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Symmetries of Sasakian Generalized Sasakian-Space-Form Admitting Generalized Tanaka-Webster Connection

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Abstract. The object of this paper is to study certain symmetric properties of Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection. We studied semi-symmetry and Ricci semi-symmetry of Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection. Further we obtain results for Ricci pseudosymmetric and Ricci-generalized pseudosymmetric Sasakian generalized Sasakian-space-form.

1 Introduction

In a Riemannian manifold, a curvature tensor given by $K(X,Y) = R(X,Y,Y,X)$ for an orthonormal pair of vectors (X,Y), is known as the sectional curvature. A Riemannian manifold with constant sectional curvature c is called a real-space-form, and its curvature tensor R satisfies

$$R(X,Y)Z = c\big\{g(Y,Z)X - g(X,Z)Y\big\}.$$

A Sasakian manifold with constant ϕ-sectional curvature c is called a Sasakian-space-form and its curvature tensor R is given by

$$\begin{aligned} R(X,Y)Z &= \frac{c+3}{4}\big[g(Y,Z)X - g(X,Z)Y\big] \\ &+ \frac{c-1}{4}\big[g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\big] \\ &+ \frac{c-1}{4}\big[\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ &+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\big]. \end{aligned} \tag{1.1}$$

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In 2004, Alegre et al. [2] generalized the Sasakian-space-form by replacing the constant quantities $\frac{c+3}{4}$ and $\frac{c-1}{4}$ with differentiable functions. Such space is called generalized Sasakian-space-form.

The generalized Sasakian-space-form have been studied by many authors such as Sarkar and De ([17, 10, 11]), Singh ([18, 19]), De and Majhi ([8, 9, 12]), Kishor et al. [15], Alegre and Carriazo [3, 4], Akbar and Sarkar [1], Sular and Ozgur [20, 21] and many others.

In 2008, Alegre and Carriazo studied structures on generalized Sasakian-space-form [4] and studied generalized Sasakian-space-form admitting trans-Sasakian structure. In this paper we studied generalized Sasakian-space-form admitting Sasakian structure and we called such manifold as Sasakian generalized Sasakian-space-form

In 1989, Tanno [23] defined the generalized Tanaka-Webster connection for contact metric manifolds, which generalized the connection given by Tanaka [22] and Webster [24]. The generalized Tanaka-Webster connection have been studied by De [7], de Dios Pérez [16] and others.

A manifold is said to be semi-symmetric and Ricci semi-symmetric [26, 27] if the Riemannian curvature tensor R and Ricci tensor S satisfies $R.R = 0$ and $R.S = 0$ respectively. That is

$$R(X, Y).R(U, V)W = 0 \tag{1.2}$$

and

$$R(X, Y).S(U, V) = 0 \tag{1.3}$$

for all $X, Y, U, V, , W \in \chi(M)$.

There are two notions of pseudosymmetric manifolds which are defined by Chaki in 1987 [6] and Deszcz in 1992 [13]. Throughout the paper we consider pseudosymmetric manifolds defined by Deszcz. An n-dimensional Riemannian manifold M, $n > 2$, is called pseudosymmetric manifolds if $R.R$ and $Q(g, R)$ are are linearly dependent, i.e.,

$$R.R = FQ(g, R), \tag{1.4}$$

holds on the set $U_R = \{x \in M : Q(g, R) \neq 0 \; at \; x\}$,where F is some function on U_R.

And the manifold is called Ricci pseudosymmetric and Ricci-generalized pseudosymmetric manifold if

$$R.S = f'Q(g, S) \tag{1.5}$$

and

$$R.R = fQ(S, R) \tag{1.6}$$

holds on the set $U_S = \{x \in M : Q(g, S) \neq 0 \; at \; x\}$ and $U_R = \{x \in M : Q(g, R) \neq 0 \; at \; x\}$ respectively, where f' and f are some function on U_S and U_R.

In this paper we studied symmetries of Sasakian generalized Sasakian-space-form admitting generalized Tanaka-Webster connection. After introduction in preliminaries section, we showed some known relation in Sasakian manifold and generalized Sasakian-space-form. In the third section, we have given the expression for curvature tensor with respect to genaralized Tanaka-Webster connection in generalized Sasakian-space-form. The next section is dedicated for the study of semi-symmetry and Ricci semi-symmetry. In the last two sections we studied Ricci pseudosymmetric and Ricci-generalized pseudosymmetric manifolds.

