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ON THREE DIMENSIONAL COSYMPLECTIC MANIFOLDS ADMITTING ALMOST RICCI SOLITONS

UDAY CHAND DE AND CHIRANJIB DEY

Abstract. In the present paper we study three dimensional cosymplectic manifolds admitting almost Ricci solitons. Among others, we prove that in a three dimensional compact orientable cosymplectic manifold M^3 without boundary, an almost Ricci soliton reduces to a Ricci soliton under certain restriction on the potential function λ. As a consequence we obtain a corollary. Moreover, we study gradient almost Ricci solitons.

1. Introduction

The study of almost Ricci solitons was introduced by Pigola et al. [19], where essentially they modified the definition of Ricci soliton by adding the condition on the parameter λ to be a variable function. More precisely, we say that a Riemannian manifold (M^n, g) is an almost Ricci soliton, if there exists a complete vector field V and a smooth soliton function $\lambda : M^n \longrightarrow \mathbb{R}$ satisfying

$$S + \frac{1}{2}\pounds_V g + \lambda g = 0, \tag{1.1}$$

where S and $\pounds$ stand, respectively, for the Ricci tensor and the Lie derivative. We shall refer to this equation as the fundamental equation of an almost Ricci soliton (M^n, g, V, λ). The soliton will be called expanding, steady or shrinking, respectively, if $\lambda > 0$, $\lambda = 0$ or $\lambda < 0$. Otherwise, it will be called indefinite. When the vector field V is the gradient of a smooth function $f : M^n \longrightarrow \mathbb{R}$, the manifold will be called gradient almost Ricci soliton. In this case, the preceding equation becomes

$$S + \nabla^2 f + \lambda g = 0, \tag{1.2}$$

where $\nabla^2 f$ stands for the Hessian of f.

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Corresponding author: Uday Chand.

We notice that when $n \geq 3$ and V is a Killing vector field, an almost Ricci soliton will be a Ricci soliton, since in this case we have an Einstein manifold, from which we can apply Schur's lemma to deduce that λ is constant. Ricci solitons have been studied by several authors such as Bejan et al. [4], Chen [6], Wang et al. ([21], [22], [23]), Deshmukh ([11], [12]), Cho [7], De et al. ([8], [9], [10]) and many others. Taking into account that the soliton function λ is not necessarily constant, certainly comparison with soliton theory will be modified. In particular, the rigidity result contained in Theorem 1.3 of [19] indicates that almost Ricci solitons should reveal a reasonably broad generalization of the fruitful concept of classical soliton.

To understand the geometry of almost Ricci soliton, in [2] Barros et al. proved that a compact non-trivial almost Ricci soliton with constant scalar curvature is isometric to a Euclidean sphere $\mathbb{S}^n$ and is gradient. Also, Barros and Ribeiro Jr. proved in [3] that a compact gradient almost Ricci soliton with non-trivial conformal vector field is isometric to a Euclidean sphere. In the same paper, they proved an integral formula for compact case, which was used to prove several rigidity results, for more details see [3].

Almost Ricci solitons have been studied by Duggal [13], Ghosh [15], Sharma [20] and many others.

The existence of almost Ricci soliton has been confirmed by Pigola et al. [19] on some certain class of warped product manifolds. Some characterizations of almost Ricci soliton on a compact Riemannian manifold can be found in ([1], [2] [3]). It is interesting to note that if the potential vector field V of the almost Ricci soliton (M^n, g, V, λ) is Killing, then the soliton becomes trivial, provided the dimension of M is > 2. Moreover, if V is conformal then, M^n is isometric to the Euclidean sphere S^n. Thus, the almost Ricci soliton can be considered as a generalization of Einstein metric as well as Ricci soliton.

The paper is organized as follows: After introduction, in section 2 we discuss some preliminaries of cosymplectic manifolds. Section 3 is devoted to prove our main result. Section 4 deals with the study of gradient almost Ricci solitons. Our main Theorems can be presented as follows:

Theorem 1.1. *In a three dimensional compact orientable cosymplectic manifold M^3 without boundary, an almost Ricci soliton reduces to a Ricci soliton, provided $\xi\lambda = 0$. Also the scalar curvature r cannot be constant.*

Theorem 1.2. *If a three dimensional cosymplectic manifold admits a gradient almost Ricci soliton, then it reduces to a gradient Ricci soliton.*

