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SOME NEW INTEGRAL INEQUALITIES

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Abstract. In the paper, some new integral inequalities are presented by using analytic methods.

1. Introduction

In [31] and its preprint [30], the following problem was posed by the third author.

Open Problem 1.1. Under what conditions does the inequality

$$\int_a^b [f(x)]^p \,\mathrm{d}x \ge \left[\int_a^b f(x)\,\mathrm{d}x\right]^{p-1} \tag{1.1}$$

hold for $p > 1$?

Since then, this problem has been stimulating much interest of many mathematicians. In recent years, the third author has collected over forty articles devoted to answering and generalizing this open problem and to applying inequalities of this type. For potential availability to interested readers, we list the collection in the list of references of this paper.

In [33] and its preprint [48], the following result was obtained.

Theorem 1.1 ([33]). *Let f be a continuous function on $[a,b]$. If*

$$\int_a^b f(x)\,\mathrm{d}x \ge (b-a)^{p-1} \tag{1.2}$$

for some $p > 1$, then the inequality (1.1) is true.

Later in [32, p. 4, Theorem 1.1], an alternative condition for the inequality (1.1) to be valid was procured.

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Theorem 1.2 ([32])**.** *Let $f(x)$ be continuous and not identically zero on $[a,b]$ and differentiable on (a,b) with $f(a)=0$, and let α,β be positive real numbers such that $\alpha>\beta>1$. If*

$$\left[f^{(\alpha-\beta)/(\beta-1)}(x)\right]' \gtreqless \frac{(\alpha-\beta)\beta^{1/(\beta-1)}}{\alpha-1} \tag{1.3}$$

for all $x\in(a,b)$, then

$$\int_a^b [f(x)]^\alpha \,\mathrm{d}x \gtreqless \left[\int_a^b f(x)\,\mathrm{d}x\right]^\beta. \tag{1.4}$$

In [3, p. 124, Theorem C], a different form of the inequality (1.1) was established, which can be reformulated as Theorem 1.3 below.

Theorem 1.3 ([3])**.** *If $f(x)$ is a continuous function on $[a,b]$ such that $f(a)\ge 0$ and $f'(x)\ge p\ge 1$ on (a,b), then*

$$\int_a^b [f(x)]^{p+2}\,\mathrm{d}x \ge \frac{1}{(b-a)^{p-1}}\left[\int_a^b f(x)\,\mathrm{d}x\right]^{p+1}. \tag{1.5}$$

The main purpose of this paper is to generalize the above-mentioned results and to present some new integral inequalities.

2. Lemmas

For generalizing Theorems 1.1 to 1.3, we need the following lemmas.

Lemma 2.1. *Let $x_1,x_2,\dots,x_n$ and $y_1,y_2,\dots,y_n$ be positive numbers. For $p,q>1$ with $\frac1p+\frac1q=1$, we have*

$$\sum_{k=1}^n \frac{x_k^p}{y_k^{p/q}} \ge \frac{\left(\sum_{k=1}^n x_k\right)^p}{\left(\sum_{k=1}^n y_k\right)^{p/q}}. \tag{2.1}$$

The equality in (2.1) holds if and only if $\frac{x_1}{y_1}=\frac{x_2}{y_2}=\cdots=\frac{x_n}{y_n}$.

Proof. Using Hölder's inequality, we have

$$\sum_{i=1}^n x_i = \sum_{i=1}^n\left(\frac{x_i}{\sqrt[q]{y_i}}\sqrt[q]{y_i}\right) \le \left[\sum_{i=1}^n\left(\frac{x_i}{\sqrt[q]{y_i}}\right)^p\right]^{1/p}\left[\sum_{i=1}^n\left(\sqrt[q]{y_i}\right)^q\right]^{1/q}$$

and

$$\left[\sum_{i=1}^n\left(\sqrt[q]{y_i}\right)^q\right]^{1/q} = \left(\sum_{i=1}^n \frac{x_i^p}{y_i^{p/q}}\right)^{1/p}\left(\sum_{i=1}^n y_i\right)^{1/q}.$$

Combining the above inequality with the above equality yields (2.1). The proof of Lemma 2.1 is complete. □

The integral form of Lemma 2.1 may be easily derived as follows.

