegö inequality for the class of functions $f \in \mathcal{A}$ satisfying

$$\operatorname{Re}\left\{f'(z)\left(\frac{z}{f(z)}\right)^{\lambda+1}\right\} > \alpha \quad (0 \le \alpha < 1, 0 < \lambda < 1). \tag{1.2}$$

The Hadamard product (or convolution) of $f(z)$, given by (1.1) and

$$g(z) = z + \sum_{n=2}^{\infty} g_n z^n \tag{1.3}$$

is defined by $(f*g)(z) := z + \sum_{n=2}^{\infty} a_n g_n z^n =: (g*f)(z)$. Using the Hadamard product, Murugusundaramoorthy et al. [9] introduced a class $\mathcal{M}_{g,h}(\phi)$ of functions f in $\mathcal{A}$ satisfying

$$\frac{(f*g)(z)}{(f*h)(z)} \prec \phi(z) \quad (g_n > 0, h_n > 0, g_n - h_n > 0),$$

where $g,h \in \mathscr{A}$, $g(z)$ is given by (1.3) and $h(z) = z + \sum_{n=2}^{\infty} h_n z^n$ and obtained the Fekete-Szegö inequality for the class $\mathscr{M}_{g,h}(\phi)$. More information on related works can be found in [1, 2, 11, 16] and references cited therein.

Motivated by the works of Ma and Minda [6] and others [9, 13, 17], in the present paper, we investigate Fekete-Szegö problem for a more general class $\mathscr{M}_{g,h}^{\alpha,\beta}(\phi)$ defined using convolution and subordination. Earlier results in [6, 9, 17] shown to be special case of our results.

Definition 1.1. Let α and β are real numbers. Assume that $g(z)$ given by (1.3) and $h(z) = z + \sum_{n=2}^{\infty} h_n z^n$ with $g_n > 0$, $h_n > 0$ and $\alpha g_n + \beta h_n > 0$. A function $f \in \mathscr{A}$ is said to be in the class $\mathscr{M}_{g,h}^{\alpha,\beta}(\phi)$, if it satisfies

$$\left(\frac{(f*g)(z)}{z}\right)^{\alpha}\left(\frac{(f*h)(z)}{z}\right)^{\beta} \prec \phi(z),$$

where the powers are principle one.

For appropriate functions g, h, ϕ and constants α and β, the class $\mathscr{M}_{g,h}^{\alpha,\beta}(\phi)$ reduces to the following classes:

(1) $\mathscr{M}_{g,h}^{1,-1}(\phi) =: \mathscr{M}_{g,h}(\phi)$.

(2) $\mathscr{M}_{\frac{z}{(1-z)^2},\frac{z}{1-z}}^{1,-1}(\phi) =: \mathscr{S}^*(\phi)$

(3) $\mathscr{M}_{\frac{z+z^2}{(1-z)^3},\frac{z}{(1-z)^2}}^{1,-1}(\phi) =: \mathscr{C}(\phi)$

(4) With $g(z) = z/(1-z)^2$, $h(z) = z/(1-z)$ and $\phi(z) = (1+z)/(1-z)$, the class $\mathscr{M}_{g,h}^{1,-(\lambda+1)}(\phi), 0 < \lambda < 1$ reduces to the class introduced by Obradović [10].

We need the following results:

Lemma 1.2 ([6]). *If $p(z) = 1 + c_1 z + c_2 z^2 + \ldots \in \mathscr{P}$, then*

$$|c_2 - v c_1^2| \le \begin{cases} -4v + 2 & (v \le 0), \\ 2 & (0 \le v \le 1), \\ 4v - 2 & (v \ge 1). \end{cases}$$

When $v < 0$ or $v > 1$, equality holds if and only if $p(z)$ is $(1+z)/(1-z)$ or one of its rotations. If $0 < v < 1$, then equality holds if and only if $p(z)$ is $(1+z^2)/(1-z^2)$ or one of its rotations. If $v = 0$, equality holds if and only if

$$p(z) = \left(\frac{1+\gamma}{2}\right)\frac{1+z}{1-z} + \left(\frac{1-\gamma}{2}\right)\frac{1-z}{1+z} \quad (0 \le \gamma \le 1, z \in \mathbb{D}) \tag{1.4}$$

or one of its rotations. While for $v = 1$, *equality holds if and only if* $p(z)$ *is the reciprocal of one of the functions such that equality holds in the case of* $v = 0$.

