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TWO DIMENSIONAL THERMO-ELASTIC WAVES DUE TO DISTURBANCE PRODUCED BY AN IMPULSIVE HEAT NUCLEUS

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Abstract. This paper deals with a two-dimensional dynamical problem of thermoelasticity generating elastic waves due to disturbance produce by a periodic heat nucleus. Initially, a detailed discussion on the theoretical part of the problem[2] is made without temperature, then the disturbance due to impulsive heat nucleus is considered.

Intrduction

In a paper, Eason, Fulton and Sneddon[3] have dealt with the disturbances produced by the distribution of stresses in an infinite elastic solid when the time dependent body force act upon certain region of the body. Strain being small, the general solution of the equation of motion for any distribution of body force is derived by the Four-dimensional Fourrier transforms[4] and thus the general solution is obtained. It is a typical two dimensional problem when the disturbance is generated due to time dependent impulsive heat nucleus.

Method of Solution

Equation of motion in two dimensions with density ρ of the medium are[1]

$$\left.\begin{aligned}\frac{\partial\tau^{xx}}{\partial x}+\frac{\partial\tau^{xy}}{\partial y}+\rho F^{x}=\rho f^{x}\\ \frac{\partial\tau^{xy}}{\partial x}+\frac{\partial\tau^{yy}}{\partial y}+\rho F^{y}=\rho f^{y}\end{aligned}\right\}$$

where F^x, F^y denote components of body force at (x, y).

The acceleration of the infinitisimal element centred at this point is denoted by (f^x, f^y). If we introduce the displacement vector components (v^x, v^y) at such a typi-

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cal point, we have

$$\left.\begin{aligned} f^x &= \frac{\partial^2 v^x}{\partial t^2} = c^2 \frac{\partial^2 v^x}{\partial \tau^2} \\ f^y &= \frac{\partial^2 v^y}{\partial t^2} = c^2 \frac{\partial^2 v^y}{\partial \tau^2} \end{aligned}\right\}$$

where t denotes time, c is some characteristic velocity and $\tau = ct$, is a space time co-ordinate determined by time. The equation of motion may, therefore, be written in the form[5]

$$\left.\begin{aligned} \frac{\partial \tau^{xx}}{\partial x} + \frac{\partial \tau^{xy}}{\partial y} + \rho F^x = \rho c^2 \frac{\partial^2 v^x}{\partial \tau^2} \\ \frac{\partial \tau^{xy}}{\partial x} + \frac{\partial \tau^{yy}}{\partial y} + \rho F^y = \rho c^2 \frac{\partial^2 v^y}{\partial \tau^2} \end{aligned}\right\} \tag{1}$$

To solve these equation we introduce Fourier transform[4] of each of the components of stress and displacement. We shall denote the Fourier transforms of a function ϕ by placing a bar over it, thus $\overline{\phi}$; in other word

$$\left.\begin{aligned} \frac{1}{(2\pi)^{\frac{3}{2}}} \int_{s_3} \phi(x_1, x_2, \tau) \exp\{i(x_p\, \xi_p + \omega\tau)\} ds = \overline{\phi}(\xi_x, \xi_y, \omega) \\ \frac{1}{(2\pi)^{\frac{3}{2}}} \int_{s_3} \left(\frac{\partial \phi}{\partial x_p}, \frac{\partial^2 \phi}{\partial \tau^2}\right) \exp\{i(x_p\, \xi_p + \omega\tau)\} ds = -(i\, \xi_p, \omega^2)\overline{\phi} \end{aligned}\right\} \tag{2}$$

where $p = 1, 2$; $ds = dx_1\ dx_2\ d\tau$ and s_3 denotes the entire $x_1\ x_2\ \tau$ plane. Applying (2) in (1), we get

$$\left.\begin{aligned} i\,\xi_x \overline{\tau}^{xx} + i\,\xi_y \overline{\tau}^{xy} - \rho \overline{F}^x = \rho c^2 \omega^2 \overline{v}^x \\ i\,\xi_x \overline{\tau}^{xy} + i\,\xi_y \overline{\tau}^{yy} - \rho \overline{F}^y = \rho c^2 \omega^2 \overline{v}^y \end{aligned}\right\} \tag{3}$$