2 Preliminaries

An n-dimensional smooth manifold M is said to be an almost contact metric manifold if it admits an almost contact metric structure (ϕ, ξ, η, g) consisting of a tensor field ϕ of type $(1, 1)$, a vector field ξ, a 1-form η and a Riemannian metric g satisfying [5]

$$\phi^2 = -I + \eta \otimes \xi, \qquad \eta(\xi) = 1, \qquad \phi\xi = 0, \qquad \eta \circ \phi = 0,$$

and

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y). \tag{2.1}$$

On such a manifold, the fundamental Φ of M is defined as

$$\Phi(X, Y) = g(\phi X, Y), \qquad X, Y \in \Gamma(TM).$$

An almost contact metric manifold is called a Sasakian manifold if and only if [25]

$$(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X, \qquad \nabla_X \xi = -\phi X. \tag{2.2}$$

On a Sasakian manifold M, the following relations are held [25]

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.3}$$

$$R(X, \xi)Y = \eta(Y)X - g(X, Y)\xi, \tag{2.4}$$

$$\eta\big(R(X, Y)Z\big) = \eta(X)g(Y, Z) - \eta(Y)g(X, Z), \tag{2.5}$$

$$\eta\big(R(X, Y)\xi\big) = 0, \tag{2.6}$$

$$S(X,\xi) = (n-1)\eta(X), \tag{2.7}$$

$$Q\xi = (n-1)\xi, \tag{2.8}$$

$$(\nabla_X \eta)Y = g(X,\phi Y). \tag{2.9}$$

In a generalized Sasakian-space-form the following properties holds [2]

$$\begin{aligned} R(X,Y)Z &= f_1\big[g(Y,Z)X - g(X,Z)Y\big] \\ &+ f_2\big[g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\big] \\ &+ f_3\big[\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi \\ &- g(Y,Z)\eta(X)\xi\big], \end{aligned} \tag{2.10}$$

$$\begin{aligned} S(X,Y) &= \big[(n-1)f_1 + 3f_2 - f_3\big]g(X,Y) \\ &- \big[3f_2 + (n-2)f_3\big]\eta(X)\eta(Y), \end{aligned} \tag{2.11}$$

$$QX = \big[(n-1)f_1 + 3f_2 - f_3\big]X - \big[3f_2 + (n-2)f_3\big]\eta(X)\xi, \tag{2.12}$$

$$S(X,\xi) = (n-1)(f_1 - f_3)\eta(X), \tag{2.13}$$

$$Q\xi = (n-1)(f_1 - f_3)\xi, \tag{2.14}$$

$$R(X,Y)\xi = (f_1 - f_3)\big\{\eta(Y)X - \eta(X)Y\big\}, \tag{2.15}$$

$$R(\xi,Y)Z = (f_1 - f_3)\big\{g(Y,Z)\xi - \eta(Z)Y\big\}, \tag{2.16}$$

$$R(\xi,Y)\xi = (f_1 - f_3)\big\{\eta(Y)\xi - Y\big\}. \tag{2.17}$$

$$r = n(n-1)f_1 + 3(n-1)f_2 - 2(n-1)f_3, \tag{2.18}$$

where $r = \sum_{i=1}^{n} S(e_i, e_i)$ is the scalar curvature.

3 Generalized Tanaka-Webster connection

Tanno [23], defined the generalized Tanaka-Webster connection $\tilde{\nabla}$ for contact metric manifolds by

$$\tilde{\nabla}_X Y = \nabla_X Y + (\nabla_X \eta)(Y)\xi - \eta(Y)\nabla_X \xi - \eta(X)\phi(Y) \tag{3.1}$$

for all $X, Y \in \chi M$, and ∇ is the Riemannian connection.