Although the above upper bound is sharp, it can be improved as follows when $0 < v < 1$:

$$|c_2 - vc_1^2| + v|c_1|^2 \le 2 \quad (0 < v \le 1/2)$$

and

$$|c_2 - vc_1^2| + (1-v)|c_1|^2 \le 2 \quad (1/2 \le v < 1).$$

Lemma 1.3 ([5](see also [15])). *If* $p(z) = 1 + c_1 z + c_2 z^2 + \ldots \in \mathscr{P}$. *Then for any complex number* v,

$$|c_2 - vc_1^2| \le 2\max\{1; |2v-1|\}$$

and the result is sharp for the functions given by

$$p(z) = \frac{1+z^2}{1-z^2} \text{ and } p(z) = \frac{1+z}{1-z}.$$

Lemma 1.4 ([12]). *If the function* $p(z) = 1 + c_1 z + c_2 z^2 + \ldots \in \mathscr{P}$, *then* $|c_n| \le 2$ *for* $n \ge 1$.

2. The Fekete-Szegö problem

We begin with the following result.

Theorem 2.1. *Let* $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$. *If* $f(z)$ *given by* (1.1) *belongs to the class* $\mathscr{M}_{g,h}^{\alpha,\beta}(\phi)$, *then, for any real number* μ,

$$|a_3 - \mu a_2^2| \le \begin{cases} \frac{B_1 A}{(\alpha g_3 + \beta h_3)} & (\mu \le \sigma_1), \\ \frac{B_1}{(\alpha g_3 + \beta h_3)} & (\sigma_1 \le \mu \le \sigma_2), \\ -\frac{B_1 A}{(\alpha g_3 + \beta h_3)} & (\mu \ge \sigma_2), \end{cases}$$

where

$$A := \frac{B_2}{B_1} - \frac{[\alpha(\alpha-1)g_2^2 + \beta(\beta-1)h_2^2 + 2\alpha\beta g_2 h_2 + 2\mu(\alpha g_3 + \beta h_3)]B_1}{2(\alpha g_2 + \beta h_2)^2},$$

$$\sigma_1 := \frac{2(B_2 - B_1)(\alpha g_2 + \beta h_2)^2 - [\alpha(\alpha-1)g_2^2 + \beta(\beta-1)h_2^2 + 2\alpha\beta g_2 h_2]B_1^2}{2(\alpha g_3 + \beta h_3)B_1^2}$$

and

$$\sigma_2 := \frac{2(B_2 + B_1)(\alpha g_2 + \beta h_2)^2 - [\alpha(\alpha-1)g_2^2 + \beta(\beta-1)h_2^2 + 2\alpha\beta g_2 h_2]B_1^2}{2(\alpha g_3 + \beta h_3)B_1^2}.$$

Proof. Let $f \in \mathscr{M}_{g,h}^{\alpha,\beta}(\phi)$. Then the function p defined by

$$p(z) = \left(\frac{(f*g)(z)}{z}\right)^{\alpha}\left(\frac{(f*h)(z)}{z}\right)^{\beta} \tag{2.1}$$
$$= 1 + b_1 z + b_2 z^2 + \cdots$$

is analytic. By a computation, we get

$$\left(\frac{(f*g)(z)}{z}\right)^{\alpha} = 1 + \alpha a_2 g_2 z + \left(\alpha a_3 g_3 + \frac{\alpha(\alpha-1)}{2} a_2^2 g_2^2\right) z^2 + \cdots$$

and

$$\left(\frac{(f*h)(z)}{z}\right)^{\beta} = 1 + \beta a_2 h_2 z + \left(\beta a_3 h_3 + \frac{\beta(\beta-1)}{2} a_2^2 h_2^2\right) z^2 + \cdots .$$

Substituting these in (2.1) and comparing coefficients, we have

$$b_1 = (\alpha g_2 + \beta h_2) a_2 \tag{2.2}$$

and

$$b_2 = (\alpha g_3 + \beta h_3) a_3 + [\alpha(\alpha-1) g_2^2 + \beta(\beta-1) h_2^2 + 2\alpha\beta g_2 h_2] \frac{a_2^2}{2}. \tag{2.3}$$

Since ϕ is univalent and $p \prec \phi$, the function $p_1(z)$ defined by

$$p_1(z) = \frac{1+\phi^{-1}(p(z))}{1-\phi^{-1}(p(z))} = 1 + c_1 z + c_2 z^2 + \cdots, \tag{2.4}$$

is analytic with positive real part in $\mathbb{D}$. Further from (2.4), we have

$$\begin{aligned} p(z) &= \phi\left(\frac{p_1(z)-1}{p_1(z)+1}\right) \\ &= \phi\left(\frac{c_1 z + c_2 z^2 + \cdots}{2 + c_1 z + c_2 z^2 + \cdots}\right) \\ &= 1 + \frac{1}{2} B_1 c_1 z + \left[\frac{1}{2} B_1 \left(c_2 - \frac{1}{2} c_1^2\right) + \frac{1}{4} B_2 c_1^2\right] z^2 + \cdots . \end{aligned}$$

