TAMKANG JOURNAL OF MATHEMATICS
Volume 35, Number 3, Autumn 2004

ON THE HOCHSCHILD COHOMOLOGY GROUPS OF ENDOMORPHISM ALGEBRAS OF EXCEPTIONAL SEQUENCES*

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Abstract. Let A be a finite dimensional associative algebra over an algebraically closed field k, and mod$A$ be the category of finite dimensional left A-module and $X_1, X_2, \ldots, X_n$ in mod$A$ be a complete exceptional sequence, then we investigate the Hochschild Cohomology groups of endomorphism algebra of exceptional sequence $\{X_1, X_2, \ldots, X_n\}$ in this paper.

1. Introduction

Let A be a finite dimensional associative algebra with identity over a field. The Hochschild cohomology groups $H^i(A, X)$ of A with coefficients in a finitely generated A-A-bimodule X were defined by Hochschild[1] in 1945. In case of $X = A$ we write $H^i(A)$ instead of $H^i(A, A)$, and $H^i(A)$ is called the i^{th}-Hochschild Cohomology group of A. The lower dimensional groups ($i \leq 2$) have a very concrete interpretation of classical algebraic structures such as derivations and extensions. It was observed by Gerstenhaber[2] that there are connections to algebraic geometry. In fact, $H^2(A, A)$ controls the deformation theory of A, and it was shown that the algebra A which satisfy $H^2(A, A) = 0$ are rigid. In [3], P. Gabriel gave the relation between $H^2(A)$ and the structure of A.

In general, it is not easy to compute the Hochschild cohomology groups of a given algebra. Computations for semi-commutative Schurian algebras and algebras arising from narrow quivers have been provided in [4] and [5] respectively. The case of monomial and truncated algebras have been studied in [6, 7]. However, the actual calculations of Hochschild cohomology groups have been fairly limited.

The aim of this paper is to study the Hochschild cohomology groups of endomorphism algebras of exceptional sequences over hereditary algebras.

Received November 5, 2002.
2000 *Mathematics Subject Classification.* 16G20, 16G60.
Key words and phrases. Hochschild cohomology groups, exceptional sequence, endomorphism algebras, tilting modules.
*Supported by the Natural Science Foundation of China (grant No. 10371036), the Natural Science Foundation of Beijing (grant No. 1042001) and the Fundamental research Fund of College of Applied Science in Beijing University of Technology.

2. Preliminaries

Let k be an algebraically closed field and A be a basic and connected finite-dimensional k-algebra. Let ${}_AX_A$ be an A-bimodule which is finite-dimensional over k. We define the Hochschild cocomplex $C^* = (C^i, d^i)_{i\in\mathbb{Z}}$ associated with this data as follows:

$$C^i = 0, \quad d^i = 0, \quad \text{for } i < 0;$$
$$C^0 = {}_AX_A, \quad d^0 : X \to \mathrm{Hom}_k(A, X), \quad \text{with } (d^0x)(a) = ax - xa,$$

for $x \in X$, $a \in A$; and $C_i = \mathrm{Hom}_k(A^{\otimes i}, X)$ for $i > 0$ where $A^{\otimes i}$ denotes the i-fold tensor product over k of A with itself, $d^i : C^i \to C^{i+1}$ with $(d^i f)(a_1 \otimes \cdots \otimes a_{i+1}) = a_1 f(a_2 \otimes \cdots \otimes a_{i+1}) + \sum_{j=1}^{i} (-1)^j f(a_1 \otimes \cdots \otimes a_j a_{j+1} \otimes \cdots \otimes a_{i+1}) + (-1)^{i+1} f(a_1 \otimes \cdots \otimes a_i) a_{i+1})$ for $f \in C^i$ and $a_1, a_2, \ldots, a_{i+1} \in A$.

Thus, we define $H^i(A, X) = H^i(C^*) = \ker d^i / \operatorname{im} d^{i-1}$, and we call it the ith cohomology group of A with coefficients in the bimodule X.