Let R and $\tilde{R}$ denotes the Riemannian curvature tensors of Sasakian manifold with respect to ∇ and $\tilde{\nabla}$ respectively. A relation between R and $\tilde{R}$ is given by [7]

$$\begin{aligned} \tilde{R}(X,Y)Z &= R(X,Y)Z + \big[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)\big]\xi \\ &- g(Y,\phi Z)\phi X + g(X,\phi Z)\phi Y + 2g(Y,\phi X)\phi Z \\ &- \eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y. \end{aligned} \tag{3.2}$$

Contracting (3.2) we obtain

$$\tilde{S}(Y,Z) = S(Y,Z) - g(Y,Z) - (n-3)\eta(X)\eta(Y). \tag{3.3}$$

Using (2.10) and (2.11) in the above equations we have

$$\begin{aligned} \tilde{R}(X,Y)Z &= (f_1 - 1)\big[g(Y,Z)X - g(X,Z)Y\big] \\ &+ f_2\big[g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\big] \\ &+ f_3\big[\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi \\ &- g(Y,Z)\eta(X)\xi\big] - g(Y,\phi Z)\phi X + g(X,\phi Z)\phi Y \\ &+ 2g(Y,\phi X)\phi Z - \eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y, \end{aligned} \tag{3.4}$$

and

$$\begin{aligned} \tilde{S}(Y,Z) &= \big[(n-1)f_1 + 3f_2 - f_3\big]g(X,Y) \\ &- \big[3f_2 + (n-2)f_3\big]\eta(X)\eta(Y) \\ &- g(Y,Z) - (n-3)\eta(X)\eta(Y). \end{aligned} \tag{3.5}$$

Now we have

$$\tilde{R}(X,Y)\xi = (f_1 - f_3 - 1)\big\{\eta(Y)X - \eta(X)Y\big\}, \tag{3.6}$$

$$\tilde{R}(\xi,X)Y = (f_1 - f_3 - 1)\big\{g(Y,Z)\xi - \eta(Z)Y\big\}, \tag{3.7}$$

$$\tilde{R}(\xi,X)\xi = (f_1 - f_3)\big\{\eta(Y)\xi - Y\big\}, \tag{3.8}$$

$$\tilde{S}(X,\xi) = (n-1)(f_1 - f_3 - 1)\eta(X), \tag{3.9}$$

$$\tilde{S}(\xi,\xi) = (n-1)(f_1 - f_3 - 1). \tag{3.10}$$

4 Semi-symmetric and Ricci semi-symmetric

Suppose that the Sasakian generalized Sasakian-space-form is semi-symmetric with respect to generalized Tanaka-Webster connection, then from (1.2) we get

$$\tilde{R}(X,Y).\tilde{R}(U,V)W = 0. \tag{4.1}$$

It is well known that

$$\begin{aligned}\tilde{R}(X,Y).\tilde{R}(U,V)W &= \tilde{R}(X,Y)\tilde{R}(U,V)W - \tilde{R}(\tilde{R}(X,Y)U,V)W \\ &- \tilde{R}(U,\tilde{R}(X,Y)V)W - \tilde{R}(U,V)\tilde{R}(X,Y)W.\end{aligned} \tag{4.2}$$

Now setting $X = U = \xi$ in (4.1) and using (4.2) we get

$$(f_1 - f_3 - 1)^2\{g(Y,W)V - g(V,W)Y\} + (f_1 - f_3 - 1)\tilde{R}(Y,V)W = 0,$$

which can be written as

$$\tilde{R}(Y,V)W = (f_1 - f_3 - 1)\{g(V,W)Y - g(Y,W)V\}, \tag{4.3}$$

provided $f_1 - f_3 - 1 \neq 0$.

Thus we have

Theorem 4.1. *In a semi-symmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection, we have*

$$\tilde{R}(Y,V)W = (f_1 - f_3 - 1)\{g(V,W)Y - g(Y,W)V\},$$

provided $f_1 - f_3 - 1 \neq 0$.

Now using (3.2) in (4.3) we get

$$\begin{aligned}R(Y,V)W &= (f_1 - f_3 - 1)\{g(V,W)Y - g(Y,W)V\} \\ &- \big[g(Y,W)\eta(Y) - g(V,W)\eta(Y)\big]\xi + g(V,\phi W)\phi Y \\ &- g(Y,\phi W)\phi V - 2g(V,\phi Y)\phi W \\ &+ \eta(V)\eta(W)Y - \eta(Y)\eta(W)V,\end{aligned} \tag{4.4}$$

provided $f_1 - f_3 - 1 \neq 0$. Thus we can state that:

Theorem 4.2. *In a semi-symmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection the Riemannian curvature tensor is given by (4.4),provided* $f_1 - f_3 - 1 \neq 0$.