Thus, we have

$$b_1 = \frac{1}{2} B_1 c_1 \tag{2.5}$$

and

$$b_2 = \frac{1}{2} B_1 (c_2 - \frac{1}{2} c_1^2) + \frac{1}{4} B_2 c_1^2. \tag{2.6}$$

Using (2.5) in (2.2), we obtain

$$a_2 = \frac{B_1 c_1}{2(\alpha g_2 + \beta h_2)}. \tag{2.7}$$

The equations (2.3) and (2.6), lead to

$$a_3 = \frac{2(\alpha g_2+\beta h_2)^2[2(c_2-\frac{1}{2}c_1^2)B_1+B_2c_1^2]-[\alpha(\alpha-1)g_2^2+\beta(\beta-1)h_2^2+2\alpha\beta g_2h_2]B_1^2c_1^2}{8(\alpha g_3+\beta h_3)(\alpha g_2+\beta h_2)^2}. \tag{2.8}$$

From (2.7) and (2.8), we have

$$|a_3-\mu a_2^2| = \frac{B_1}{2(\alpha g_3+\beta h_3)}[c_2-v c_1^2],$$

where

$$v := \frac{1}{2}\left[1-\frac{B_2}{B_1}+\frac{[\alpha(\alpha-1)g_2^2+\beta(\beta-1)h_2^2+2\alpha\beta g_2h_2+2\mu(\alpha g_3+\beta h_3)]B_1}{2(\alpha g_2+\beta h_2)^2}\right]. \tag{2.9}$$

The result now follows by an application of Lemma 1.2. □

If $\sigma_1 \leq \mu \leq \sigma_2$, then the above result can be improved by bifurcating the interval as follows:

Remark 2.2. Let

$$\sigma_3 := \frac{2B_2(\alpha g_2+\beta h_2)^2-[\alpha(\alpha-1)g_2^2+\beta(\beta-1)h_2^2+2\alpha\beta g_2h_2]B_1^2}{2(\alpha g_3+\beta h_3)B_1^2}.$$

If $\sigma_1 \leq \mu \leq \sigma_3$, then

$$|a_3-\mu a_2^2|+R_1 \leq \frac{B_1}{\alpha g_3+\beta h_3},$$

where

$$R_1 := \frac{[2(B_1-B_2)(\alpha g_2+\beta h_2)^2+[\alpha(\alpha-1)g_2^2+\beta(\beta-1)h_2^2+2\alpha\beta g_2h_2+2\mu(\alpha g_3+\beta h_3)]B_1^2]}{2(\alpha g_3+\beta h_3)B_1^2}|a_2|^2.$$

Similarly if $\sigma_3 \leq \mu \leq \sigma_2$, then

$$|a_3-\mu a_2^2|+R_2 \leq \frac{B_1}{\alpha g_3+\beta h_3},$$

where

$$R_2 := \frac{[2(B_2+B_1)(\alpha g_2+\beta h_2)^2+[\alpha(\alpha-1)g_2^2+\beta(\beta-1)h_2^2+2\alpha\beta g_2h_2+2\mu(\alpha g_3+\beta h_3)]B_1^2]}{2(\alpha g_3+\beta h_3)B_1^2}|a_2|^2.$$

Remark 2.3. For $\alpha = 1$ and $\beta = -1$, Theorem 2.1 reduces to [9, Theorem 2.1] due to Murugusundaramoorthy et al. [9].

Theorem 2.4. *Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$ be analytic in $\mathbb{D}$ and $B_1 > 0, B_2 \in \mathbb{R}$. If $f(z)$ given by (1.1) belongs to the class $\mathcal{M}^{\alpha,\beta}_{g,h}(\phi)$, then*

$$|a_2| \leq \frac{B_1}{\alpha g_2 + \beta h_2} \tag{2.10}$$

and for any complex number μ

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{2(\alpha g_2 + \beta h_2)} \max\{1; |R|\},$$

where

$$R := \frac{[\alpha(\alpha-1)g_2^2 + \beta(\beta-1)h_2^2 + 2\alpha\beta g_2 h_2 + 2\mu(\alpha g_3 + \beta h_3)]B_1}{2(\alpha g_2 + \beta h_2)^2} - \frac{B_2}{B_1}.$$

Proof. The inequality (2.10) follows from (2.7) and Lemma 1.4. Using (2.9) one can easily verify that

$$2v - 1 = \frac{B_1[\alpha(\alpha-1)g_2^2 + \beta(\beta-1)h_2^2 + 2\alpha\beta g_2 h_2 + 2\mu(\alpha g_3 + \beta h_3)]}{2(\alpha g_2 + \beta h_2)^2} - \frac{B_2}{B_1}.$$

Now an application of Lemma 1.3 completes the proof. □

Here below, we discuss some applications of Theorem 2.1:

