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AN ECOLOGICAL MODEL INVOLVING NONLOCAL OPERATOR AND REACTION DIFFUSION

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Abstract. Using the method of sub-super solutions, we study the existence of positive solutions for a class of infinite semipositone problems involving nonlocal operator.

1. Introduction

In this paper, we study the following nonlinear reaction diffusion problem:

$$\begin{cases} -A\left(\int_\Omega |\nabla u|^p\,dx\right)\Delta_p u = au^{p-1} - f(u) - \frac{c}{u^\alpha}, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where Δ_p denotes the p-Laplacian operator defined by $\Delta_p z = \mathrm{div}(|\nabla z|^{p-2}\nabla z)$, $p > 1$, Ω is a bounded domain of $\mathbb{R}^N$ with smooth boundary, $\alpha \in (0,1)$, a and c are positive constants and $f : [0,\infty) \to \mathbb{R}$ is a continuous function. This model arises in the studies of population biology of one species with u representing the concentration of the species. We discuss the existence of positive solution when f satisfies certain additional conditions. We make the following assumptions:

(H_1) There exist $L > 0$ and $\beta > 0$ such that $f(u) \leq Lu^\beta$, for all $u \geq 0$.

(H_2) There exists a constant $S > 0$ such that $au^{p-1} < f(u) + S$ for all $u \geq 0$.

(H_3) $A : [0,\infty) \to \mathbb{R}$ is a continuous and increasing function such that $0 < A_0 \leq A(t) \leq A_\infty$ for all t.

More recently, reaction diffusion models have been used to describe spatiotemporal phenomena in disciplines other than ecology, such as physics, chemistry, and biology (see [4], [18], [22]). In addition, most ecological systems have some form of predation or harvesting

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of the population. For example, hunting or fishing is often used as an effective means of wildlife management. This model describes the dynamics of the fish population with predation. In such cases u denotes the population density and the term $\frac{c}{u^\alpha}$ corresponds to predation. So, the study of positive solutions of (1.1) has more practical meanings. In recent years, problems involving Kirchhoff type operators have been studied in many papers; we refer to [1, 2, 3, 5, 6, 8, 10, 17, 23, 24] in which the authors have used variational method and topological method to get the existence of solutions for (1.1). In this paper, motivated by the ideas introduced in [21] and the properties of Kirchhoff type operators in [7, 9, 13], we study problem (1.1) in semipositone case (i.e., $\lim_{u\to 0} F(u) := -\infty$; $F(u) := au^{p-1} - f(u) - \frac{c}{u^\alpha}$); see [11, 14, 15, 16, 20]. Our approach is based on the method of sub- and super-solutions (see [11]). Our result in this note improves the previous one [21] in which $M(t) \equiv 1$. To our best knowledge, this is a new research topic for nonlocal problems; see [1, 13].

2. Preliminaries and main result

Let $W_0^{1,s} = W_0^{1,s}(\Omega)$, $s > 1$, denote the usual Sobolev space. To precisely state our existence result we consider the eigenvalue problem

$$\begin{cases} -\Delta_p \phi = \lambda |\phi|^{p-2}\phi, & x \in \Omega, \\ \phi = 0, & x \in \partial\Omega. \end{cases} \tag{2.2}$$

Let $\phi_{1,p}$ be the eigenfunction corresponding to the first eigenvalue $\lambda_{1,p}$ of (2.2) such that $\phi_{1,p}(x) > 0$ in Ω, and $\|\phi_{1,p}\|_\infty = 1$, where $\|\phi_{1,p}\|_\infty$ denotes the essential supremum of $\phi_{1,p}$. It can be shown that $\frac{\partial \phi_{1,p}}{\partial n} < 0$ on $\partial\Omega$. Here n is the outward normal. We will also consider the unique solution, $\zeta(x) \in C^1(\overline{\Omega})$, of the boundary value problem

$$\begin{cases} -\Delta_p \zeta = 1 & x \in \Omega, \\ \zeta = 0, & x \in \partial\Omega, \end{cases}$$

to discuss our existence result. It is known that $\zeta(x) > 0$ in Ω and $\frac{\partial \zeta(x)}{\partial n} < 0$ on $\partial\Omega$.

Now, we give the definitions of sub- and super-solutions of (1.1).