From stress-strain relations in two dimensional thermoelastic problem for a temperature distribuion T[5], we have

$$\left.\begin{aligned} \tau^{xx} &= E_1\left(\frac{\partial v^x}{\partial x} + \nu \frac{\partial v^y}{\partial y} - \alpha_1\, T\right) \\ \tau^{yy} &= E_1\left(\nu \frac{\partial v^x}{\partial x} + \frac{\partial v^y}{\partial y} - \alpha_1\, T\right) \\ \tau^{xy} &= \mu\left(\frac{\partial v^x}{\partial y} + \nu \frac{\partial v^y}{\partial x}\right) \end{aligned}\right\} \tag{4}$$

where $E_1 = \frac{E}{1-\nu^2}$, $\alpha_1 = (1+\nu)\alpha$, $\nu = \frac{E}{2(1+\nu)}$, E and α denote Young's modulus and coefficient of linear thermal expansion.
Using (2) over (4) we get

$$\left.\begin{aligned} \overline{\tau}^{xx} &= -i\, E_1(\xi_x \overline{v}^x + \nu\, \xi_y \overline{v}^y - i\, \alpha_1\, \overline{T}) \\ \overline{\tau}^{yy} &= -i\, E_1(\nu \xi_x \overline{v}^x + \xi_y \overline{v}^y - i\, \alpha_1\, \overline{T}) \\ \overline{\tau}^{xy} &= -i\, \mu(\xi\, \overline{v}^x + \xi \overline{v}^y). \end{aligned}\right\} \tag{5}$$

Substituting (5) in (3), and solving for $\overline{v}^x$ and $\overline{v}^y$ we get,

$$\left.\begin{aligned}\overline{v}^x = &\Big\{\beta^2(\xi_x^2+\xi_y^2-\omega^2)\overline{F}^x-(\beta^2-1)(\xi_x^2\overline{F}^x+\xi_y\,\xi_x\,\overline{F}^y)\\ &+\frac{2i\,c_1^2}{\beta^2}(\beta^2-1)\alpha\,\overline{T}\xi_x(\xi_x^2+\xi_y^2-\omega^2)\Big\}\\ &\cdot\Big\{c_1^2(\xi_x^2+\xi_y^2-\beta^2\omega^2)(\xi_x^2+\xi_y^2-\omega^2)\Big\}^{-1}\end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned}\overline{v}^y = &\Big\{\beta^2(\xi_x^2+\xi_y^2-\omega^2)\overline{F}^y-(\beta^2-1)(\xi_x\xi_y\overline{F}^x+\xi_y^2\overline{F}^y)\\ &+\frac{2i\,c_1^2}{\beta^2}(\beta^2-1)\alpha\,\overline{T}\xi_y(\xi_x^2+\xi_y^2-\omega^2)\Big\}\\ &\cdot\Big\{c_1^2(\xi_x^2+\xi_y^2-\beta^2\omega^2)(\xi_x^2+\xi_y^2-\omega^2)\Big\}^{-1}\end{aligned}\right\} \tag{7}$$

where $\beta^2=\frac{4(\lambda+\mu)}{\lambda+2\mu}\cdot c_1^2=\frac{4\mu(\lambda+\mu)}{\rho(\lambda+2\mu)}$ is chosen for our characteristic velociry c.
Now, we find from (5)

$$\begin{aligned}\overline{\tau}^{xx} = &-i\,\rho\Big\{\beta^2(\xi_x^2+\xi_y^2-\omega^2)(\xi_x\overline{F}^x+\nu\xi_y\overline{F}^y)-(\beta^2-1)(\xi_x^2+\nu\,\xi_y^2)(\xi_x\overline{F}^x+\xi_y\overline{F}^y)\\ &+\frac{2i\,c_1^2}{\beta^2}(\beta^2-1)\alpha\overline{T}(\xi_x^2+\xi_y^2-\beta^2\omega^2)[(\nu-1)\xi_y^2+\omega^2]\Big\}\\ &\cdot\{(\xi_x^2+\xi_y^2-\beta^2\omega^2)(\xi_x^2+\xi_y^2-\omega^2)\}^{-1}\end{aligned} \tag{8}$$