2. Cosymplectic manifolds

In this section, we shall collect some fundamental results regarding cosymplectic manifolds (for more details see Blair [5], Goldberg and Yano [16]). A $(2n+1)$-dimensional manifold M is said to admit an almost contact structure if it admits a tensor field ϕ of type $(1,1)$, a vector field ξ and a 1-form η satisfying ([5])

$$\text{(a)}\ \ \phi^2 = -I + \eta \otimes \xi,\ \ \text{(b)}\ \ \eta(\xi) = 1,\ \ \text{(c)}\ \ \phi\xi = 0,\ \ \text{(d)}\ \ \eta \circ \phi = 0. \tag{2.1}$$

An almost contact structure is said to be normal if the almost complex structure J on the product manifold $M \times \mathbb{R}$ defined by

$$J\Big(X, f\frac{d}{dt}\Big) = \Big(\phi X - f\xi, \eta(X)\frac{d}{dt}\Big)$$

is integrable, where X is tangent to M, t is the coordinate of $\mathbb{R}$ and f is a smooth function on $M \times \mathbb{R}$. If g is a compatible Riemannian metric with the almost contact metric structure (ϕ, ξ, η), that is,

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

then M becomes an almost contact metric structure (ϕ, ξ, η, g). From (2.2) it can be easily seen that

$$\text{(a)} \quad g(X, \phi Y) = -g(\phi X, Y), \quad \text{(b)} \quad g(X, \xi) = \eta(X), \tag{2.3}$$

for all vector fields X, Y on M. An almost contact metric structure becomes a contact metric structure if

$$g(X, \phi Y) = d\eta(X, Y), \tag{2.4}$$

for all vectors fields X, Y. In this case, the 1-form η is called a contact metric form and ξ is its characteristic vector field. We define a $(1,1)$-tensor field h by $h = \frac{1}{2}\pounds_\xi \phi$, where $\pounds$ denote the Lie derivative. Then h is symmetric and satisfies the conditions $h\phi = -\phi h$, $Tr.h = Tr.\phi h = 0$ and $h\xi = 0$. Also

$$\nabla_X \xi = -\phi X - \phi h X, \tag{2.5}$$

holds in a contact metric manifold.

An almost contact metric manifold is a Sasakian manifold if and only if

$$(\nabla_X \phi)(Y) = g(X, Y)\xi - \eta(Y)X, \tag{2.6}$$

where $X, Y \in \chi(M)$ and ∇ is the Levi-Civita connection of the Riemannian metric g. Remark that a normal contact metric manifold is a Sasakian manifold. A contact metric manifold $M^{2n+1}(\phi, \xi, \eta, g)$ for which ξ is a Killing vector field is said to be a K-contact metric manifold. Following Blair [5], an almost contact metric manifold satisfying $d\eta = 0$ and $d\Phi = 0$ where

$\Phi(X,Y) = g(X,\phi Y)$ is called an almost cosymplectic manifold. In particular, an almost cosymplectic manifold is said to be a cosymplectic manifold if the associated almost contact structure is normal, which is also equivalent to $\nabla\phi = 0$.

It is well-known that the Riemannian product of the real line and a Kähler manifold admits a cosymplectic structure. However, there exist some examples of cosymplectic manifolds which are not globally the product of a Kähler manifold and the real line (see Olszak [17]). Moreover, on a cosymplectic manifold we have the following relation (see Goldberg and Yano [16]):

$$\nabla\xi = 0 \quad (\Leftrightarrow \nabla\eta = 0), \tag{2.7}$$

this implies that ξ is a Killing vector field. By (2.7), it follows directly that

$$R(\cdot,\cdot)\xi = 0 \quad (\Rightarrow Q\xi = 0), \tag{2.8}$$

where Q denotes the Ricci operator.

3. Proof of the Theorem 1.1

Suppose that (M^3,ϕ,ξ,η,g) is a three dimensional cosymplectic manifold. It is known that the curvature tensor of a 3-dimensional Riemannian manifold is given by

$$R(X,Y)Z = [S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] - \frac{r}{2}[g(Y,Z)X - g(X,Z)Y], \tag{3.1}$$

where S and r are the Ricci tensor and the scalar curvature respectively and Q is the Ricci operator defined by $g(QX,Y) = S(X,Y)$.

If we replace both Y and Z by ξ in (3.1) and use (2.8), then the Ricci operator can be written as

$$QX = \frac{r}{2}X - \frac{r}{2}\eta(X)\xi, \tag{3.2}$$

for all vector fields X. This means that M^3 is an η-Einstein manifold.