Lemma 2.2. *Let $f(x)$ and $g(x)$ be positive and integrable functions on $[a,b]$. If $p>1$ and the functions $f^p(x)$ and $g^{p-1}(x)$ are integrable on $[a,b]$, then*

$$\int_a^b \frac{f^p(x)}{g^{p-1}(x)}\,dx \geq \frac{\left[\int_a^b f(x)\,dx\right]^p}{\left[\int_a^b g(x)\,dx\right]^{p-1}}. \tag{2.2}$$

The equality in (2.2) holds if and only if $f(x)=kg(x)$.

Lemma 2.3. *Let $f_k(x)$ and $g_k(x)$ for $k=1,2\dots,n$ be positive and integrable functions on $[a,b]$. If $1<p\leq 2$ and the functions $f_k^p(x)$ and $g_k^{p-1}(x)$ are integrable on $[a,b]$, then*

$$\sum_{k=1}^{n} \frac{\int_a^b f_k^p(x)\,dx}{\int_a^b g_k^{p-1}(x)\,dx} \geq \frac{\int_a^b \left[\sum_{k=1}^n f_k(x)\right]^p dx}{\int_a^b \left[\sum_{k=1}^n g_k(x)\right]^{p-1} dx}. \tag{2.3}$$

Proof. We prove the inequality (2.3) by mathematical induction.

When $n=1$, the inequality (2.3) is trivial.

When $n=2$, the inequality (2.3) may be rewritten as

$$\frac{\int_a^b [f_1(x)+f_2(x)]^p\,dx}{\int_a^b [g_1(x)+g_2(x)]^{p-1}\,dx} \leq \frac{\int_a^b f_1^p(x)\,dx}{\int_a^b g_1^{p-1}(x)\,dx} + \frac{\int_a^b f_2^p(x)\,dx}{\int_a^b g_2^{p-1}(x)\,dx}. \tag{2.4}$$

Setting

$$\begin{aligned}
A_1 &= \|f_1(x)\|_p = \left[\int_a^b f_1^p(x)\,dx\right]^{1/p},\\
A_2 &= \|f_2(x)\|_p = \left[\int_a^b f_2^p(x)\,dx\right]^{1/p},\\
B_1 &= \|g_1(x)\|_{p-1} = \left[\int_a^b g_1^{p-1}(x)\,dx\right]^{1/(p-1)},\\
B_2 &= \|g_2(x)\|_{p-1} = \left[\int_a^b g_2^{p-1}(x)\,dx\right]^{1/(p-1)}.
\end{aligned}$$

Then, the inequality (2.4) becomes

$$\frac{A_1^p}{B_1^{p-1}} + \frac{A_2^p}{B_2^{p-1}} \geq \frac{\|f_1(x)+f_2(x)\|_p^p}{\|g_1(x)+g_2(x)\|_{p-1}^{p-1}}. \tag{2.5}$$

By Lemma 2.2, we obtain

$$\frac{A_1^p}{B_1^{p-1}} + \frac{A_2^p}{B_2^{p-1}} \geq \frac{(A_1+A_2)^p}{(B_1+B_2)^{p-1}}. \tag{2.6}$$

So, in order to prove the inequality (2.5), it is sufficient to show

$$\frac{\left[\|f_1(x)\|_p+\|f_2(x)\|_p\right]^p}{\left[\|g_1(x)\|_{p-1}+\|g_2(x)\|_{p-1}\right]^{p-1}} \geq \frac{\|f_1(x)+f_2(x)\|_p^p}{\|g_1(x)+g_2(x)\|_{p-1}^{p-1}}. \tag{2.7}$$

Since $1 < p \le 2$, then $0 < p-1 \le 1$. Applying Minkowski's inequality leads to

$$\|f_1(x)\|_p + \|f_2(x)\|_p \ge \|f_1(x)+f_2(x)\|_p \tag{2.8}$$

and

$$\|g_1(x)\|_{p-1} + \|g_2(x)\|_{p-1} \le \|g_1(x)+g_2(x)\|_{p-1}. \tag{2.9}$$

Considering the ratio between inequalities (2.8) and (2.9) results in (2.7).