$$\begin{aligned}\overline{\tau}^{yy} = &-i\,\rho\Big\{\beta^2(\xi_x^2+\xi_y^2-\omega^2)(\nu\xi_x\overline{F}^x+\xi_y\overline{F}^y)-(\beta^2-1)(\nu\xi_x^2+\,\xi_y^2)(\xi_x\overline{F}^x+\xi_y\overline{F}^y)\\ &+\frac{2i\,c_1^2}{\beta^2}(\beta^2-1)\alpha\overline{T}(\xi_x^2+\xi_y^2-\beta^2\omega^2)[(\nu-1)\xi_y^2+\omega^2]\Big\}\\ &\cdot\{(\xi_x^2+\xi_y^2-\beta^2\omega^2)(\xi_x^2+\xi_y^2-\omega^2)\}^{-1}\end{aligned} \tag{9}$$

$$\begin{aligned}\overline{\tau}^{xy} = &\frac{-i\,\rho}{\beta^2}\Big\{\beta^2(\xi_x^2+\xi_y^2-\omega^2)(\xi_y\overline{F}^x+\xi_x\overline{F}^y)-2(\beta^2-1)\xi_x\,\xi_y(\xi_x\overline{F}^x+\xi_y\overline{F}^y)\\ &+\frac{2i\,c_1^2}{\beta^2}(\beta^2-1)\alpha\overline{T}(\xi_x^2+\xi_y^2)(\xi_x^2+\xi_y^2-\beta^2\omega^2)\Big\}\\ &\cdot\{(\xi_x^2+\xi_y^2-\beta^2\omega^2)(\xi_x^2+\xi_y^2-\omega^2)\}^{-1}\end{aligned} \tag{10}$$

