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CERTAIN CLASSES OF MEROMORPHIC p-VALENT FUNCTIONS WITH POSITIVE COEFFICIENTS

MOA'ATH NASSER AND MASLINA DARUS

Abstract. We introduce the new class $L(\alpha, \beta, \lambda, p)$ of meromorphic p-valent functions. The aim of the paper is to obtain coefficient inequalities, growth and distortion, radii of convexity and starlikeness and the convex linear combinations for the class $L(\alpha, \beta, \lambda, p)$.

1. Introduction

Let Σ_p denotes the class of the form

$$f(z) = \frac{1}{z^p} + \sum_{n=1} a_{p+n-1} z^{p+n-1} \tag{1.1}$$

which are analytic and p-valent in a punctured disc $D = \{z : 0 < |z| < 1\}$. Further let $\Sigma_p^*(\alpha)$ be the class of Σ_p consisting of function f which satisfies the inequality

$$Re\left(-\frac{zf'(z)}{f(z)} \right) > \alpha, \quad (0 \leq \alpha < p).$$

And let $\Sigma_p^C(\alpha)$ be the class of Σ_p consisting of function f which satisfies the inequality

$$Re\left(-1 - \frac{zf''(z)}{f'(z)} \right) > \alpha, \quad (0 \leq \alpha < p).$$

Clearly, we have

$$f \in \Sigma_p^C(\alpha) \iff zf'(z) \in \Sigma_p^*(\alpha), (0 \leq \alpha < p, p \in N).$$

This condition is obviously analogous to the well-known Alexander equivalent (see for details [1]). Many important properties and characteristics of various interesting subclasses of the class Σ_p of meromorphic p-valent functions, including the classes $\Sigma_p^*(\alpha)$

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and $\Sigma_p^C(\alpha)$ defined above, were studied rather extensively by (among others) Aouf et al. ([4], [5]), Kulkarni et al. [8], Mogra ([6], [7]) and Srivastava et al. ([2], [3]).

A function f given by (1.1) is said to be a member of the class $L(\alpha,\beta,\lambda,p)$ if it satisfies

$$\left|\frac{z^{p+1}f'(z)+p}{\alpha z^{p+1}f'(z)+[p+(\lambda-\alpha)(p-\alpha)]}\right|<\beta \tag{1.2}$$

where $0<\beta\le 1$, $0<\lambda+p-\sqrt{(\lambda+p)^2-4p(1+\lambda)}<\alpha<\lambda+p+\sqrt{(\lambda+p)^2-4p(1+\lambda)}\le p$, $(\lambda+p)^2>4p(1+\lambda)$ and $\lambda\ge\frac{p}{2}-2$ for all $z\in D$.

In this paper sharp results concerning coefficients, distortion theorem and the radius of convexity for the class $L(\alpha,\beta,\lambda,p)$ are determined. Finally we prove that the class is closed under convex linear combination.

2. Coefficient Inequalities

Our first result for functions $f\in L(\alpha,\beta,\lambda,p)$ is given as the following theorem.

Theorem 2.1. *If $f\in\sum_p$ given by (1.1) satisfies*

$$\sum_{n=1}^{\infty}(p+n-1)(1+\alpha\beta)|a_{p+n-1}|\le\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]. \tag{2.3}$$

Then $f\in L(\alpha,\beta,\lambda,p)$, where $0<\beta\le 1$, $0<\lambda+p-\sqrt{(\lambda+p)^2-4p(1+\lambda)}<\alpha<\lambda+p+\sqrt{(\lambda+p)^2-4p(1+\lambda)}\le p$, $(\lambda+p)^2>4p(1+\lambda)$ and $\lambda\ge\frac{p}{2}-2$, for all $z\in D$.