In view of equation (3.2), the Ricci tensor is given by

$$S(X,Y) = \frac{r}{2}g(X,Y) - \frac{r}{2}\eta(X)\eta(Y). \tag{3.3}$$

By our hypothesis (M^3,ϕ,ξ,η,g) admits an almost Ricci soliton. Therefore, (1.1) becomes

$$\begin{aligned}(\pounds_V g)(Y,Z) &= -2S(Y,Z) - 2\lambda g(Y,Z)\\ &= -(2\lambda + r)g(Y,Z) + r\eta(Y)\eta(Z).\end{aligned} \tag{3.4}$$

Taking covariant differentiation of $£_V g$ with respect to X, we get

$$(\nabla_X £_V g)(Y,Z) = -[2(X\lambda)+(Xr)]g(Y,Z)+(Xr)\eta(Y)\eta(Z), \tag{3.5}$$

for any vector field X, Y, Z on M. Following Yano ([24], pp.23), the following formula holds

$$(£_V\nabla_X g - \nabla_X £_V g - \nabla_{[V,X]} g)(Y,Z) = -g((£_V\nabla)(X,Y),Z) - g((£_V\nabla)(X,Z),Y)$$

for any vector fields X, Y, Z on M. As g is parallel with respect to the Levi-Civita connection ∇, then the above relation becomes

$$(\nabla_X £_V g)(Y,Z) = g((£_V\nabla)(X,Y),Z)+g((£_V\nabla)(X,Z),Y) \tag{3.6}$$

for any vector fields X, Y, Z on M. Since $£_V\nabla$ is a symmetric tensor of type $(1,2)$, i.e., $(£_V\nabla)(X,Y) = (£_V\nabla)(Y,X)$, it follows from (3.6) that

$$g((£_V\nabla)(X,Y),Z) = \frac{1}{2}(\nabla_X £_V g)(Y,Z)+\frac{1}{2}(\nabla_Y £_V g)(X,Z)-\frac{1}{2}(\nabla_Z £_V g)(X,Y). \tag{3.7}$$

Using (3.5) in (3.7) we obtain

$$\begin{aligned} 2g((£_V\nabla)(X,Y),Z) = &-[2(X\lambda)+(Xr)]g(Y,Z)+(Xr)\eta(Y)\eta(Z) \\ &-[2(Y\lambda)+(Yr)]g(X,Z)+(Yr)\eta(X)\eta(Z) \\ &+[2(Z\lambda)+(Zr)]g(X,Y)-(Zr)\eta(X)\eta(Y). \end{aligned} \tag{3.8}$$

Removing Z from the above equation, we have

$$\begin{aligned} 2(£_V\nabla)(X,Y) = &-[2(X\lambda)+(Xr)]Y+(Xr)\eta(Y)\xi \\ &-[2(Y\lambda)+(Yr)]X+(Yr)\eta(X)\xi \\ &+g(X,Y)[2(D\lambda)+(Dr)]-\eta(X)\eta(Y)(Dr), \end{aligned} \tag{3.9}$$

where $X\alpha = g(D\alpha,X)$, D denotes the gradient operator with respect to g.
Taking the covariant derivative of $£_V\nabla$ with respect to X, we get

$$\begin{aligned} 2(\nabla_X £_V\nabla)(Y,Z) = &-[2g(\nabla_X(D\lambda),Y)+g(\nabla_X(Dr),Y)]Z+g(\nabla_X(Dr),Y)\eta(Z)\xi \\ &-[2g(\nabla_X(D\lambda),Z)+g(\nabla_X(Dr),Z)]Y \\ &+g(\nabla_X(Dr),Z)\eta(Y)\xi \\ &+g(Y,Z)[2\nabla_X D\lambda+\nabla_X Dr]-\eta(Y)\eta(Z)\nabla_X Dr. \end{aligned} \tag{3.10}$$