Again suppose that the Sasakian generalized Sasakian-space-form is Ricci semi-symmetric with respect to generalized Tanaka-Webster connection, then from (1.3) we get

$$\tilde{R}(X,Y).\tilde{S}(U,V) = 0. \tag{4.5}$$

It implies

$$\tilde{S}(\tilde{R}(X,Y).U,V) + \tilde{S}(U,\tilde{R}(X,Y)V) = 0. \tag{4.6}$$

Setting $X = U = \xi$ in (4.6) we get

$$(f_1 - f_3 - 1)\{(n-1)(f_1 - f_3 - 1)g(Y,V) - S(Y,V)\} = 0.$$

Which implies

$$S(Y,V) = (n-1)(f_1 - f_3 - 1)g(Y,V), \tag{4.7}$$

provided $f_1 - f_3 - 1 \neq 0$.

We have

Theorem 4.3. *A semi-symmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection is Einstein manifold provided* $f_1 - f_3 - 1 \neq 0$.

5 Ricci-generalized pseudosymmetric manifold

Suppose that the Sasakian generalized Sasakian-space-form is Ricci-generalized pseudosymmetric with respect to generalized Tanaka-Webster connection, then from (1.6)

$$\tilde{R}(X,Y).\tilde{R}(U,V)W = fQ(\tilde{S},\tilde{R})(U,V,W;X,Y).$$

This is equivalent to

$$\tilde{R}(X,Y).\tilde{R}(U,V)W = f\{\big((X \wedge_{\tilde{S}} Y).\tilde{S}\big)(U,V)\}, \tag{5.1}$$

where $\big((X \wedge_{\tilde{S}} Y)\big)Z = \tilde{S}(Y,Z)X - \tilde{S}(X,Z)Y$ for all X, Y, Z.

Thus we get

$$\begin{aligned} \tilde{R}(X,Y).\tilde{R}(U,V)W &- \tilde{R}(\tilde{R}(U,V)X,Y)W - \tilde{R}(X,\tilde{R}(U,V)Y)W \\ -\tilde{R}(X,Y)\tilde{R}(U,V)W &= f\{(X \wedge_{\tilde{S}} Y)\tilde{R}(U,V)W - \tilde{R}((X \wedge_{\tilde{S}} Y)U,V)W \\ &- \tilde{R}(U,(X \wedge_{\tilde{S}} Y)V)W - \tilde{R}(U,V)(X \wedge_{\tilde{S}} Y)W)\}. \end{aligned}$$

or

$$\begin{aligned}\tilde{R}(X,Y).\tilde{R}(U,V)W &- \tilde{R}(\tilde{R}(U,V)X,Y)W - \tilde{R}(X,\tilde{R}(U,V)Y)W \\ -\tilde{R}(X,Y)\tilde{R}(U,V)W &= f\{\tilde{S}(Y,\tilde{R}(U,V)W)X - \tilde{S}(X,\tilde{R}(U,V)W)Y \\ &- \tilde{S}(Y,U)\tilde{R}(X,V)W + \tilde{S}(X,U)\tilde{R}(Y,V)W \\ &- \tilde{S}(Y,V)\tilde{R}(U,X)W + \tilde{S}(X,V)\tilde{R}(U,Y)W \\ &- \tilde{S}(Y,W)\tilde{R}(U,V)X + \tilde{S}(X,W)\tilde{R}(U,V)Y\}. \end{aligned} \tag{5.2}$$

Setting $X = U = \xi$ in (5.2) we get

$$\begin{aligned}(f_1 - f_3 - 1)^2 \quad \{ \quad & g(Y,W)V - g(V,W)Y\} + (f_1 - f_3 - 1)\tilde{R}(Y,V)W \\ = \quad & f\Big[(n-1)(f_1 - f_3 - 1)\big\{\tilde{R}(Y,V)W - g(V,W)Y \\ + \quad & g(Y,W)\eta(V)\xi + g(V,Y)\eta(W)\xi\big\} \\ - \quad & \tilde{S}(Y,V)\eta(W)\xi - \tilde{S}(Y,W)\big\{\eta(V)\xi - V\big\}\Big]. \end{aligned} \tag{5.3}$$