Proof. Let us suppose that

$$\sum_{n=1}^{\infty}(p+n-1)(1+\alpha\beta)|a_{p+n-1}|\le\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]$$

for $f\in\sum_p$. Consider the expression

$$M(f,f')=|z^{p+1}f'(z)+p|<\beta|\alpha z^{p+1}f'(z)+p+(\lambda-\alpha)(p-\alpha)|.$$

Then for $0<|z|=r<1$ we have

$$M(f,f')=\left|\sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}z^{2p+n-1}\right|$$
$$-\beta\left|\sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}z^{2p+n-1}+p(1-\alpha)+(\lambda-\alpha)(p-\alpha)\right|.$$

$$\begin{aligned}\frac{M(f,f')}{r^p} &\leq \left(\sum_{n=1}^{\infty}(p+n-1)|a_{p+n-1}|r^{p+n-1}\right)\\ &\quad -\beta\left(-\sum_{n=1}^{\infty}(p+n-1)|a_{p+n-1}|r^{p+n-1}+\frac{p(1-\alpha)+(\lambda-\alpha)(p-\alpha)}{r^p}\right)\\ &\leq \sum_{n=1}^{\infty}(p+n-1)(1+\alpha\beta)|a_{p+n-1}|r^{p+n-1}-\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)].\end{aligned}$$

The inequality above holds true for all $r(0 < r < 1)$. Thus by letting $r \to 1^-$ into the inequality above, we obtain $M(f,f') \leq 0$. Hence it follows that (1.2) true, and so $f \in L(\alpha,\beta,\lambda,p)$.

The result is sharp for functions f of the form

$$f(z) = z^{-p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}z^{p+n-1} \quad (n \geq 1). \tag{2.4}$$

Corollary 2.1. *Let the function be defined by* (1.1). *If* $f \in L(\alpha,\beta,\lambda,p)$,then

$$|a_{p+n-1}| \leq \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}, \qquad (n \geq 1).$$

The result is sharp for functions f given by (2.4).

3. Distortion Theorem

A distortion property for functions in the class $f \in L(\alpha,\beta,\lambda,p)$, is given as follows:

Theorem 3.1. *If the function f given by* (1.1) *is in the class $f \in L(\alpha,\beta,\lambda,p)$, then for $0 < |z| = r < 1$ we have*

$$\frac{1}{r^p} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)}r^p \leq \left|f(z)\right| \leq \frac{1}{r^p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)}r^p$$

with equality for

$$f(z) = \frac{1}{z^p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)}z^p, \qquad z = (ir, r)$$

and

$$\frac{p}{r^{p+1}} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}r^{p-1} \leq \left|f'(z)\right| \leq \frac{p}{r^{p+1}} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}r^{p-1}.$$

With equality for,

$$f(z) = \frac{1}{z^p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)} z^p, \qquad z = (\pm ir, \pm r)$$

Proof. Since $f \in L(\alpha, \beta, \lambda, p)$, Theorem 2.1 yields the inequality

$$\sum_{n=1}^{\infty} |a_{p+n-1}| \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)} \leq 1 \tag{3.5}$$

Thus, for $0 < |z| = r < 1$, and making use of (2.3) we have

$$\begin{aligned} |f(z)| &= \left| \frac{1}{z^p} + \sum_{n=1}^{\infty} a_{p+n-1} z^{p+n-1} \right| \\ &\leq |z|^{-p} + \sum_{n=1}^{\infty} a_{p+n-1} |z|^{p+n-1} \\ &\leq r^{-p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}, \qquad \text{(we substitute in (3.5) when n=1)} \\ &\leq r^{-p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)} \\ &= \frac{1}{r^p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)}. \end{aligned}$$

And

$$\begin{aligned} |f(z)| &= \left| \frac{1}{z^p} + \sum_{n=1}^{\infty} a_{p+n-1} z^{p+n-1} \right| \\ &\geq |z|^{-p} - \sum_{n=1}^{\infty} a_{p+n-1} |z|^{p+n-1} \\ &\geq r^{-p} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}, \qquad \text{(we substitute in (3.5) when n=1)} \\ &\geq r^{-p} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)} \\ &= \frac{1}{r^p} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{p(1+\alpha\beta)}. \end{aligned}$$

Also from Theorem 2.1, it follows that

$$\sum_{n=1}^{\infty} (p+n-1)|a_{p+n-1}| \leq \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}. \tag{3.6}$$

Thus

$$\begin{aligned}|f'(z)| &= \left| -pz^{-p-1} + \sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}z^{p+n-2}\right|,\\ &\leq pr^{-p-1} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}r^{p+n-2},\\ &\leq \frac{p}{r^{p+1}} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}r^{p-1},\end{aligned}$$

and

$$\begin{aligned}|f'(z)| &= \left| -pz^{-p-1} + \sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}z^{p+n-2}\right|,\\ &\geq pr^{-p-1} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}r^{p+n-2},\\ &\geq \frac{p}{r^{p+1}} - \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}r^{p-1}.\end{aligned}$$

Hence completes the proof of Theorem 3.1.