A function ψ is said to be a subsolution of problem (1.1) if it is in $W^{1,p}(\Omega)$ such that $\psi \le 0$ on $\partial\Omega$ and satisfies

$$A\left(\int_\Omega |\nabla \psi|^p \, dx\right) \int_\Omega |\nabla \psi|^{p-2} \nabla\psi \cdot \nabla w \, dx \le \int_\Omega \left(a\psi^{p-1} - f(\psi) - \frac{c}{\psi^\alpha}\right) w \, dx, \quad \forall w \in W, \tag{2.3}$$

where $W := \left\{ w \in C_0^\infty(\Omega) : \ w \ge 0 \text{ in } \Omega \right\}$. A non-negative function z is called a super-solution of (1.1) if it satisfies $z \ge 0$ on $\partial\Omega$ and satisfies

$$A\left(\int_\Omega |\nabla z|^p \, dx\right) \int_\Omega |\nabla z|^{p-2} \nabla z \cdot \nabla w \, dx \ge \int_\Omega \left(a z^{p-1} - f(z) - \frac{c}{z^\alpha}\right) w \, dx, \quad \forall w \in W. \tag{2.4}$$

Then the following result holds.

Lemma 2.1 ([11]). *Suppose there exist sub- and super-solutions ψ and z respectively of* (1.1) *such that $\psi \le z$. Then* (1.1) *has a solution u such that $\psi \le u \le z$.*

We are now ready to give our existence result.

Theorem 2.2. *Let (H_1), (H_2) and (H_3) hold. If $\frac{a}{A_\infty} > (\frac{p}{p-1+\alpha})\lambda_{1,p}$, then there exists $c_0 > 0$ such that if $0 < c < c_0$, then the problem* (1.1) *admits a positive solution.*

Proof. We start with the construction of a positive sub-solution for (1.1). To get a positive sub-solution, we can apply an anti-maximum principle (see [12]), from which we know that there exist a $\delta_1 > 0$ and a solution z_λ of

$$\begin{cases} -\Delta_p z - \lambda z^{p-1} = -1 & x \in \Omega, \\ \zeta = 0, & x \in \partial\Omega, \end{cases} \tag{2.5}$$

for $\lambda \in (\lambda_{1,p}, \lambda_{1,p} + \delta_1)$. Fix $\widehat{\lambda} \in (\lambda_{1,p}, \min\{(\frac{p-1+\alpha}{p})a, \lambda_{1,p} + \delta_1\})$. Let $\gamma = \|z_\lambda\|_\infty$ and z_λ be the solution of (2.5) when $\lambda = \widehat{\lambda}$. It is well known that $z_{\widehat{\lambda}} > 0$ in Ω and $\frac{\partial z_{\widehat{\lambda}}}{\partial n} < 0$ on $\partial\Omega$, where n is the outer unit normal to Ω. Hence there exist positive constants ϵ, δ, σ such that

$$|\nabla z_{\widehat{\lambda}}|^p \ge \epsilon, \qquad x \in \overline{\Omega}_\delta, \tag{2.6}$$

$$z_{\widehat{\lambda}} \ge \sigma, \qquad x \in \Omega_0 = \Omega \setminus \overline{\Omega}_\delta, \tag{2.7}$$

where $\overline{\Omega}_\delta = \{x \in \Omega \,|\, d(x, \partial\Omega) \le \delta\}$.

We construct a sub-solution ψ of (1.1) using $z_{\widehat{\lambda}}$. Define $\psi = M(\frac{p-1+\alpha}{p}) z_{\widehat{\lambda}}^{\frac{p}{p-1+\alpha}}$, where

$$M = \min\left\{ \left(\frac{A_\infty \left(\frac{p}{(p-1+\alpha)}\right)^\beta}{L\gamma^{\frac{p\beta-(1-\alpha)(p-1)}{p-1+\alpha}}} \right)^{\frac{1}{\beta-p+1}}, \left(\frac{\left(\frac{p-1}{Lp}\right)\left[\left(\frac{p-1+\alpha}{p}\right)^{p-1} a - A_\infty \widehat{\lambda}\right]}{\left(\frac{p-1+\alpha}{p}\right)^\beta \gamma^{\frac{p\beta-p(p-1)}{p-1+\alpha}}} \right)^{\frac{1}{\beta-p+1}} \right\}.$$