Theorem 2.5. *Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$ be analytic in $\mathbb{D}$ and $B_1 > 0, B_2 \in \mathbb{R}$. Assume that*

$$g(z) = z + \sum_{n=2}^{\infty} \frac{n\Gamma(n+1)\Gamma(2-\delta)}{\Gamma(n-\delta+1)} z^n \text{ and } h(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\delta)}{\Gamma(n-\delta+1)} z^n.$$

If $f(z)$ given by (1.1) belongs to the class $M^{\alpha,\beta}_{g,h}(\phi)$, then for any real number μ

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{(2-\delta)(3-\delta)AB_1}{6(3\alpha+\beta)} & (\mu \leq \sigma_1), \\ \frac{(2-\delta)(3-\delta)B_1}{6(3\alpha+\beta)} & (\sigma_1 \leq \mu \leq \sigma_2), \\ -\frac{(2-\delta)(3-\delta)AB_1}{6(3\alpha+\beta)} & (\mu \geq \sigma_2), \end{cases}$$

where

$$A := \frac{B_2}{B_1} - \frac{[(4\alpha(\alpha-1) + \beta(\beta-1) + 4\alpha\beta)(3-\delta) + 3\mu(2-\delta)(3\alpha+\beta)]B_1}{(2\alpha+\beta)^2(3-\delta)},$$

$$\sigma_1 := \frac{(3-\delta)[2(B_1 - B_2)(2\alpha+\beta)^2 - (4\alpha(\alpha-1) + \beta(\beta-1) + 4\alpha\beta)B_1^2]}{3(2-\delta)(3\alpha+\beta)B_1^2}$$

and

$$\sigma_2 := \frac{(3-\delta)[2(B_1 + B_2)(2\alpha+\beta)^2 - (4\alpha(\alpha-1) + \beta(\beta-1) + 4\alpha\beta)B_1^2]}{3(2-\delta)(3\alpha+\beta)B_1^2}.$$

Remark 2.6. If we set $\alpha = 1$ and $\beta = -1$ in Theorem 2.5, it reduces to [9, Corollary 3.2] of Murugusundaramoorthy *et al.* For $\alpha = 1$, $\beta = -1$, $B_1 = \frac{8}{\pi^2}$, $B_2 = \frac{16}{3\pi^2}$ and $\delta = 1$, Theorem 2.5 reduces to the result [7, Theorem 2] of Ma and Minda.

Setting $g(z) = \frac{z}{(1-z)^2}$, $h(z) = \frac{z}{1-z}$, $\alpha = 1$ and $\beta = -\lambda - 1$, $\lambda < 1$ in Theorem 2.4, we deduce the following result:

Corollary 2.7. *Let $f(z)$ given by (1.1) and satisfies*

$$f'(z)\left(\frac{z}{f(z)}\right)^{\lambda+1} \prec \frac{1+Cz}{1+Dz} \quad (\lambda < 1),$$

then $|a_2| \le \frac{C-D}{1-\lambda}$ and for any complex number μ, we have

$$|a_3 - \mu a_2^2| \le \frac{C-D}{2-\lambda} \max\left\{1; \left|D + \frac{(1+\lambda-2\mu)(\lambda-2)(C-D)}{(1-\lambda)^2}\right|\right\}.$$

Remark 2.8. For $C = 1-2a, 0 \le a < 1, 0 < \lambda < 1$ and $D = -1$, Corollary 2.7, reduces to the result [17, Theorem 1] of Tuneski and Darus. Note that our proof is quite different from that one given by Tuneski and Darus [17]. It is necessary to make it clear that there was a typographical error in the assertion of [17, Theorem 1]; however the following is the correct one.

Example 2.9 ([17], Theorem 1)**.** Let $0 \le a < 1$, $0 < \lambda < 1$. If $f \in \mathscr{A}$ satisfies

$$\operatorname{Re}\left\{f'(z)\left(\frac{z}{f(z)}\right)^{\lambda+1}\right\} > a,$$

then $|a_2| \le \frac{2(1-a)}{1-\lambda}$ and for any complex number μ

$$|a_3 - \mu a_2^2| \le \frac{2(1-a)}{2-\lambda} \max\left\{1; \left|1 + \frac{(1+\lambda-2\mu)(2-\lambda)(1-a)}{(1-\lambda)^2}\right|\right\}.$$

Remark 2.10. For $a = 0$, Example 2.9 reduces to [17, Corollary 1] of Tuneski and Darus. Setting $C = k$ $(0 < k \le 1)$, $D = 0$, in Corollary 2.7, we obtain the result of Tuneski and Darus [17, Theorem 2].

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Department of Applied Mathematics, Delhi Technological University, Delhi-110042, India.
E-mail: spkumar@dce.ac.in

Department of Applied Mathematics, Delhi Technological University, Delhi-110042, India.
E-mail: vktmaths@yahoo.in