Using the foregoing equation in the following formula (see [24])

$$(£_V R)(X,Y)Z = (\nabla_X £_V\nabla)(Y,Z)-(\nabla_Y £_V\nabla)(X,Z), \tag{3.11}$$

we get

$$\begin{aligned}2(\pounds_V R)(X,Y)Z =&-[2g(\nabla_X D\lambda, Z)+g(\nabla_X Dr, Z)]Y+g(\nabla_X Dr, Z)\eta(Y)\xi\\&+g(Y,Z)[2\nabla_X D\lambda+\nabla_X Dr]-\eta(Y)\eta(Z)\nabla_X Dr\\&+[2g(\nabla_Y D\lambda, Z)+g(\nabla_Y Dr, Z)]X-g(\nabla_Y Dr, Z)\eta(X)\xi\\&-g(X,Z)[2\nabla_Y D\lambda+\nabla_Y Dr]+\eta(X)\eta(Z)\nabla_Y Dr.\end{aligned} \tag{3.12}$$

Since ξ is Killing, $\xi r=0$. Applying $\xi r=0$ and $\nabla\xi=0$, contracting X in (3.12), we infer

$$\begin{aligned}2(\pounds_V S)(Y,Z) =&2g(\nabla_Y D\lambda, Z)+[2\Delta\lambda+\Delta r]g(Y,Z)\\&-g(\nabla_\xi Dr, Z)\eta(Y)-\Delta r\eta(Y)\eta(Z),\end{aligned} \tag{3.13}$$

where Δ denotes the Laplacian. Moreover, from (3.3) follows directly that

$$\begin{aligned}(\pounds_V S)(Y,Z) =&\frac{(Vr)}{2}g(Y,Z)-\frac{(Vr)}{2}\eta(Y)\eta(Z)+\frac{r}{2}[g(\nabla_Y V, Z)+g(Y,\nabla_Z V)]\\&-\frac{r}{2}\eta(\nabla_Y V)\eta(Z)-\frac{r}{2}\eta(\nabla_Z V)\eta(Y).\end{aligned} \tag{3.14}$$

Equating (3.13) and (3.14) yields that

$$\begin{aligned}&2g(\nabla_Y D\lambda, Z)+[2\Delta\lambda+\Delta r]g(Y,Z)-g(\nabla_\xi Dr, Z)\eta(Y)-\Delta r\eta(Y)\eta(Z)\\&=(Vr)g(Y,Z)-(Vr)\eta(Y)\eta(Z)+r[g(\nabla_Y V, Z)+g(Y,\nabla_Z V)]\\&\quad-r\eta(\nabla_Y V)\eta(Z)-r\eta(\nabla_Z V)\eta(Y).\end{aligned} \tag{3.15}$$

Then substituting $Y=\xi$ and $Z=\xi$ in the foregoing equation we get

$$\xi(\xi\lambda)+\Delta\lambda=0. \tag{3.16}$$

Now we assume that $\xi\lambda=0$. Then (3.16) implies that the Laplacian of the smooth soliton function λ is zero, that is, λ is harmonic. Thus we can state the following:

Proposition 3.1. *In a three dimensional cosymplectic manifold M^3 with $\xi\lambda=0$, admitting almost Ricci solitons, the soliton function λ is harmonic.*

Now we state the Hopf's Lemma:

Lemma 3.1 ([14])**.** *If $\Delta f=0$ for a smooth function f on a compact orientable Riemannian manifold M without boundary, then f is constant on M.*

In view of Lemma 3.1 and (3.16) we can conclude that in a three dimensional compact orientable cosymplectic manifold M^3 without boundary admitting almost Ricci solitons, the soliton function λ is constant. Also, Barros et al. [2] proved that a compact non-trivial almost Ricci soliton with constant scalar curvature is isometric to a Euclidean sphere $\mathbb{S}^n$ and is gradient. This completes the proof.

In a recent paper Wang [23] proved that if a three dimensional cosymplectic manifold M^3 admits a Ricci soliton, then either M^3 is locally flat or the potential vector field is an infinitesimal contact transformation. Hence, we can state the following:

Corollary 3.1. *If a three dimensional compact orientable cosymplectic manifold M^3 without boundary with $\xi\lambda = 0$ admits an almost Ricci soliton, then either M^3 is locally flat or the potential vector field is an infinitesimal contact transformation.*

Now we have justified the assumption $\xi\lambda = 0$.
Taking Lie derivative of the equation (1.1) along the vector field ξ, we have

$$\pounds_\xi \pounds_V g + 2(\xi\lambda)g = 0. \tag{3.17}$$

But $\pounds_V \pounds_\xi g - \pounds_\xi \pounds_V g = \pounds_{[V,\xi]} g$. So using this relation in the above equation we obtain

$$\pounds_{[V,\xi]} g = 2(\xi\lambda)g. \tag{3.18}$$

Now we have considered two cases:

`Case 1`: Let V be point-wise orthogonal to ξ. From equation (1.1) and using (2.7) we get

$$g(\nabla_\xi V, X) + 2\lambda g(\xi, X) = 0. \tag{3.19}$$

Removing X from both sides of the above equation we have $\nabla_\xi V = -2\lambda\xi$. This implies $[V,\xi] = 2\lambda\xi$. Putting this relation in (3.18) and contracting, we get $2\xi\lambda = 3\xi\lambda$. Hence $\xi\lambda = 0$.

