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AN ERRATUM TO: "A NOTE ON COMMON FIXED POINTS BY ALTERING DISTANCES"

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Abstract. The aim of this remark is to provide a correction to an error in the paper of Pant *et al.* [1].

The original paper by Pant *et al.* [1] contains one mistake. In this remark, we provide a minor correction for that mistake in the proof of Theorem 2.1 of [1]. We start with the following Definition and Theorem (2.1 of [1]).

Definition 1. A control function Ψ is defined as $\Psi : \Re^+ \to \Re^+$ which is continuous at zero, monotonically increasing, $\Psi(2t) \leq 2\Psi(t)$ and $\Psi(t) = 0$ if, and only if $t = 0$. It is noted that this function Ψ need not be sub additive [2].

Theorem 1. ([1]): *Let (A, S) and (B, T) be weakly commuting pairs of self mappings of a complete metric space (X, d) and the function Ψ be as in definition (1) satisfying*

(i) *$AX \subset TX$, $BX \subset SX$ and*
(ii) *There exists h in $[0, 1)$ such that $\Psi(d(Ax, By)) \leq hM_\Psi(x, y)$ for all x, y in X.*

Suppose that A and S are Ψ-compatible and A is continuous. Then A, B, S and T have a unique common fixed point.

The error occurs in line 13 on page 61 to line 5 on page 62 (from above) which claim that $Az = Sz = Tw = Bw$. But the given proof in [1] is valid only when S is assumed to be continuous. This leads to contradiction to our assumption on A in [1]. To overcome this problem, the theorem can be proved along the similar lines as given in the original one with minor changes in accordance with the following steps.

Since $AX \subset TX$, $Az = Tw$ for some w in X and corresponding to each x_{2n}, there exists a w_{2n} such that $AAx_{2n} = Tw_{2n}$. Thus we have $AAx_{2n} = Tw_{2n} \to Tw$ and $SAx_{2n} \to Tw$. Also, sicne $BX \subset SX$, corresponding to each w_{2n}, there corresponds u_{2n} such that $Bw_{2n} = Su_{2n}$. Thus, we have $Bw_{2n} = Su_{2n} \to Tw$ and $Tw_{2n} \to Tw$.

Now, we claim that $Au_{2n} \to Tw$ as $n \to \infty$.

For this,

$$\begin{aligned}\Psi(d(Au_{2n}, Bw_{2n})) &\leq hM_\Psi(u_{2n}, w_{2n})\\ &= h\max\{\Psi(d(Su_{2n}, Tw_{2n})), \Psi(d(Au_{2n}, Su_{2n})), \Psi(d(Bw_{2n}, Tw_{2n})),\\ &\qquad [\Psi(d(Au_{2n}, Tw_{2n})) + \Psi(d(Su_{2n}, Bw_{2n}))]/2\}.\end{aligned}$$

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Taking $n \to \infty$, we get $\Psi(d(Au_{2n}, Tw)) < h\Psi(d(Au_{2n}, Tw)) < \Psi(d(Au_{2n}, Tw))$, a contradiction. Thus we have $Au_{2n} \to Tw$ as $n \to \infty$.

Also, we claim that $Bw = Tw$.

If possible, suppose $Bw \neq Tw$. Then, as $n \to \infty$, the inequality

$$\begin{aligned}\Psi(d(Au_{2n}, Bw)) &\leq hM_\Psi(u_{2n}, w)\\ &= h\max\{\Psi(d(Su_{2n}, Tw)), \Psi(d(Au_{2n}, Su_{2n})), \Psi(d(Bw, Tw)),\\ &\qquad [\Psi(d(Au_{2n}, Tw)) + \Psi(d(Su_{2n}, Bw))]/2\},\end{aligned}$$

yields $\Psi(d(Tw, Bw)) < h\Psi(d(Tw, Bw)) < \Psi(d(Tw, Bw))$, a contradiction.

Hence, we get $Tw = Bw$. Thus we have $Az = Tw = Bw$.

Again, since $BX \subset SX$, so there exists u in X such that $Bw = Su$; that is, $Su = Bw = Tw$. Finally, we assert that $Au = Su$.

If $Au \neq Su$. Then by virtue of (ii) of Theorem 1, we get

$$\begin{aligned}\Psi(d(Au, Su)) &= \Psi(d(Au, Bw))\\ &< hM_\Psi(u, w)\\ &= h\max\{\Psi(d(Su, Tw)), \Psi(d(Au, Su)), \Psi(d(Bw, Tw)),\\ &\qquad [\Psi(d(Au, Tw)) + \Psi(d(Su, Bw))]/2\},\\ &= h\Psi(d(Au, Su)) < \Psi(d(Au, Su)), \text{ a contradiction.}\end{aligned}$$

Thus, $Au = Su$ and hence we have $Au = Su = Bw = Tw$. (1.1)

Since A and S are weakly commuting, we have by (1.1), $ASu = SAu$ and hence

$$AAu = ASu = SAu = SSu. \tag{1.2}$$

Also, applying the weakly commuting property of B and T, we get

$$BBw = BTw = TBw = TTw. \tag{1.3}$$

We now finally show that $AAu = Au$.

Suppose on the contrary that $AAu \neq Au$. Then by (ii), we get

$$\begin{aligned}\Psi(d(Au, AAu)) &= \Psi(d(AAu, Bw))\\ &\leq hM_\Psi(Au, w) = h\Psi(d(Au, AAu)), \text{ (using (1.2) and (1.3))},\end{aligned}$$

a contradiction. Hence, we must have $AAu = Au$. Therefore, Au is a common fixed point of A and S.

The remaining part of Theorem 2.1 in [1] remains unaltered. This result along with [1] provides a complete affirmative answer to the open problem posed by Sastry *et al.* [2].

References

[1] R. P. Pant, K. Jha and A. B. Lohani, *A note on common fixed points by altering distances*, Tamkang J. Maths. **34**(2003), 59-62.

[2] K. P. R. Sastry, S. V. R. Naidu, G. V. R. Babu and G. A. Naidu, *Generalization of common fixed point theorems for weakly commuting mappings by altering distances*, Tamkang J. Math. **31**(2000), 243-250.

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