Applying Fourier inversion theorem[4] over (6)−(10), we have

$$\left.\begin{aligned}v^x = &\frac{1}{c_1^2(2\pi)^{\frac{3}{2}}}\int_{W_3}\Big\{\beta^2(\xi_x^2+\xi_y^2-\omega^2)\overline{F}^x-(\beta^2-1)(\xi_x^2\overline{F}^x+\xi_x\xi_y\overline{F}^y)\\ &+\frac{2i\,c_1^2}{\beta^2}(\beta^2-1)\alpha\,\overline{T}\,\xi_x(\xi_x^2+\xi_y^2-\omega^2)\Big\}\cdot\exp[-i(x\,\xi_x+y\,\xi_y+\omega\tau)]\\ &\cdot\Big\{(\xi_x^2+\xi_y^2-\beta^2\omega^2)(\xi_x^2+\xi_y^2-\omega^2)\Big\}^{-1}\cdot dW\end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned} v^x = \frac{1}{c_1^2(2\pi)^{\frac{3}{2}}} \int_{W_3} & \Big\{\beta^2(\xi_x^2+\xi_y^2-\omega^2)\overline{F}^y - (\beta^2-1)(\xi_x^2\overline{F}^y + \xi_x\xi_y\overline{F}^x) \\ & + \frac{2i\,c_1^2}{\beta^2}(\beta^2-1)\alpha\,\overline{T}\,\xi_y(\xi_x^2+\xi_y^2-\omega^2)\Big\}\cdot \exp[-i(x\,\xi_x + y\,\xi_y + \omega\tau)] \\ & \cdot \Big\{(\xi_x^2+\xi_y^2-\beta^2\omega^2)(\xi_x^2+\xi_y^2-\omega^2)\Big\}^{-1}\cdot dW \end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} \tau^{xx} = \frac{-i\rho}{(2\pi)^{\frac{3}{2}}} \int_{W_3} & \Big\{\beta^2(\xi_x^2+\xi_y^2-\omega^2)(\xi_x\overline{F}^x + \nu\xi_y\overline{F}^y) - (\beta^2-1)(\xi_x^2+\nu\xi_y^2) \\ & \cdot(\xi_x\overline{F}^x + \xi_y\overline{F}^y) + \tfrac{2i\,c_1^2}{\beta^2}(\beta^2-1)\alpha\overline{T}(\xi_x^2+\xi_y^2-\beta^2\omega^2)[(v-1)\xi_x^2+\omega^2]\Big\} \\ & \cdot\Big\{(\xi_x^2+\xi_y^2-\beta^2\omega^2)(\xi_x^2+\xi_y^2-\omega^2)\Big\}^{-1}\cdot dW \end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned} \tau^{yy} = -i\rho\frac{1}{(2\pi)^{\frac{3}{2}}} \int_{W_3} & \Big\{\beta^2(\xi_x^2+\xi_y^2-\omega^2)(\nu\,\xi_x\overline{F}^x + \xi_y\overline{F}^y) - (\beta^2-1) \\ & \cdot(\nu\,\xi_x^2+\xi_y^2)(\xi_x\overline{F}^x + \xi_y\overline{F}^y) + \tfrac{2i\,c_1^2}{\beta^2}(\beta^2-1)\alpha\overline{T}(\xi_x^2+\xi_y^2-\beta^2\omega^2) \\ & \cdot[(\nu-1)\xi_x^2+\omega^2]\Big\}\cdot\exp[-i(x\,\xi_x + y\xi_y + \omega\tau)] \\ & \cdot\Big\{(\xi_x^2+\xi_y^2-\beta^2\omega^2)\cdot(\xi_x^2+\xi_y^2-\omega^2)\Big\}^{-1}\cdot dW \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} \tau^{xy} = \frac{-i\rho}{\beta^2}\frac{1}{(2\pi)^{\frac{3}{2}}} \int_{W_3} & \Big\{\beta^2(\xi_x^2+\xi_y^2-\omega^2)(\xi_y\overline{F}^x + \xi_x\overline{F}^y) - 2(\beta^2-1)\xi_y\,\xi_x \\ & \cdot(\xi_x\overline{F}^x + \xi_y\overline{F}^y) + \frac{2i\,c_1^2}{\beta^2}(\beta^2-1)\alpha\overline{T}(\xi_x^2+\xi_y^2)(\xi_x^2+\xi_x^2-\beta^2\omega^2)]\Big\} \\ & \cdot\exp[-i(x\xi_x + y\xi_y + \omega\tau)]\cdot\Big\{(\xi_x^2+\xi_y^2-\beta^2\omega^2)\cdot(\xi_x^2+\xi_y^2-\omega^2)\Big\}^{-1}\cdot dW \end{aligned}\right\} \tag{15}$$

where W is the $\xi_x\ \xi_y\ \omega$ space and $dW = d\xi_x\, d\xi_y d\omega$.
Now, from the set of relations $\tau^{xx}+\tau^{yy}$, $\tau^{xx}-\tau^{yy}$, normal stresses τ^{xx} and τ^{yy} can be obtained easily.

Solution of the Problem

We consider the solution of the equation of motion when the time-dependent body force[3] and temperature T, acting at the origin in the direction of x-increasing, varies harmonically with time period $\frac{2\pi}{\rho}$. For such case we may write

$$X = \frac{F^x}{\rho}\delta(x)\delta(y)\delta(t) \tag{16}$$

$$T = \frac{T_0}{2\mu}\delta(x)\delta(y)\delta(t) \tag{17}$$

which gives us for $\overline{X}$ and $\overline{T}$, the relation

$$\overline{X} = \frac{F^x c_1}{\rho(2\pi)^{\frac{3}{2}}} \tag{18}$$

$$\overline{T} = \frac{T_0 c_1}{2\mu(2\pi)^{\frac{3}{2}}} \tag{19}$$

since $\delta(t) = c_1\delta(\tau)$. Here we have chosen for the shake of simplicity $F^x = X$, $F^y = 0$. We shall adopt the usual notation for this problem, *i.e.*

$$v^x = u_T, \quad v^y = v_T, \quad \tau^{xx} = (\sigma_x)_T, \quad \tau^{yy} = (\sigma_y)_T, \quad \tau^{xy} = (\tau_{xy})_T.$$