Of particular interest to us is the case of ${}_AX_A = {}_AA_A$. In this case $H^i(A, A)$ is denoted by $H^i(A)$. We call it the i-th Hochschild cohomology group of A and define the cohomology algebra $H(A) = \underset{i\in\mathbb{Z}}{\oplus} H^i(A)$. The multiplication is induced by the Yoneda product. In this way $H(A)$ is a $\mathbb{Z}$-grade algebra.

Let A be a finite-dimensional hereditary algebra and T_A be a tilting module in mod$A$, then $B = \mathrm{End}(T_A)$ is called a tilted algebra.

We have the following Lemma for the tilted algebra from [12].

Lemma 1. *Let A be an algebra and T be a tilting module over A, $B = \mathrm{End}(T_A)$ be the endomorphism algebra of T. Then $gl.\dim A - 1 \leq gl.\dim B \leq gl.\dim A + 1$.*

In [4], Happel showed:

Lemma 2. *Let $\vec{\triangle}$ be a finite quiver without oriented cycle and B be a finite-dimensional k-algebra which is tiltable to $k\vec{\triangle}$. Then $H^0(B) = k$, $\dim H^1(B) = \dim H^1(k\vec{\triangle})$, and $H^i(B) = 0$, for $i \geq 2$.*

For a hereditary algebra $A = k\vec{\triangle}$, where $\vec{\triangle}$ is a finite connected quiver without oriented cycle and α is an arrow in $\vec{\triangle}$. Let $\nu(\alpha) = \dim_k s(\alpha)Ae(\alpha)$, and let n be the number of vertices in $\vec{\triangle}$.

Lemma 3. *$H^0(k\vec{\triangle}) = k$, $\dim H^1(k\vec{\triangle}) = 1 - n + \sum_{\alpha\in\triangle_1} \nu(\alpha)$, $H^i(k\vec{\triangle}) = 0$ for $i \geq 2$.*

Lemma 4. *Let $\vec{\triangle}$ be a finite quiver without oriented cycle. Then $H^1(k\vec{\triangle}) = 0$ if and only if $\vec{\triangle}$ is a tree.*

Considering two arbitrary modules, there is the following result in [14].

Lemma 5. *Let X_1, X_2 be indecomposable A-modules and* $\mathrm{Ext}^1_A(X_i, X_i) = 0$ *for* $1 \leq i \leq 2$. *Let* $\mathrm{Ext}^1_A(X_2, X_1) = 0$. *If* $\mathrm{Hom}_A(X_1, X_2) \neq 0$, *then* $\mathrm{Ext}^1_A(X_1, X_2) = 0$.

Proof. Since $\mathrm{Ext}^1_A(X_2, X_1) = 0$, then the non-zero map $f \in \mathrm{Hom}_A(X_1, X_2)$ is either injective or surjective (see [9]). If f is surjective, f induced a surjective $\mathrm{Ext}^1_A(X_1, X_1) \to \mathrm{Ext}^1_A(X_1, X_2)$. So $\mathrm{Ext}^1_A(X_1, X_1) = 0$ implies $\mathrm{Ext}^1_A(X_1, X_2) = 0$. If f is injective, f induced a surjective $\mathrm{Ext}^1_A(X_2, X_2) \to \mathrm{Ext}^1_A(X_1, X_2)$. Then $\mathrm{Ext}^1_A(X_2, X_2) = 0$ implies $\mathrm{Ext}^1_A(X_1, X_2) = 0$. In a word, if $\mathrm{Hom}_A(X_1, X_2) \neq 0$, then $\mathrm{Ext}^1_A(X_1, X_2) = 0$.

Now we let A be a finite dimensional hereditary algebra over an algebraically field k, mod$A$ be the category of finite dimensional left A-modules. An indecomposable A-module X is said to be exceptional if $\mathrm{Ext}^1_A(X, X) = 0$. Note that $\mathrm{End}(X)$ is the field k in this case. A set of exceptional modules $X_1, X_2, \ldots, X_r$ is said to be an exceptional sequence if $\mathrm{Hom}(X_j, X_i) = 0$ and $\mathrm{Ext}^1_A(x_j, X_i) = 0$ when $j > i$. We call $\{X_1, X_2, \ldots, X_r\}$ a complete exceptional sequence if $r = n$ is the number of isomorphic classes of simple modules in mod$A$. $\mathrm{End}_A(X_1 \oplus X_2 \oplus \cdots \oplus X_n) = \mathrm{Hom}_A(X_1 \oplus X_2 \oplus \cdots \oplus X_n, X_1 \oplus X_2 \oplus \cdots \oplus X_n)$ is said to be the endomorphism algebra of the complete exceptional sequence $\{X_1, X_2, \ldots, X_n\}$, $\mathrm{End}(X_1 \oplus X_2 \oplus \cdots \oplus X_n)$ is said to be an endomorphism algebra of type A_n or $\tilde{A}_n$ if A is a hereditary algebra of A_n or $\tilde{A}_n$.