Now assume that the inequality (2.3) is true for some $n = m \in \mathbb{N}$. Letting

$$\begin{cases} p_i(x) = f_i(x) \\ p_m(x) = f_m(x) + f_{m+1}(x) \end{cases} \quad \text{and} \quad \begin{cases} q_i(x) = g_i(x) \\ q_m(x) = g_m(x) + g_{m+1}(x), \end{cases}$$

where $i = 1, 2, \dots, m-1$. By the inductive hypothesis and the inequality (2.4), we obtain

$$\begin{aligned} \frac{\int_a^b \left[\sum_{k=1}^{m+1} f_k(x)\right]^p \mathrm{d}x}{\int_a^b \left[\sum_{k=1}^{m+1} g_k(x)\right]^{p-1} \mathrm{d}x} &= \frac{\int_a^b \left[\sum_{k=1}^{m} p_k(x)\right]^p \mathrm{d}x}{\int_a^b \left[\sum_{k=1}^{m} q_k(x)\right]^{p-1} \mathrm{d}x} \le \sum_{k=1}^{m} \frac{\int_a^b p_k^p(x)\,\mathrm{d}x}{\int_a^b q_k^{p-1}(x)\,\mathrm{d}x} \\ = \sum_{k=1}^{m-1} \frac{\int_a^b f_k^p(x)\,\mathrm{d}x}{\int_a^b g_k^{p-1}(x)\,\mathrm{d}x} &+ \frac{\int_a^b \left[f_m(x)+f_{m+1}(x)\right]^p \mathrm{d}x}{\int_a^b \left[g_m(x)+g_{m+1}(x)\right]^{p-1} \mathrm{d}x} \le \sum_{k=1}^{m+1} \frac{\int_a^b f_k^p(x)\,\mathrm{d}x}{\int_a^b g_k^{p-1}(x)\,\mathrm{d}x}. \end{aligned}$$

This means that the inequality (2.3) holds for $n = m+1$. Lemma 2.3 is thus proved inductively. □

The discrete version of the inequality (2.3) in Lemma 2.3 may be stated as a corollary below.

Corollary 2.4. *Let $x_{k,i}$ for $k = 1, 2, \dots, m$ and $i = 1, 2, \dots, n$ be positive numbers. If $1 < p \le 2$, then*

$$\sum_{i=1}^{n} \frac{\sum_{k=1}^{m} x_{k,i}^p}{\sum_{k=1}^{m} y_{k,i}^{p-1}} \ge \frac{\sum_{k=1}^{m} \left(\sum_{i=1}^{n} x_{k,i}\right)^p}{\sum_{k=1}^{m} \left(\sum_{i=1}^{n} y_{k,i}\right)^{p-1}}. \tag{2.10}$$

Lemma 2.5. *Let a_k for $k = 1, 2, \dots, n$ be nonnegative numbers. If $p \ge 1$, then*

$$\left(\sum_{k=1}^{n} a_k\right)^p \le n^{p-1} \sum_{k=1}^{n} a_k^p. \tag{2.11}$$

Proof. This follows from the convexity of the function $f(x) = x^p$ for $p \ge 1$ and the well known Jensen's inequality. □

3. Main results

Now we are in a position to generalize Theorems 1.1 to 1.3.