Putting $F^x = X$, $F^y = 0$ and using the usual notations we get from (11) and (12),

$$\left.\begin{aligned} u_T = \frac{1}{c_1^2(2\pi)^{\frac{3}{2}}} \int_{W_3} \Bigg[& \frac{\beta^2(\xi_x^2 + \xi_y^2 - \omega) - (\beta^2 - 1)\xi_x^2}{(\xi_x^2 + \xi_y^2 - \beta^2\omega^2)(\xi_x^2 + \xi_y^2 - \omega^2)} \overline{X} \\ & + \frac{2i\, c_1^2}{\beta^2} \frac{(\beta^2 - 1)\alpha\xi_x}{(\xi_x^2 + \xi_y^2 - \beta^2\omega^2)} \overline{T} \Bigg] \cdot \exp\{-i(x\,\xi_x + y\xi_y + \omega\,\tau)\} \cdot dW \end{aligned}\right\}$$

$$\left.\begin{aligned} v_T = -\frac{1}{c_1^2(2\pi)^{\frac{3}{2}}} \int_{W_3} \Bigg[& \frac{(\beta^2 - 1)\xi_x\,\xi_y}{(\xi_x^2 + \xi_y^2 - \beta_y^2\omega^2)(\xi_x^2 + \xi_y^2 - \omega^2)} \overline{X} \\ & + \frac{2i\, c_1^2}{\beta^2} \frac{\alpha\xi_x}{(\xi_x^2 + \xi_y^2 - \beta^2\omega^2)} \overline{T} \Bigg] \cdot \exp\{-i(x\,\xi_x + y\xi_y + \omega\,\tau)\} \cdot dW \end{aligned}\right\}$$

Rewriting u_T and v_T we get,

$$\left.\begin{aligned} u_T = \frac{1}{c_1^2(2\pi)^{\frac{3}{2}}} \int_{W_3} \Bigg[& \frac{\overline{X}}{\xi_x^2 + \xi_y^2} \left\{ \frac{\xi_x^2}{\xi_x^2 + \xi_y^2 - \omega^2} + \frac{\beta^2\xi_y^2}{\xi_x^2 + \xi_y^2 - \beta^2\omega^2} \right\} \\ & + 2i\frac{\mu}{\rho}(\beta^2 - 1)\alpha\,\overline{T} \frac{\xi_x}{(\xi_x^2 + \xi_y^2 - \beta^2\omega^2)} \Bigg] \cdot \exp\{-i(x\,\xi_x + y\,\xi_y + \omega\,\tau)\} \cdot dW \end{aligned}\right\} \tag{20}$$

$$\left.\begin{aligned} v_T = \frac{1}{c_1^2(2\pi)^{\frac{3}{2}}} \int_{W_3} \Bigg[& \frac{\overline{X}\xi_x\,\xi_y}{\xi_x^2 + \xi_y^2} \left\{ \frac{1}{\xi_x^2 + \xi_x^2 - \omega^2} - \frac{\beta^2}{\xi_x^2 + \xi_y^2 - \beta^2\omega^2} \right\} \\ & - 2i\frac{\mu}{\rho}(\beta^2 - 1)\alpha\,\overline{T} \frac{\xi_y}{(\xi_x^2 + \xi_y^2 - \beta^2\omega^2)} \Bigg] \cdot \exp\{-i(x\,\xi_x + y\,\xi_y + \omega\,\tau)\} \cdot dW \end{aligned}\right\} \tag{21}$$

Subsitiuting the value of $\overline{X}$ and $\overline{T}$ form (18) and (19) into (20) and (21), we get

$$\left.\begin{aligned} u_T = & -\frac{1}{8\pi^3}\frac{Fc_1}{\mu\beta^2}\left(\frac{\partial^2 I_1}{\partial x^2} + \beta^2\frac{\partial^2 I_2}{\partial y^2}\right) \\ & - \frac{1}{8\pi^3}\frac{(\beta^2 - 1)}{\mu\beta^2}\alpha T_0 c_1 \frac{\partial}{\partial x}\left(\frac{\partial^2 I_2}{\partial x^2} + \frac{\partial^2 I_2}{\partial y^2}\right) \end{aligned}\right\} \tag{22}$$