let $\mathcal{C}$ be a set of modules in mod$A$, then

$$\mathcal{C}^{\perp} = \{M \mid \mathrm{Hom}_A(X, M) = 0, \mathrm{Ext}^1_A(X, M) = 0, M \in \mathrm{mod}A, X \in \mathcal{C}\}$$

and

$$^{\perp}\mathcal{C} = \{M \mid \mathrm{Hom}_A(M, X) = 0, \mathrm{Ext}^1_A(M, X) = 0, M \in \mathrm{mod}A, X \in \mathcal{C}\}$$

are respectively called the right and left perpendicular categories detemined by $\mathcal{C}$. When $\mathcal{C}$ contains only one module X we write $^{\perp}\mathcal{C} =^{\perp} X$, $\mathcal{C}^{\perp} = X^{\perp}$.

Let $H = kQ$, where Q is a quiver with n vertices, $\varepsilon = (X_1, \ldots, X_r)$ be an exceptional sequence in mod$A$. Then by [14] we know that $\mathcal{C}^{\perp}$ and $^{\perp}\mathcal{C}$ are respectively equivalent to mod$kQ(\varepsilon^{\perp})$ and mod$kQ(^{\perp}\varepsilon)$ where $Q(\varepsilon^{\perp})$ and $Q(^{\perp}\varepsilon)$ are quivers containing $n - r$ vertices without oriented cycles. Especially, if X is an exceptional module in mod$H$, then $X^{\perp} = \mathrm{mod}H_r$ and $^{\perp}X = \mathrm{mod}H_l$ where $H_r = kQ_r$ and $H_l = kQ_l$ with Q_r and Q_l each having one vertex less than Q.

3. The Main Theorems and Their Proofs

In this section we will state our main results on the Hochschild cohomology of endomorphism algebras of compete exceptional sequences, and give their proofs. Firstly, we have the following fact in [10].

Lemma 6. *Let k be an algebraically closed field, $\vec{\triangle}$ be a quiver of type A_n. Then the endomorphism algebras of complete exceptional sequences over $A = k\vec{\triangle}$ are direct sums of finitely many tilted algebras of type A_m with $m \leq n$, and thus they are representation-finite.*

The next Theorem is a direct consequence of the above Lemma.

Theorem 1. *Let $E = (X_1, X_2, \ldots, X_n)$ be a complex exceptional sequence over a hereditary algebra of type A_n and $B = \mathrm{End}(X_1 \oplus \cdots \oplus X_n)$. Then there exists $1 \leq l \leq n$, such that $H^0(B) = k^l$ and $H^i(B) = 0$ for $i \geq 1$.*

Proof. It is obvious that $B = \mathrm{End}(X_1 \oplus \cdots \oplus X_n)$ is a direct sum of finitely many tilted algebras of type A_m with $m \leq n$. We suppose $B = B_{m_1} \oplus \cdots \oplus B_{m_l}$ $(1 \leq n)$, where B_{m_i} is a finite-dimensional algebra which is tiltable to $k\overrightarrow{\triangle}_{m_i}$ and $m_i \leq n$ for $1 \leq i \leq l$. Following Lemma 2, we have $H^0(B_{m_i}) = k$ for every m_i $(1 \leq i \leq l)$; $H^j(B_{m_i}) = 0$ for $j \geq 2$, and $\dim_k H^1(B_{m_i}) = \dim_k H^1(A_{m_i})$ where $A_{m_i} = k\overrightarrow{\triangle}_{m_i}$. Since $\overrightarrow{\triangle}_{m_i}$ is a tree, we have $H^1(A_{m_i}) = 0$ using Lemma 4, then $H^1(B_{m_i}) = 0$. According to the formula $H(B) = H(B_{m_i}) \oplus \cdots \oplus H(B_{m_l})$ (see [13]), we get $H^0(B) = H^0(B_{m_1}) \oplus \cdots \oplus H^0(B_{m_l}) = k^l$ and $H^i(B) = H^i(B_{m_1} \oplus \cdots \oplus H^i(B_{m_l}) = 0$ for $i \geq 1$.