`Case 2`: Let V be point-wise colllinear with ξ, that is, $V = f\xi$, where f is a non zero smooth function. Then from (1.1), we can easily deduce $\xi f = -\lambda$. Now, using $V = f\xi$ in (1.1) and contracting we obtain

$$r + \xi f + 3\lambda = 0.$$

Substituting $\xi f = -\lambda$ in the above relation, we get $r = -2\lambda$ and therefore $\xi r = -2\xi\lambda$. But $\xi r = 0$. So, we have $\xi\lambda = 0$.

4. Proof of the Theorem 1.2

This section is devoted to study three dimensional cosymplectic manifold M^3 admitting gradient almost Ricci solitons. For a gradient almost Ricci soliton, we have

$$\nabla_Y Df = -\lambda Y - QY, \tag{4.1}$$

where D denotes the gradient operator of g.

Then

$$\nabla_{[X,Y]} Df = -\lambda[X,Y] - Q[X,Y]. \tag{4.2}$$

Differentiating (4.1) covariantly in the direction of X yields

$$\nabla_X \nabla_Y Df = -d\lambda(X)Y - \lambda\nabla_X Y - \nabla_X QY. \tag{4.3}$$

Similarly, we get

$$\nabla_Y \nabla_X Df = -d\lambda(Y)X - \lambda\nabla_Y X - \nabla_Y QX. \tag{4.4}$$

In view of (4.2), (4.3) and (4.4) we have

$$\begin{aligned} R(X,Y)Df =& \nabla_X\nabla_Y Df - \nabla_Y\nabla_X Df - \nabla_{[X,Y]}Df \\ =& (\nabla_Y Q)X - (\nabla_X Q)Y + (Y\lambda)X - (X\lambda)Y. \end{aligned} \tag{4.5}$$

From (3.2) we get

$$QY = \frac{r}{2}Y - \frac{r}{2}\eta(Y)\xi. \tag{4.6}$$

Differentiating (4.6) covariantly in the direction of X and using (2.7), we get

$$(\nabla_X Q)Y = \frac{(Xr)}{2}Y - \frac{(Xr)}{2}\eta(Y)\xi. \tag{4.7}$$

In view of (4.5) and (4.7), we get

$$\begin{aligned} R(X,Y)Df =& \frac{1}{2}[(Yr)X - (Yr)\eta(X)\xi] - \frac{1}{2}[(Xr)Y - (Xr)\eta(Y)\xi] \\ &+ (Y\lambda)X - (X\lambda)Y, \end{aligned} \tag{4.8}$$

which implies

$$R(X,\xi)Df = (\xi\lambda)X - (X\lambda)\xi. \tag{4.9}$$

Also, from (3.1) we have

$$R(X,\xi)Df = 0. \tag{4.10}$$

Taking $Y = \xi$ in (4.8) and using (4.10) we get

$$(\xi\lambda)X = (X\lambda)\xi, \tag{4.11}$$

for any vector field X on M.
Contracting X in (4.11) we get $\xi\lambda = 0$ and hence from (4.11) we obtain λ is constant on M. This completes the proof.

For a Kähler-Einstein manifold N and the real line $\mathbb{R}$, the cosymplectic manifold $N \times \mathbb{R}$ is a gradient Ricci soliton with $f = \lambda \frac{t^2}{2}$, where $t \in \mathbb{R}$. Such a gradient Ricci soliton is rigid [18].

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Department of Pure Mathematics, University of Calcutta, 35, Ballygaunge Circular Road, Kolkata 700019, West Bengal, India.

E-mail: uc_de@yahoo.com

Dhamla Jr. High School, Vill-Dhamla, P.O.-Kedarpur, Dist-Hooghly, Pin-712406, West Bengal, India.

E-mail: dey9chiranjib@gmail.com