4. Radii of Starlikeness and Convexity

The radii of starlikeness and convexity for the class $L(\alpha,\beta,\lambda,p)$, is given by the following theorem:

Theorem 4.1. *If the function f defined by* (1.1) *is in the class $L(\alpha,\beta,\lambda,p)$, then f is starlike of order $\rho(0 \leq \rho < p)$, in the disk $|z| < r_1(\alpha,\beta,\lambda,p,\rho)$, where $r_1(\alpha,\beta,\lambda,p,\rho)$,is the largest value for which*

$$r_1 = r_1(\alpha,\beta,\lambda,p,\rho) = \inf_{n\geq 1}\left(\frac{(p-\rho)[(p+n-1)(1+\alpha\beta)]}{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}\right)^{\frac{1}{2p+n-1}}. \tag{4.7}$$

The result is sharp for functions$f$ given by (2.4).

Proof. It suffices to show that

$$\left|\frac{zf'(z)}{f(z)} + p\right| \leq (p-\rho). \tag{4.8}$$

For $|z| \leq r_1$, we have

$$\left|\frac{zf'(z)}{f(z)} + p\right| = \left|\frac{-pz^{-p} + \sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}z^{p+n-1}}{z^{-p} + \sum_{n=1}^{\infty}a_{p+n-1}z^{p+n-1}} + p\right| \tag{4.9}$$

which gives

$$\left|\frac{\sum_{n=1}^{\infty}(2p+n-1)a_{p+n-1}z^{p+n-1}}{z^{-p}+\sum_{n=1}^{\infty}a_{p+n-1}z^{p+n-1}}\right| = \left|\frac{\sum_{n=1}^{\infty}(2p+n-1)a_{p+n-1}z^{2p+n-1}}{1-\sum_{n=1}^{\infty}a_{p+n-1}z^{2p+n-1}}\right|$$
$$\leq \frac{\sum_{n=1}^{\infty}\frac{(2p+n-1)\beta[p(1-p)+(\lambda-\alpha)(p-\alpha)])}{(p+n-1)(1+\alpha\beta)}|z|^{2p+n-1}}{1-\sum_{n=1}^{\infty}\frac{(2p+n-1)\beta[p(1-p)+(\lambda-\alpha)(p-\alpha)])}{(p+n-1)(1+\alpha\beta)}|z|^{2p+n-1}}$$
$$\leq (p-\rho).$$

The inequality above holds true if

$$\sum_{n=1}^{\infty}\frac{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}|z|^{2p+n-1} \leq (p-\rho),$$

and it follows that

$$|z| \leq \left(\frac{(p-\rho)[(p+n-1)(1+\alpha\beta)]}{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}\right)^{\frac{1}{2p+n-1}}, \qquad n \geq 1.$$

Then we have

$$r_1(\alpha,\beta,\lambda,p,\rho) = \inf_{n\geq 1}\left(\frac{(p-\rho)[(p+n-1)(1+\alpha\beta)]}{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}\right)^{\frac{1}{2p+n-1}}$$

as required.

Theorem 4.2. *If the function f defined by (1.1) is in the class $L(\alpha,\beta,\lambda,p)$, then f is convex of order $\rho(0 \leq \rho < p)$, in the disk $|z| < r_2(\alpha,\beta,\lambda,p,\rho)$, where $r_2(\alpha,\beta,\lambda,p,\rho)$,is the largest value for which*

$$r_2 = r_2(\alpha,\beta,\lambda,p,\rho) = \inf_{n\geq 1}\left(\frac{p(p-\rho)(1+\alpha\beta)}{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}\right)^{\frac{1}{2p+n-1}}. \tag{4.10}$$

The result is sharp for functions f given by (2.4).