On the endomorphism algebras of complete exceptional sequences over hereditary algebra of type $\tilde{A}_m$. We have the following result from [11].

Lemma 7. *Let $A = k\overrightarrow{\triangle}$ be a hereditary algebra of type $\tilde{A}_m$ with the corresponding quiver containing no oriented cycle. Let $E = (X_1, \ldots, X_n)$ be a complete exceptional sequence in modA. Then $B = \mathrm{End}(X_1 \oplus \cdots \oplus X_n)$ is either a direct sum of tilted algebras of type $\tilde{A}_m$ with $m \leq n$ and tilted algebras of type A_l with $l < n - m$, of a direct sum of tilted algebras of type A_l with $l \leq n + 1$.*

Following Lemma 7, we have

Theorem 2. *Let $A = k\overrightarrow{\triangle}$ be a hereditary algebra of type $\tilde{A}_m$ with $\overrightarrow{\triangle}$ containing no oriented cycle. Let $E = (X_1, \ldots, X_n)$ be a complete exceptional sequence in modA, $B = \mathrm{End}(X_1 \oplus \cdots \oplus X_n)$. Then there are integers $n_1 \geq 0$, $n_2 \geq 0$, such that $H^0(B) = k^{(n_1+n_2)}$, $H^1(B) = k^{n_1}$ and $H^i(B) = 0$ for $i \geq 2$.*

Proof. According to Lemma 7, we may assume that $B = B_{m_1} \oplus \cdots \oplus B_{m_{n_1}} \oplus \cdots \oplus B_{m_{(n_1+n_2)}}$, where B_{m_i} $(1 \leq i \leq n_1)$ is a tilted algebra of type $\tilde{A}_{m_i}$ $(m_i \leq n)$ and B_{m_i} $(n_1 + 1 \leq i \leq n_1 + n_2)$ is a tilted algebra of type A_{m_i} $(i \leq n - \bigoplus_{i=n_1+1}^{n_1+n_2} m_i)$. For the tilted algebra B_{m_i} $(n_1 + 1 \leq i \leq n_1 + n_2)$, we know that $H^0(B_{m_l}) = k$ and $H^i(B_{m_l}) = 0$ for $i \geq 1$ by Theorem 1; and for the tilted algebra B_{m_i} $(1 \leq i \leq n_1)$, we have $H^0(B_{m_i}) = k$, $H^1(B_{m_i}) = 0$ for $i \geq 2$ by Lemmas 2 and 3. Since $B = B_{m_1} \oplus \cdots \oplus B_{m_{n_1}} \oplus \cdots \oplus B_{m_{(n_1+n_2)}}$, then $H^0(B) = k^{n_1+n_2}$, $H^1(B) = k^{n_1}$ and $H^i(B) = 0$ for $i \geq 2$.

For the Hochschild cohomology group of complete exceptional sequence of algebra A with two simple modules, we have the next conclusion.

Theorem 3. *Let $A = k\overrightarrow{\triangle}$, where $\overrightarrow{\triangle}$ is a finite quiver without oriented cycle and with two vertices. Let $E = (X_1, X_2)$ be a complete exceptional sequence in modA and $B = \mathrm{End}(X_1 \oplus X_2)$.*

(1). *If B is a non-connected algebra, $H^0(B) = k^2$ and $H^i(B) = 0$ for $i \geq 1$;*

(2). *If B is a connected algebra, $H^0(B) = k$ and $H^i(B) = 0$ for $i \geq 2$; $H^1(B) = k^{m-1}$, where m is the number of the arrows in $\vec{\triangle}$.*

Proof. Firstly, we have $B \cong \begin{pmatrix} k & \mathrm{Hom}(X_1, X_2) \\ 0 & k \end{pmatrix}$. Considering it in the following cases.