Again setting $V = \xi$ in (5.3) we get

$$\begin{aligned} & f(f_1 - f_3 - 1)\big[g(Y,W)\xi - \eta(W)Y\big] \\ = \quad & f(f_1 - f_3 - 1)^2\big[g(Y,W)\xi - \eta(W)Y\big]. \end{aligned} \tag{5.4}$$

We have either

$$(f_1 - f_3 - 1) = 0, \tag{5.5}$$

or

$$(f_1 - f_3 - 1) = 1, \tag{5.6}$$

provided $f \neq 0$.

Setting $W = \xi$ in (5.3) and using (5.5) we get

$$S(Y,V) = 0, \tag{5.7}$$

for all $Y, V \in \chi M$, provided $f \neq 0$ and $f_1 - f_3 - 1 \neq 1$.

Thus we have

Theorem 5.1. *A Ricci-generalized pseudosymmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection is Ricci flat provided* $f \neq 0$ *and* $f_1 - f_3 - 1 \neq 1$.

Again setting $W = \xi$ in (5.3) and using (5.6) we get

$$S(Y, V) = (n-1)g(V, Y), \tag{5.8}$$

for all $Y, V \in \chi M$, provided $f \neq 0$ and $f_1 - f_3 - 1 \neq 0$.

We have

Theorem 5.2. *A Ricci-generalized pseudosymmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection is Einstein manifold provided $f \neq 0$ and $f_1 - f_3 - 1 \neq 0$.*

6 Ricci-pseudosymmetric manifold

Suppose that the Sasakian generalized Sasakian-space-form is Ricci-pseudosymmetric with respect to generalized Tanaka-Webster connection, then from (1.5)

$$\tilde{R}(X, Y).\tilde{S}(U, V) = f' Q(g, \tilde{R})(U, V; X, Y).$$

This is equivalent to

$$\tilde{R}(X, Y).\tilde{S}(U, V) = f'\{\big((X \wedge_g Y).\tilde{S}\big)(U, V)\}, \tag{6.1}$$

where $\big((X \wedge_g Y)\big)Z = g(Y, Z)X - g(X, Z)Y$ for all X, Y, Z.

Thus we get

$$\tilde{S}(\tilde{R}(X, Y).U, V) + \tilde{S}(U, \tilde{R}(X, Y)V) = f'\{\tilde{S}\big((X \wedge_g Y)U, V\big) + \tilde{S}\big(U, (X \wedge_g Y)V\big)\}.$$

or

$$\begin{aligned}\tilde{S}(\tilde{R}(X, Y).U, V) + \tilde{S}(U, \tilde{R}(X, Y)V) = f'\{&g(Y, U)\tilde{S}(X, V) - g(X, U)\tilde{S}(Y, V)\\ &+ g(Y, V)\tilde{S}(U, X) - g(X, V)\tilde{S}(U, Y)\}.\end{aligned} \tag{6.2}$$

Setting $X = U = \xi$ in (6.2) we get

$$(f_1 - f_3 - f' - 1)\{S(Y, V) - (n-1)(f_1 - f_3 - 1)g(Y, V)\} = 0.$$

Which implies

$$S(Y, V) = (n-1)(f_1 - f_3 - 1)g(Y, V), \tag{6.3}$$

for all $Y, V \in \chi M$, provided $(f_1 - f_3 - f' - 1) \neq 0$.

We have

Theorem 6.1. *A Ricci-pseudosymmetric Sasakian generalized Sasakian-space-form with respect to generalized Tanaka-Webster connection is Einstein manifold provided* $f_1 - f_3 - f' - 1 \neq 0$.

Now using Theorem 4.2 of [4] and (6.3) we get the following corollary

Corollary 6.2. *An* n*-dimensional connected Sasakian generalized Sasakian-space-form,* $(n \geq 5)$*, which is Ricci-pseudosymmetric with respect to generalized Tanaka-Webster connection is Ricci flat provided* $f' \neq 0$.

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