Let $w \in W$. Then a calculation shows that

$$\nabla\psi = M z_{\widehat{\lambda}}^{\frac{1-\alpha}{p-1+\alpha}} \nabla z_{\widehat{\lambda}}$$

$$\begin{aligned} &A\left(\int_{\overline{\Omega}_\delta} |\nabla\psi|^p\,dx\right) \int_{\overline{\Omega}_\delta} |\nabla\psi|^{p-2}\nabla\psi\nabla w\,dx \\ &= A\left(\int_{\overline{\Omega}_\delta} |\nabla\psi|^p\,dx\right) M^{p-1} \int_{\overline{\Omega}_\delta} z_{\widehat{\lambda}}^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}} |\nabla z_{\widehat{\lambda}}|^{p-2} \nabla z_{\widehat{\lambda}} \nabla w\,dx \\ &= A\left(\int_{\overline{\Omega}_\delta} |\nabla\psi|^p\,dx\right) M^{p-1} \int_{\overline{\Omega}_\delta} |\nabla z_{\widehat{\lambda}}|^{p-2} \nabla z_{\widehat{\lambda}} \left[\nabla\left(z_{\widehat{\lambda}}^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}} w\right) - |\nabla z_{\widehat{\lambda}}|^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}} w\right] dx \\ &= A\left(\int_{\overline{\Omega}_\delta} |\nabla\psi|^p\,dx\right) M^{p-1} \int_{\overline{\Omega}_\delta} \left[z_{\widehat{\lambda}}^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}} \left(\widehat{\lambda} z_{\widehat{\lambda}}^{p-1} - 1\right) - \frac{(1-\alpha)(p-1)}{p-1+\alpha} \frac{|\nabla z_{\widehat{\lambda}}|^p}{z_{\widehat{\lambda}}^{\frac{\alpha p}{p-1+\alpha}}} \right] w\,dx \end{aligned}$$

$$\leq A_\infty M^{p-1}\int_{\overline{\Omega}_\delta}\left[\widehat{\lambda} z_{\widehat{\lambda}}^{\frac{p(p-1)}{p-1+\alpha}} - z_{\widehat{\lambda}}^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}} - \frac{(1-\alpha)(p-1)}{p-1+\alpha}\frac{|\nabla z_{\widehat{\lambda}}|^p}{z_{\widehat{\lambda}}^{\frac{\alpha p}{p-1+\alpha}}}\right] w\,dx, \tag{2.8}$$

and

$$\int_{\overline{\Omega}_\delta}\left[a\psi^{p-1} - f(\psi) - \frac{c}{\psi^\alpha}\right] w\,dx$$
$$= \left[aM^{p-1}\Big(\frac{p-1+\alpha}{p}\Big)^{p-1} z_{\widehat{\lambda}}^{\frac{p(p-1)}{p-1+\alpha}} - f\Big(M\Big(\frac{p-1+\alpha}{p}\Big)z_{\widehat{\lambda}}^{\frac{p}{p-1+\alpha}}\Big) - \frac{c}{M^\alpha(\frac{p-1+\alpha}{p})^\alpha z_{\widehat{\lambda}}^{\frac{\alpha p}{p-1+\alpha}}}\right] w\,dx. \tag{2.9}$$

Let $c_0 = M^{p-1+\alpha}\min\{\frac{A_\infty(1-\alpha)(p-1)}{p-1+\alpha}(\frac{p-1+\alpha}{p})^\alpha\epsilon, \frac{1}{p}(\frac{p-1+\alpha}{p})^\alpha\sigma^p[(\frac{p-1+\alpha}{p})a - A_\infty\widehat{\lambda}]\}$. Hence by (2.3), ψ is a sub-solution of (1.1) if

$$A_\infty M^{p-1}\widehat{\lambda} z_{\widehat{\lambda}}^{\frac{p(p-1)}{p-1+\alpha}} \leq aM^{p-1}\Big(\frac{p-1+\alpha}{p}\Big)^{p-1} z_{\widehat{\lambda}}^{\frac{p(p-1)}{p-1+\alpha}},$$
$$-A_\infty M^{p-1} z_{\widehat{\lambda}}^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}} \leq -f\Big(M\Big(\frac{p-1+\alpha}{p}\Big)z_{\widehat{\lambda}}^{\frac{p}{p-1+\alpha}}\Big),$$

and

$$-A_\infty M^{p-1}\frac{(1-\alpha)(p-1)}{p-1+\alpha}\frac{|\nabla z_{\widehat{\lambda}}|^p}{z_{\widehat{\lambda}}^{\frac{\alpha p}{p-1+\alpha}}} \leq -\frac{c}{M^\alpha(\frac{p-1+\alpha}{p})^\alpha z_{\widehat{\lambda}}^{\frac{\alpha p}{p-1+\alpha}}},$$

for all $x\in\Omega$ and $c<c_0$. First we consider the case when $x\in\overline{\Omega}_\delta$. Since $(\frac{p}{p-1+\alpha})^{p-1}\widehat{\lambda} \leq \frac{a}{A_\infty}$, we have