(1). If $\mathrm{Hom}(X_1, X_2) = 0$, then $B = k \oplus k$. It is easy to check the conclusion.

(2). If $\mathrm{Hom}(X_1, X_2) \neq 0$, then we have $\mathrm{Ext}^1_A(X_1, X_2) = 0$. Thus $E = (X_1, X_2)$ is a tilting sequence and $B = \mathrm{End}(X_1 \oplus X_2)$ is a tilted algebra over A. By Lemma 2 we know that $H^0(B) = k$ and $H^i(B) = 0$ for $i \geq 2$, $\dim H^1(B) = \dim H^1(A)$. Let $A = k\vec{\triangle}$, where $\vec{\triangle}$ is a finite quiver without oriented cycle. If there are m arrows in $\vec{\triangle}$, then $\dim H^1(B) = \dim H^1(A) = 1 - 2 + m = m - 1$ by Lemma 3. That is to say $H^1(B) = k^{m-1}$.

Now we want to give the statement of our main result and its proof. Firstly, we shall need the following result proved in [9].

Lemma 8. *Let A be a finite-dimensional k-algebra and $M \in modA$, $B = A[M]$ be a one-point extension of A. Then there exists the following long exact sequence connecting the Hochschild cohomology groups of A and B.*

$$
\begin{aligned}
0 &\to H^0(B) \to H^0(A) \to \mathrm{Hom}_A(M, M)/k \to H^1(B) \to H^1(A) \to \mathrm{Ext}^1_A(M, M) \\
&\to \cdots \to \mathrm{Ext}^i_A(M, M) \to H^{i+1}(B) \to H^{i+1}(A) \to \mathrm{Ext}^{i+1}_A(M, M) \to \cdots
\end{aligned}
$$

For the lower Hochschild cohomology group, we have that $H^0(A) = Z(A)$ which is the center of A and that $H^1(A) \cong \mathrm{Der}(A, A)/\mathrm{Der}^0(A, A)$, where $\mathrm{Der}(A, A) = \{\delta \in \mathrm{Hom}_k(A, A) | \delta(ab) = a\delta(b) + \delta(a)b\}$ is the space of k-linear derivations of A, and $\mathrm{Der}^0(A, A) = \{\delta_x \in \mathrm{Hom}_k(A, A) | \delta_x(a) = ax - xa\}$ is the subspace of inner derivations of A.

Theroem 4. *Let $A = k\vec{\triangle}$, where $\vec{\triangle}$ is a finite quiver without oriented cycle and with three vertices. $E = (X_1, X_2, X_3)$ is a complete exceptional sequence in $modA$, $B = \mathrm{End}(X_1 \oplus X_2 \oplus X_3)$. Then we have $H^i(B) = 0$ for $i \geq 4$.*

Proof. We consider the computing in the four cases as follows. (1). If $\mathrm{Hom}(X_i, X_j) = 0$ for $i < j$ (where i, $j = 1, 2, 3$), then $B = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix} = k \oplus k \oplus k$. In this case, $H^0(B) = k^3$ and $H^i(B) = 0$ for $i \geq 0$.

(2) If there exists either $\mathrm{Hom}(X_1, X_2) = 0 = \mathrm{Hom}(X_1, X_3)$ or $\mathrm{Hom}(X_2, X_3) = 0 = \mathrm{Hom}(X_1, X_3)$, then $B = k \oplus C$, where $C = \begin{pmatrix} k & \mathrm{Hom}(X_2, X_3) \\ 0 & k \end{pmatrix}$, or $C = \begin{pmatrix} k & \mathrm{Hom}(X_1, X_2) \\ 0 & k \end{pmatrix}$. Then we get either $\mathrm{Ext}^1_A(X_2, X_3) = 0$ since $\mathrm{Hom}^1_A(X_2, X_3) \neq 0$

or $\mathrm{Ext}_A^1(X_1, X_2) = 0$ since $\mathrm{Hom}(X_1, X_2) \neq 0$, which illustrates that C is a tilted algebra with two simple modules. From Theorem 3, we know that $H^0(C) = k$, and there is an integer m, such that $H^1(C) = k^m$, and $H^i(C) = 0$ for $i \geq 2$. Thus we have $H^0(B) = k^2$; $H^1(B) = k^m$; and $H^i(B) = 0$ for $i \geq 2$.