Theorem 3.1. *Let $f_k(x)$ for $k=1,2,\dots,n$ be positive integrable functions on $[a,b]$. If $1<p\le 2$ and*

$$\sum_{k=1}^{n}\int_a^b f_k(x)\,\mathrm{d}x \ge [n(b-a)]^{p-1}, \tag{3.1}$$

then

$$\sum_{k=1}^{n}\int_a^b f_k^p(x)\,\mathrm{d}x \ge \left[\sum_{k=1}^{n}\int_a^b f_k(x)\,\mathrm{d}x\right]^{p-1}. \tag{3.2}$$

Proof. Letting $g_k(x)=1$ in Lemma 2.3 results in

$$\sum_{k=1}^{n}\frac{\int_a^b f_k^p(x)\,\mathrm{d}x}{\int_a^b 1^{p-1}\,\mathrm{d}x} \ge \frac{\int_a^b \left[\sum_{k=1}^n f_k(x)\right]^p\mathrm{d}x}{\int_a^b \left(\sum_{k=1}^n 1\right)^{p-1}\mathrm{d}x}.$$

Furthermore, by Lemma 2.2 and the condition (3.1), we have

$$\begin{aligned}\sum_{k=1}^{n}\int_a^b f_k^p(x)\,\mathrm{d}x &\ge \frac{1}{n^{p-1}}\int_a^b\left[\sum_{k=1}^n f_k(x)\right]^p\mathrm{d}x = \frac{1}{n^{p-1}}\int_a^b \frac{\left[\sum_{k=1}^n f_k(x)\right]^p}{1^{p-1}}\,\mathrm{d}x\\ &\ge \frac{1}{n^{p-1}}\frac{\left[\int_a^b\sum_{k=1}^n f_k(x)\,\mathrm{d}x\right]^p}{\left(\int_a^b 1\,\mathrm{d}x\right)^{p-1}} \ge \left[\int_a^b\sum_{k=1}^n f_k(x)\,\mathrm{d}x\right]^{p-1}.\end{aligned}$$

The proof is completed. □

Remark 3.1. If $n=1$, Theorem 3.1 generalizes Theorem 1.1 in relatively strong condition.

Theorem 3.2. *Let $f_k(x)$ for $k=1,2,\dots,n$ be nonnegative continuous functions on $[a,b]$. If $\sum_{k=1}^n f_k(x)$ is increasing on $[a,b]$ and*

$$\sum_{k=1}^{n} f_k(x) \ge (p-1)(b-a)^{p-2}n^{p-1}, \tag{3.3}$$

then the inequality (3.2) holds true for all $p>1$.

Proof. For all $x\in[a,b]$, let

$$H(x)=\sum_{k=1}^{n}\int_a^x f_k^p(t)\,\mathrm{d}t-\left[\int_a^x\sum_{k=1}^n f_k(t)\,\mathrm{d}t\right]^{p-1}.$$

A simple computation yields

$$H'(x)=\sum_{k=1}^n f_k^p(x)-(p-1)\left[\int_a^x\sum_{k=1}^n f_k(t)\,\mathrm{d}t\right]^{p-2}\sum_{k=1}^n f_k(x).$$

Since $\sum_{k=1}^n f_k(x)$ is increasing on $[a,b]$, then

$$0\le\int_a^x\sum_{k=1}^n f_k(t)\,\mathrm{d}t\le(b-a)\sum_{k=1}^n f_k(x). \tag{3.4}$$

Further by Lemma 2.5, we have

$$\begin{aligned}H'(x) &\ge \sum_{k=1}^{n} f_k^p(x) - (p-1)(b-a)^{p-2}\left[\sum_{k=1}^{n} f_k(x)\right]^{p-1}\\ &\ge \frac{1}{n^{p-1}}\left[\sum_{k=1}^{n} f_k(x)\right]^{p} - (p-1)(b-a)^{p-2}\left[\sum_{k=1}^{n} f_k(x)\right]^{p-1}\\ &= \left[\sum_{k=1}^{n} f_k(x)\right]^{p-1}\frac{\sum_{k=1}^{n} f_k(x) - (p-1)n^{p-1}(b-a)^{p-2}}{n^{p-1}}\\ &\ge 0.\end{aligned}$$

Thus, the function $H(x)$ is increasing on $[a,b]$. In particular, $H(b) \ge H(a) = 0$, which gives the desired inequality (3.2). □