$$A_\infty M^{p-1}\widehat{\lambda} z_{\widehat{\lambda}}^{\frac{p(p-1)}{p-1+\alpha}} \leq aM^{p-1}\Big(\frac{p-1+\alpha}{p}\Big)^{p-1} z_{\widehat{\lambda}}^{\frac{p(p-1)}{p-1+\alpha}}, \tag{2.10}$$

and from the choice of M, we know that

$$\frac{L}{A_\infty}M^{\beta-p+1}\gamma^{\frac{p\beta-(1-\alpha)(p-1)}{p-1+\alpha}} \leq \Big(\frac{p}{p-1+\alpha}\Big)^\beta. \tag{2.11}$$

By (2.11) and (H_1) we have

$$\begin{aligned} -A_\infty M^{p-1} z_{\widehat{\lambda}}^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}} &\leq -LM^\beta\Big(\frac{p-1+\alpha}{p}\Big)^\beta z_{\widehat{\lambda}}^{\frac{p\beta}{p-1+\alpha}} \\ &\leq -f\Big(M\Big(\frac{p-1+\alpha}{p}\Big)z_{\widehat{\lambda}}^{\frac{p}{p-1+\alpha}}\Big). \end{aligned} \tag{2.12}$$

Next, from (2.6) and definition of c_0, we have

$$A_\infty M^{p-1}\frac{(1-\alpha)(p-1)}{p-1+\alpha}|\nabla z_{\widehat{\lambda}}|^p \geq \frac{c}{M^\alpha(\frac{p-1+\alpha}{p})^\alpha},$$

and

$$-A_\infty M^{p-1}\frac{(1-\alpha)(p-1)}{p-1+\alpha}\frac{|\nabla z_{\widehat{\lambda}}|^p}{z_{\widehat{\lambda}}^{\frac{\alpha p}{p-1+\alpha}}} \leq -\frac{c}{M^\alpha(\frac{p-1+\alpha}{p})^\alpha z_{\widehat{\lambda}}^{\frac{\alpha p}{p-1+\alpha}}}. \tag{2.13}$$

Hence by using (2.11), (2.12) and (2.13) for $c \le c_0$, we have

$$A\Big(\int_{\overline{\Omega}_\delta} |\nabla\psi|^p\,dx\Big)\int_{\overline{\Omega}_\delta} |\nabla\psi|^{p-2}|\nabla\psi|\cdot\nabla w\,dx \le \int_{\overline{\Omega}_\delta}\Big[a\psi^{p-1}-f(\psi)-\frac{c}{\psi^\alpha}\Big]w\,dx. \tag{2.14}$$

On the other hand, on $\Omega_0 = \Omega\setminus\overline{\Omega}_\delta$, we have $z_{\widehat\lambda} \ge \sigma$, for some $0<\sigma<1$, and from the definition of c_0, for $c \le c_0$ we have

$$\begin{aligned}\frac{c}{M^\alpha}\Big(\frac{p-1+\alpha}{p}\Big)^\alpha &\le \frac{1}{p}M^{p-1}\sigma^p\Big[\Big(\frac{p-1+\alpha}{p}\Big)a - A_\infty\widehat\lambda\Big]\\ &\le \frac{1}{p}M^{p-1}z_{\widehat\lambda}^p\Big[\Big(\frac{p-1+\alpha}{p}\Big)a - A_\infty\widehat\lambda\Big].\end{aligned} \tag{2.15}$$

Also from the choice of M, we have

$$LM^{\beta-p+1}\Big(\frac{p-1+\alpha}{p}\Big)^\beta z_{\widehat\lambda}^{\frac{p\beta-p(p-1)}{p-1+\alpha}} \le \frac{p-1}{p}\Big[\Big(\frac{p-1+\alpha}{p}\Big)^{p-1}a - A_\infty\widehat\lambda\Big]. \tag{2.16}$$