(3) If $\mathrm{Hom}(X_2, X_3) \neq 0$ and at most one of $\mathrm{Hom}(X_1, X_2)$ and $\mathrm{Hom}(X_1, X_3)$ is zero. Then $\mathrm{Ext}_A^1(X_2, X_3) = 0$, and $B = \begin{pmatrix} k & \mathrm{Hom}(X_1, X_2) & \mathrm{Hom}(X_1, X_3) \\ 0 & k & \mathrm{Hom}(X_2, X_3) \\ 0 & 0 & k \end{pmatrix}$. Let $C = \begin{pmatrix} k & \mathrm{Hom}(X_2, X_3) \\ 0 & k \end{pmatrix}$ and $M = (\mathrm{Hom}(X_1, X_2), \mathrm{Hom}(X_1, X_3))$, then we have $B = \begin{pmatrix} k & M \\ 0 & C \end{pmatrix}$. It is easy to verify that M is a C-module.

Since $\mathrm{Ext}_A^1(X_2, X_3) = 0$, we have that $C = \mathrm{End}(X_2 \oplus X_3)$ is a tilted algebra over a hereditary algebra $A' = \vec{\triangle}'$ from the last paragraph in the last section where ${}^{\perp}X_1$ is naturally equivalent to $\mathrm{mod}A'$, where $\vec{\triangle}'$ is a finite quiver without oriented cycle and with two vertices. According to Lemma 8, there exists the following exact sequence between the Hochschild cohomology group of B and C, $0 \to H^0(B) \to H^0(C) \to \mathrm{Hom}_C(M, M)/k \to H^1(B) \to H^1(C) \to \mathrm{Ext}_C^1(M, M) \to \cdots$.

But $H^0(C) = k$; and $H^i(C) = 0$ for $i \geq 2$ from Lemma 3. Then we get the following exact sequence:

$0 \to H^0(B) \to H^0(C) \to \mathrm{Hom}_C(M, M)/k \to H^1(B) \to H^1(C) \to \mathrm{Ext}_C^1(M, M) \to H^2(B) \to 0$ with $\mathrm{Ext}_C^i(M, M) \cong H^{i+1}(B)$ for $i \geq 2$. By Lemma 1, we get $gl.\dim C \leq gl.\dim A' + 1 = 2$. Thus $\mathrm{Ext}_C^i(M, M) = 0$ for $i \geq 3$. Hence $H^i(B) = 0$ for $i \geq 4$ and $H^0(B) = k$.

(4). If $\mathrm{Hom}(X_2, X_3) = 0$ and $\mathrm{Hom}(X_1, X_2) \neq 0 \neq \mathrm{Hom}(X_1, X_3)$, then we obtain $B = \begin{pmatrix} k & \mathrm{Hom}(X_1, X_2) & \mathrm{Hom}(X_1, X_3) \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$. In this case $C = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = k \oplus k$, $M = (\mathrm{Hom}(X_1, X_2)\mathrm{Hom}(X_1, X_3))$. According to Lemma 3, we have $H^0(C) = k^2$ and $H^i(C) = 0$ for $i \geq 2$.

It is easy to prove that M is a C-module, then B is a one-point extension of C and $B = C[M]$. Following Lemma 7, we get a exact sequence as follows

$$0 \to H^0(B) \to H^0(C) \to \mathrm{Hom}_C(M, M)/k \to H^1(B) \to 0$$

with $\mathrm{Ext}_C^i(M, M) \cong H^{i+1}(B)$ for $i \geq 1$. Hence $\mathrm{Ext}_C^i(M, M) = 0$ for $i \geq 1$ since $\mathrm{Ext}_k^i(N, N) = 0$ for $i \geq 1$ and $C = k \oplus k$. Thus $H^i(B) = 0$ for $i \geq 2$, and $H^0(B) = k$. This completes the proof.

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