Proof. It suffices to show that

$$\left|\frac{zf''(z)}{f'(z)}+(1+p)\right| \leq (p-\rho). \tag{4.11}$$

For $|z| \leq r_2$, we have

$$\left|\frac{zf''(z)}{f'(z)} + (1+p)\right|$$

$$= \left|\frac{(p^2+p)z^{-p-1} + \sum_{n=1}^{\infty}(p+n-1)(p+n-2)a_{p+n-1}z^{p+n-2}}{-pz^{-p-1} + \sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}z^{p+n-2}} + (p+1)\right|$$

$$= \left|\frac{\sum_{n=1}^{\infty}(2p+n-1)(p+n-1)a_{p+n-1}z^{2p+n-1}}{-p + \sum_{n=1}^{\infty}(p+n-1)a_{p+n-1}z^{2p+n-1}}\right|$$

$$\leq \frac{\sum_{n=1}^{\infty}(2p+n-1)(p+n-1)\frac{\beta[p(1-p)+(\lambda-\alpha)(p-\alpha)])}{(p+n-1)(1+\alpha\beta)}|z|^{2p+n-1}}{p - \sum_{n=1}^{\infty}(p+n-1)\frac{\beta[p(1-p)+(\lambda-\alpha)(p-\alpha)])}{(p+n-1)(1+\alpha\beta)}|z|^{2p+n-1}} \leq (p-\rho).$$

The inequality above holds true if

$$\sum_{n=1}^{\infty}\frac{[(3p+n-\rho-1)]\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(1+\alpha\beta)}|z|^{2p+n-1} \leq p(p-\rho),$$

and it follows that

$$|z| \leq \left(\frac{p(p-\rho)(1+\alpha\beta)}{[(3p+n-\rho-1)]\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}\right)^{\frac{1}{2p+n-1}}, \qquad n \geq 1.$$

Then we have

$$r_2(\alpha,\beta,\lambda,p,\rho) = \inf_{n\geq 1}\left(\frac{p(p-\rho)(1+\alpha\beta)}{(3p+n-\rho-1)\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}\right)^{\frac{1}{2p+n-1}}$$

as required.

5. Convex Linear Combination

Our next result involves a linear combination of function f of the type (2.4).

Theorem 5.1 *Let*

$$f_p(z) = z^{-p},$$

and

$$f_{p+n-1}(z) = z^{-p} + \frac{\beta[p(1-\alpha)+(\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}z^{p+n-1}, \qquad (n \geq 1). \qquad (5.12)$$

Then $f \in L(\alpha, \beta, \lambda, p)$ if and only if it can be expressed in the form

$$f(z) = \sum_{n=1}^{\infty} \lambda_{p+n-1} f_{p+n-1}(z), \tag{5.13}$$

$$\text{where} \quad \lambda_{p+n-1} \geq 0 \qquad \textit{and} \qquad \sum_{n=1}^{\infty} \lambda_{p+n-1} = 1. \tag{5.14}$$

Proof. From (5.12), (5.13) and (5.14), it is easily seen that

$$\begin{aligned} f(z) &= \sum_{n=1}^{\infty} \lambda_{p+n-1} f_{p+n-1}(z), \\ &= \sum_{n=1}^{\infty} \lambda_{p+n-1} \left(z^{-p} + \frac{\beta[p(1-\alpha) + (\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)} z^{p+n-1} \right), \\ &= z^{-p} + \sum_{n=1}^{\infty} \frac{\beta[p(1-\alpha) + (\lambda-\alpha)(p-\alpha)]\lambda_{p+n-1}}{(p+n-1)(1+\alpha\beta)} z^{p+n-1}. \end{aligned}$$

Then it follows that

$$\sum_{n=1}^{\infty} \frac{(p+n-1)(1+\alpha\beta)}{\beta[p(1-\alpha) + (\lambda-\alpha)(p-\alpha)]} \frac{\beta[p(1-\alpha) + (\lambda-\alpha)(p-\alpha)]\lambda_{p+n-1}}{(p+n-1)(1+\alpha\beta)} \tag{5.15}$$

and

$$\sum_{n=2}^{\infty} \lambda_{p+n-1} = 1 - \lambda_p \leq 1. \tag{5.16}$$

So, by Theorem 2.1 we have $f \in L(\alpha, \beta, \lambda, p)$. Conversely, let us suppose that $f \in L(\alpha, \beta, \lambda, p)$. Then

$$a_{p+n-1} \leq \frac{\beta[p(1-\alpha) + (\lambda-\alpha)(p-\alpha)]}{(p+n-1)(1+\alpha\beta)}, \qquad (n \geq 1). \tag{5.17}$$