Hence from (2.15) and (2.16) we have

$$\begin{aligned}
&A\Big(\int_{\Omega_0}|\nabla\psi|^p\,dx\Big)\int_{\Omega_0}|\nabla\psi|^{p-2}\nabla\psi\nabla w\,dx\\
&= A\Big(\int_{\Omega_0}|\nabla\psi|^p\,dx\Big)\int_{\Omega_0}\Big[M^{p-1}\widehat\lambda z_{\widehat\lambda}^{\frac{p(p-1)}{p-1+\alpha}} - M^{p-1}z_{\widehat\lambda}^{\frac{(1-\alpha)(p-1)}{p-1+\alpha}} - M^{p-1}\frac{(1-\alpha)(p-1)}{p-1+\alpha}\frac{|\nabla z_{\widehat\lambda}^p|}{z_{\widehat\lambda}^{\frac{\alpha p}{p-1+\alpha}}}\Big]w\,dx\\
&\le \int_{\Omega_0} A_\infty M^{p-1}\widehat\lambda z_{\widehat\lambda}^{\frac{p(p-1)}{p-1+\alpha}}w\,dx = \int_{\Omega_0}\frac{A_\infty}{z_{\widehat\lambda}^{\frac{\alpha p}{p-1+\alpha}}}\Big[\frac{1}{p}\widehat\lambda M^{p-1}z_{\widehat\lambda}^p + \frac{p-1}{p}\widehat\lambda M^{p-1}z_{\widehat\lambda}^p\Big]w\,dx\\
&\le \int_{\Omega_0}\frac{1}{z_{\widehat\lambda}^{\frac{\alpha p}{p-1+\alpha}}}\Big[\Big(\frac{1}{p}M^{p-1}\Big(\frac{p-1+\alpha}{p}\Big)^{p-1}az_{\widehat\lambda}^p - \frac{c}{M^\alpha(\frac{p-1+\alpha}{p})^\alpha}\Big) + M^{p-1}z_{\widehat\lambda}^p\Big(\frac{p-1+\alpha}{p}\Big)^{p-1}\\
&\quad\times\Big(\frac{(p-1)a}{p} - LM^{\beta-p+1}\Big(\frac{p-1+\alpha}{p}\Big)^{\beta-p+1}z_{\widehat\lambda}^{\frac{p\beta-p(p-1)}{p-1+\alpha}}\Big)\Big]w\,dx\\
&= \int_{\Omega_0}\Big[aM^{p-1}\Big(\frac{p-1+\alpha}{p}\Big)^{p-1}z_{\widehat\lambda}^{\frac{p(p-1)}{p-1+\alpha}} - LM^\beta\Big(\frac{p-1+\alpha}{p}\Big)^\beta z_{\widehat\lambda}^{\frac{p\beta}{p-1+\alpha}} - \frac{cz_{\widehat\lambda}^{\frac{-\alpha p}{p-1+\alpha}}}{M^\alpha(\frac{p-1+\alpha}{p})^\alpha}\Big]w\,dx\\
&\le \int_{\Omega_0}\Big[aM^{p-1}\Big(\frac{p-1+\alpha}{p}\Big)^{p-1}z_{\widehat\lambda}^{\frac{p(p-1)}{p-1+\alpha}} - f\Big(M\Big(\frac{p-1+\alpha}{p}\Big)z_{\widehat\lambda}^{\frac{p}{p-1+\alpha}}\Big) - \frac{c}{M^\alpha(\frac{p-1+\alpha}{p})^\alpha z_{\widehat\lambda}^{\frac{\alpha p}{p-1+\alpha}}}\Big]w\,dx\\
&= \int_{\Omega_0}\Big[a\psi^{p-1} - f(\psi) - \frac{c}{\psi^\alpha}\Big]w\,dx.
\end{aligned} \tag{2.17}$$

By using (2.14) and (2.17) we see that ψ is a sub-solution of (1.1). Next, we construct a super-solution z of (1.1) such that $z \ge \psi$. By (H_2), we can choose a large constant S^* such that

$au^{p-1} - f(u) - \frac{c}{u^\alpha} \le S^* A_0$ for all $u > 0$. Let $z = (S^*)^{\frac{1}{p-1}}\zeta(x)$. We shall verify that z is a super-solution of (1.1). To this end, let $w(x) \in W_0^{1,p}(\Omega)$ with $w \ge 0$. Then we have

$$A\left(\int_\Omega |\nabla z|^p\,dx\right)\int_\Omega |\nabla z|^{p-2}\nabla z \nabla w\,dx = A\left(\int_\Omega |\nabla z|^p\,dx\right) S^* \int_\Omega w\,dx \ge S^* A_0 \int_\Omega w\,dx$$
$$\ge \int_\Omega \left[az^{p-1} - f(z) - \frac{c}{z^\alpha}\right] w\,dx. \tag{2.18}$$

Thus z is a super-solution of (1.1). Finally, we can choose $S^* \gg 1$ (where $\gg 1$ means large enough) such that $\psi \le z$ in Ω. Hence, for $c \le c_0$ by Lemma 2.1 there exists a positive solution u of (1.1) such that $\psi \le u \le z$. This completes the proof. □

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