$$\left.\begin{aligned} v_T = & -\frac{1}{8\pi^3}\frac{Fc_1}{\mu\beta^2}\frac{\partial^2}{\partial x \partial y}(I_1 - \beta^2 I_2) \\ & + \frac{1}{8\pi^3}\frac{(\beta^2 - 1)}{\mu\beta^2}\alpha\, T_0\, c_1 \frac{\partial}{\partial y}\left(\frac{\partial^2 I_2}{\partial x^2} + \frac{\partial^2 I_2}{\partial y^2}\right) \end{aligned}\right\} \tag{23}$$

where

$$I_1 = \int_{W_3} \frac{\exp\{-i(x\,\xi_x + y\,\xi_y + \omega\tau)\}}{(\xi_x^2+\xi_y^2)(\xi_x^2+\xi_y^2-\omega^2)} \cdot dW \tag{24}$$

$$I_2 = \int_{W_3} \frac{\exp\{-i(x\,\xi_x + y\,\xi_y + \omega\tau)\}}{(\xi_x^2+\xi_y^2)(\xi_x^2+\xi_y^2-\beta^2\omega^2)} \cdot dW \tag{25}$$

Putting $x = r\cos\theta$, $y = r\sin\theta$, $\xi_x = \rho\cos\phi$, $\xi_y = \rho\,\sin\phi$ in (24) and (25).
After making necessary substitution and then integrating we find that $\frac{\partial I_2}{\partial r}$ is a function of r only, and that[4]

$$\frac{\partial I_2}{\partial_r} = -\frac{4\pi^2}{\beta}\int_0^\infty \frac{\sin(\frac{\rho\tau}{\beta})J_1(\rho\,r)}{\rho}d\rho$$
$$= \begin{cases} -\dfrac{4\pi^2\tau}{\beta^2 r}, & (\tau \le \beta r) \\ -\dfrac{4\pi^2}{\beta^2 r}\Big\{\tau - \sqrt{(\tau^2-\beta^2 r^2)}\Big\}, & (\tau \ge \beta r) \end{cases} \tag{26}$$

A similar expression can be obtained for $\frac{\partial I_1}{\partial r}$ by putting $\beta = 1$ in equation (26).
Substituting these values into equation (22) and (23) we obtain the formula

$$u_T = \begin{cases} 0, & (r > \tau) \\ \dfrac{Fc_1}{2\pi\mu\beta^2}\Big[\dfrac{x^2}{r^2}(\tau^2-r^2)^{-\frac{1}{2}} + \dfrac{x^2-y^2}{r^4}(\tau^2-r^2)^{\frac{1}{2}}\Big], & (\tau' < r < \tau) \\ \dfrac{Fc_1}{2\pi\mu\beta^2}\Big[\dfrac{x^2}{r^2}(\tau^2-r^2)^{-\frac{1}{2}} + \dfrac{\beta y^2}{r^2}(\tau'^2-r^2)^{-\frac{1}{2}} & \\ \qquad + \dfrac{x^2-y^2}{r^4}\Big\{(\tau^2-r^2)^{\frac{1}{2}} - \beta(\tau'^2-r^2)^{\frac{1}{2}}\Big\}\Big] & \\ \qquad + \dfrac{T_0\,\alpha\,c_1}{2\pi\,\mu\,\beta^3}(\beta^2-1)x(\tau'^2-r^2)^{-\frac{3}{2}}, & (r < \tau) \end{cases} \tag{27}$$

$$v_T = \begin{cases} 0, & (r > \tau) \\ \dfrac{Fc_1}{2\pi\mu\beta^2}\dfrac{xy}{r^2}\Big\{(\tau^2-r^2)^{-\frac{1}{2}} + \dfrac{2}{r^2}(\tau^2-r^2)^{\frac{1}{2}}\Big\}, & (\tau' < r < \tau) \\ -\dfrac{T_0\,c_1}{2\pi\,\mu}\dfrac{\alpha}{\beta^3}(\beta^3-1)y(\tau'^2-r^2)^{\frac{-3}{2}} & \\ +\dfrac{xy}{r^2}\dfrac{Fc_1}{2\pi\mu\beta^2}\Big\{(\tau'^2-r^2)^{-\frac{1}{2}} - \beta(\tau'^2-r^2)^{-\frac{1}{2}}\Big\} & \\ +\dfrac{2}{r^2}\Big[(\tau^2-r^2)^{\frac{1}{2}} - \beta(\tau'^2-r^2)^{\frac{1}{2}}\Big]\Big\}, & (r < \tau') \end{cases} \tag{28}$$