Theorem 3.3. *Let $f_k(x)$ for $k = 1,2,\dots,n$ be nonnegative continuous on $[a,b]$ and differentiable on (a,b), such that*

$$\sum_{k=1}^{n} f_k(a) \ge 0 \quad \text{and} \quad \sum_{k=1}^{n} f_k'(x) \ge p. \tag{3.5}$$

Then we have

$$\sum_{k=1}^{n}\int_a^b f_k^{p+2}(x)\,\mathrm{d}x \ge \frac{1}{n^{p+1}(b-a)^{p-1}}\left[\sum_{k=1}^{n}\int_a^b f_k(x)\,\mathrm{d}x\right]^{p+1} \tag{3.6}$$

for all $p > 1$.

Proof. Set

$$G(x) = \sum_{k=1}^{n}\int_a^x f_k^{p+2}(t)\,\mathrm{d}t - \frac{1}{n^{p+1}(b-a)^{p-1}}\left[\int_a^x \sum_{k=1}^{n} f_k(t)\,\mathrm{d}t\right]^{p+1}$$

for all $x \in [a,b]$. Simple computations and utilization of Lemma 2.5 and (3.4) yield

$$\begin{aligned}G'(x) &= \sum_{k=1}^{n} f_k^{p+2}(x) - \frac{p+1}{n^{p+1}(b-a)^{p-1}}\left[\int_a^x \sum_{k=1}^{n} f_k(t)\,\mathrm{d}t\right]^{p}\sum_{k=1}^{n} f_k(x)\\ &\ge \frac{1}{n^{p+1}}\left[\sum_{k=1}^{n} f_k(x)\right]^{p+2} - \frac{p+1}{n^{p+1}(b-a)^{p-1}}\left[\int_a^x \sum_{k=1}^{n} f_k(t)\,\mathrm{d}t\right]^{p}\sum_{k=1}^{n} f_k(x)\\ &= \frac{\sum_{k=1}^{n} f_k(x)}{n^{p+1}}h(x)\end{aligned}$$

and

$$\begin{aligned}h'(x) = (p+1)\left[\sum_{k=1}^{n} f_k(x)\right]^{p}\sum_{k=1}^{n} f_k'(x) - \frac{(p+1)p}{(b-a)^{p-1}}\left[\int_a^x \sum_{k=1}^{n} f_k(t)\,\mathrm{d}t\right]^{p-1}\sum_{k=1}^{n} f_k(x)\\ \ge (p+1)\left[\sum_{k=1}^{n} f_k(x)\right]^{p}\left[\sum_{k=1}^{n} f_k'(x) - p\right] \ge 0.\end{aligned}$$

Since $h(x)$ is increasing on $[a,b]$ and $h(a) = \left[\sum_{k=1}^{n} f_k(a)\right]^{p+1} \ge 0$, the function $G(x)$ is also increasing on $[a,b]$. Especially, $G(b) \ge G(a) = 0$, which gives the desired inequality (3.6). □

Remark 3.2. If $n = 1$, Theorem 3.3 is just Theorem 1.3.

Corollary 3.4. *Under conditions of Theorem 3.3, when $[a,b] = [0,1]$, we have*

$$\sum_{k=1}^{n}\int_0^1 f_k^{p+2}(x)\,\mathrm{d}x \geq \frac{1}{n^{p+1}p}\left[\sum_{k=1}^{n}\int_0^1 f_k(x)\,\mathrm{d}x\right]^{p+1}. \tag{3.7}$$

Proof. This follows from respectively replacing $f_k(x)$ by $pf_k(x)$ and $[a,b]$ by $[0,1]$ in Theorem 3.3. □

Remark 3.3. If $n = 1$, Corollary 3.4 becomes [3, p. 124, Corollary 3.1].

By the similar method as above, we may prove following Theorem 3.5.