Setting

$$\lambda_{p+n-1} = \frac{(p+n-1)(1+\alpha\beta)}{\beta[p(1-\alpha) + (\lambda-\alpha)(p-\alpha)]} a_{p+n-1} \qquad (n \geq 1) \tag{5.18}$$

and $\lambda_p = 1 - \sum_{n=2}^{\infty} \lambda_{p+n-1}$. It follows that

$$f(z) = \sum_{n=1}^{\infty} \lambda_{p+n-1} f_{p+n-1}(z) \tag{5.19}$$

and the proof of the theorem is complete. Finally we prove

Theorem 5.2. *The class $L(\alpha, \beta, \lambda, p)$ is closed under convex linear combinations.*

Proof. Suppose that the functions f_1 and f_2 defined by,

$$f_i(z) = z^{-p} + \sum_{n=1}^{\infty} a_{p+n-1,i} z^{p+n-1} \qquad (i = 1, 2; z \in D) \tag{5.20}$$

are in the class $L(\alpha, \beta, \lambda, p)$.

Setting $h(z) = \mu f_1(z) + (1 - \mu) f_2(z)$ we want to show that $f \in L(\alpha, \beta, \lambda, p)$.

For $0 \leq \mu \leq 1$, we can write

$$\begin{aligned} h(z) &= z^{-p} + \mu \sum_{n=1}^{\infty} a_{p+n-1,1} + (1 - \mu) \sum_{n=1}^{\infty} a_{p+n-1,2} z^{p+n-1}, \\ &= z^{-p} + \sum_{n=1}^{\infty} \Big\{ \mu a_{p+n-1,1} + (1 - \mu) a_{p+n-1,2} \Big\} z^{p+n-1}, \qquad (i = 1, 2; z \in D). \end{aligned}$$

In view of Theorem 2.1, we have

$$\begin{aligned} &\sum_{n=1}^{\infty} [(p + n - 1)(1 + \alpha\beta)] \Big\{ \mu a_{p+n-1,1} + (1 - \mu) a_{p+n-1,2} \Big\} \\ &= \mu \sum_{n=1}^{\infty} [(p + n - 1)(1 + \alpha\beta)] a_{p+n-1,1} + (1 - \mu) \sum_{n=1}^{\infty} [(p + n - 1)(1 + \alpha\beta)] a_{p+n-1,2} \\ &\leq \mu\beta[p(1 - \alpha) + (\lambda - \alpha)(p - \alpha)] + (1 - \mu)\beta[p(1 - \alpha) + (\lambda - \alpha)(p - \alpha)] \\ &= \beta[p(1 - \alpha) + (\lambda - \alpha)(p - \alpha)] \end{aligned}$$

which shows that $f \in L(\alpha, \beta, \lambda, p)$. Hence the theorem.

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References

[1] P. L. Duren, Univalent functions, In Grundlehren der Mathematischen Wissenschaften, 259, Springer-Verlag, New York, 1983.

[2] H. M. Srivastava, H. M. Hossen and M. K. Aouf., *A unified presentation of some classes meromorphically multivalent functions*, Comp. Math. with App. **38**(1999), 63-70.

[3] H. M. Srivastava and M. K. Aouf, *Some applications of fractional calculus operators to certain subclasses of prestarlike functions with negative coefficients*, Comp. Math. with App. **30**,(1995), 53-61.

[4] M. K. Aouf, *On a class meromorphic multivalent functions with positive coefficients*, Math. Japan. **35**, (1990), 603-608.

[5] M. K. Aouf, *A generalization of meromorphic multivalent functions with positive coefficients*, Math. Japan. **35**, (1990), 609-614.

[6] M. L. Mogra, *Meromorphic multivalent functions with positive coefficients I*, Math. Japan. **35**, (1990), 1-11.
[7] M. L. Mogra, *Meromorphic multivalent functions with positive coefficients II*, Math. Japan. **35**, (1990), 1089-1098.
[8] S. R. Kulkarni, U. H. Naik and H. M. Srivastava, *A certain class of meromorphically p-valent, quasi-convex functions*, Pan Amer. Math. J. **8** (1998), 57-64.

School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, Bangi 43600 Selangor D.E., Malaysia.

E-mail: maslina@pkrisc.cc.ukm.my