In these formula $\tau = c_1\,t$, $\tau' = c_2\,t = \frac{c_1\,t}{\beta}$.
These determine the components of displacement vector.
Differentiating u_T and v_T, and substituting these values in (4) we obtain,

$$\frac{\tau\beta^2[(\sigma_x)_T-(\sigma_y)_T]}{Fc_1}=\begin{cases}\frac{T_0\alpha}{F}\cdot\frac{(\beta^2-1)}{\beta}\Big\{2(\tau'^2-r^2)^{-\frac{3}{2}}+3r^2(\tau'^2-r^2)^{-\frac{5}{2}}\Big\}\\ \quad+(\tau^2-r^2)^{-\frac{3}{2}}-\frac{3y^2}{r^2}\Big\{(\tau^2-r^2)^{-\frac{3}{2}}-\beta(\tau'^2-r^2)^{-\frac{3}{2}}\Big\}\\ \quad-\frac{2}{r^4}(x^2-3y^2)\Big\{(\tau^2-r^2)^{-\frac{1}{2}}-\beta(\tau'^2-r^2)^{-\frac{1}{2}}\Big\}\\ \quad-\frac{4}{r^6}(x^2-3y^2)\Big\{(\tau^2-r^2)^{\frac{1}{2}}-\beta(\tau'^2-r^2)^{\frac{1}{2}}\Big\}, & (r<\tau'),\\ \frac{x^2-y^2}{r^2}(\tau^2-r^2)^{-\frac{3}{2}}-\frac{2(x^2-3y^2)}{r^4}(\tau^2-r^2)^{-\frac{1}{2}}\\ \quad-\frac{4}{r^6}(x^2-3y^2)(\tau^2-r^2)^{\frac{1}{2}}, & (\tau'<r<\tau),\\ 0, & (r>\tau)\end{cases} \tag{29}$$

$$\frac{2\pi\beta^2[(\sigma_x)_T-(\sigma_y)_T]}{(\beta^2-)c_1F}=\begin{cases}x(\tau^2-r^2)^{-\frac{3}{2}}, & (r<\tau)\\ 0, & (r>\tau),\end{cases} \tag{30}$$

$$\frac{2\pi\beta^2(\tau_{xy})_T}{y\,c_1\,F}=\begin{cases}\beta^2(\tau'^2-r^2)^{-\frac{3}{2}}+\frac{2x^2}{r^2}\Big\{(\tau'^2-r^2)^{-\frac{3}{2}}-\beta(\tau'^2-r^2)^{-\frac{3}{2}}\Big\}\\ \frac{2(y^2-3x^2)}{r^4}\Big\{(\tau^2-r^2)^{-\frac{1}{2}}-\beta(\tau'^2-r^2)^{-\frac{1}{2}}\Big\}\\ \quad-\frac{4}{r^6}(x^2-3y^2)\Big\{(\tau^2-r^2)^{-\frac{1}{2}}-\beta(\tau'^2-r^2)^{-\frac{1}{2}}\Big\}, & (r<\tau'),\\ \frac{2x^2}{r^2}(\tau^2-r^2)^{-\frac{3}{2}}+\frac{2(y^2-3x^2)}{r^4}(\tau^2-r^2)^{-\frac{1}{2}},\\ \quad+\frac{4}{r^6}(x^2-3y^2)(\tau^2-r^2)^{\frac{1}{2}}, & (\tau'<r<\tau),\\ 0 & (r>\tau)\end{cases} \tag{31}$$

From (29) and (30) principal stresses $((\sigma_x)_T,\,(\sigma_y)_T)$ can be determined easily.

From the expressions it is clear that the distribution is propagated outwards from the centre with velocities c_1 and $c_2 = \frac{c_1}{\beta}$. These waves are known in sismology as the P-waves, and s-waves, respectively (Bullen 1947, p.47). The wave fronts are circles, centre the origin and radii $\tau = c_1 t$, $\tau' = c_2 t$.

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