Theorem 3.5. *Let $f_k(x)$ for $k = 1,2,\ldots,n$ be nonnegative and continuous on $[a,b]$ and be differentiable on (a,b), such that*

$$\sum_{k=1}^{n} f_k(a) \geq 0 \quad \textit{and} \quad \sum_{k=1}^{n} f_k'(x) \geq \frac{2n^{2p}}{p+1}. \tag{3.8}$$

Then

$$\sum_{k=1}^{n}\int_a^b f_k^{2p+1}(x)\,\mathrm{d}x \geq \left\{\int_a^b \left[\sum_{k=1}^{n} f_k(x)\right]^p \mathrm{d}x\right\}^2 \tag{3.9}$$

for all $p > 1$.

Remark 3.4. If $n = 1$, Theorem 3.5 is the same as [3, p. 124, Proposition 1.1].

Theorem 3.6. *Let $f_k(x)$ for $k = 1,2,\ldots,n$ be nonnegative, continuous, and not identically zero on $[a,b]$ with $f_k(a) = 0$, and let α,β be positive real numbers such that $\alpha > \beta > 1$. If*

$$\left\{\left[\sum_{k=1}^{n} f_k(x)\right]^{(\alpha-\beta)/(\beta-1)}\right\}' \geq \frac{(\alpha-\beta)\left(n^{\alpha-1}\beta\right)^{1/(\beta-1)}}{\alpha-1} \tag{3.10}$$

for all $x \in (a,b)$, then

$$\sum_{k=1}^{n}\int_a^b f_k^{\alpha}(x)\,\mathrm{d}x \geq \left[\sum_{k=1}^{n}\int_a^b f_k(x)\,\mathrm{d}x\right]^{\beta}. \tag{3.11}$$

Proof. Utilizing Lemma 2.5 and Cauchy's mean value theorem consecutively yields

$$\begin{aligned}\frac{\left[\int_a^b \sum_{k=1}^{n} f_k(x)\,\mathrm{d}x\right]^{\beta}}{\sum_{k=1}^{n}\int_a^b f_k^{\alpha}(x)\,\mathrm{d}x} &= \frac{\beta\left[\int_a^{\xi}\sum_{k=1}^{n} f_k(x)\,\mathrm{d}x\right]^{\beta-1}\sum_{k=1}^{n} f_k(\xi)}{\sum_{k=1}^{n} f_k^{\alpha}(\xi)} \\ &\leq \frac{\beta n^{\alpha-1}\left[\int_a^{\xi}\sum_{k=1}^{n} f_k(x)\,\mathrm{d}x\right]^{\beta-1}}{\left[\sum_{k=1}^{n} f_k(\xi)\right]^{\alpha-1}}\end{aligned}$$

$$
\begin{aligned}
&= \left\{ \frac{\left(\beta n^{\alpha-1}\right)^{1/(\beta-1)} \int_a^\xi \sum_{k=1}^n f_k(x)\,\mathrm{d}x}{\left[\sum_{k=1}^n f_k(\xi)\right]^{(\alpha-1)/(\beta-1)}} \right\}^{\beta-1} \\
&= \left\{ \frac{\left(\beta n^{\alpha-1}\right)^{1/(\beta-1)} \sum_{k=1}^n f_k(\theta)}{\frac{\alpha-1}{\beta-1}\left[\sum_{k=1}^n f_k(\theta)\right]^{\frac{\alpha-\beta}{\beta-1}} \sum_{k=1}^n f_k'(\theta)} \right\}^{\beta-1} \\
&= \left\{ \frac{\frac{\alpha-\beta}{\alpha-1}\left(\beta n^{\alpha-1}\right)^{1/(\beta-1)}}{\left[\left(\sum_{k=1}^n f_k(\theta)\right)^{(\alpha-\beta)/(\beta-1)}\right]'} \right\}^{\beta-1} \\
&\le 1.
\end{aligned}
$$

Thus, the inequality (3.11) follows. □

Remark 3.5. If $n = 1$, Theorem 3.6 is reduced to [32, p. 7, Theorem 1.1].

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