



Realized Multi-Power Variation Process for Jump Detection in the Nigerian All Share Index

Mabel Eruore Adeosun and Olabisi Oreofe Ugbebor

Abstract. In this paper, we studied the particular cases of higher-order realized multipower variation process, their asymptotic properties comprising the probability limits and limit distributions were highlighted. The respective asymptotic variances of the limit distributions were obtained and jump detection models were developed from the asymptotic results. The models were obtained from the particular cases of the higher-order of the realized multipower variation process, in a class of continuous stochastic volatility semimartingale process. These are extensions of the method of jump detection by Barndorff-Nielsen and Shephard (2006), for large discrete data. An Empirical Application of the models to the Nigerian All Share Index (NASI) data shows that the models are robust to jumps and suggest that stochastic models with added jump components will give a better representation of the NASI price process.

1 Introduction

Jumps are sudden discontinuities that occur spontaneously in the trajectories of price processes, which could be as a result of sudden and unpredictable inflows of information into the market. The effect on the dynamics of a price process cannot be overemphasized; it can lead to additional risk parameters in the process of a risky asset as well as a sporadic change in the dynamics of the process. In recent times, several attempts have been made by researchers to suggest empirical models that best describe the behavior of the Nigerian stock market price process (see [1, 2, 3, 4, 5, 6, 7]). However, detecting the presence or absence of jumps in discontinuous paths in the Nigerian stock market price process is rarely investigated in the literature. To determine the dynamics of any process, there is the need to examine and estimate some very important asymptotic features, the presence or absence of jumps in available discretely observed price data. In this paper, we

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Corresponding author: Mabel Eruore Adeosun.

,therefore, establish jump detection models via the realised multipower variation process. We then applied the models to the Nigerian All Share Index (NASI) discretely observed data by considering the NASI log- price process as $X = \{X_t\}_{t \geq 0}$ defined on a given interval $[0, t]$ such that the observations of this process are made for all discrete times $0 = t_0, t_1, \dots, t_n = t$. The j^{th} observed time is given as:

$$j\Delta = j \left[\frac{t}{n} \right] \implies n = \left[\frac{t}{\Delta} \right], \quad j = 1, 2, \dots, n \quad (1.1)$$

where Δ is the time interval between two successive observations, assumed to be of equal distance, and n is the number of observations. These observations are of utmost importance since they give the kind of stochastic process governed by an underlying stock. The challenge of capturing jumps in discretely observed processes, in the face of available high-frequency data on the Internet (especially when n is so large that Δ is vanishingly small) is on the increase.

Given that Δ is the equal time interval between two successive observations within $[0, t]$, and given a positive real constant r , the r^{th} - order realized power variation of such a process is given as:

$$\{X_\Delta\}^{[r]} = \Delta^{1-r/2} \sum_{j=1}^{\lceil t/\Delta \rceil} |X_{j\Delta} - X_{(j-1)\Delta}|^r \quad (1.2)$$

where $X_{j\Delta}$ is the j^{th} observed log-return price for $j = 1, 2, \dots, n$.

The r^{th} - order power variation process is given as the limit in probability of the r^{th} - order realized power variation (RPV):

$$\{X\}^{[r]} = \mathbb{P} - \lim_{\Delta \rightarrow 0} \{X_\Delta\}^{[r]}. \quad (1.3)$$

An estimation of the Quadratic Variation (QV) denoted by $[X]_t$ is obtained for $r = 2$ of the realized variation in (2). In [8], the Realized Variance (RV) is proven to be a consistent estimator (as $n \rightarrow \infty$) of $[X]_t$, when X is assumed to be a class of continuous stochastic volatility semimartingales (Svsm^c).

Given that X_t belongs to the class of continuous stochastic volatility semimartingales (Svsm^c), ([9, 10]) such that:

$$X_t = \int_0^t \alpha_s ds + \int_0^t \sigma_s dW_s \quad (1.4)$$

where $A_t = \int_0^t \alpha_s ds$ is an adapted, càdlàg process with "finite variation" which implies that the variation of each path $t \rightarrow A_t$ is bounded over each finite interval in $[0, t]$ and $M_t = \int_0^t \sigma_s dW_s$ is a continuous local martingale and an Ito integral of the spot volatility process $\sigma_t > 0$ with respect

to a standard Brownian motion W_t . The Integrated Volatility (IV) process is also assumed finite. Then, the limit distribution of functions of (3) above, its convergence in probability and the central limit theorem results were obtained in [11, 12]. Based on the procedure given in [11, 12], a jump detection method was achieved in [13]. The main limitation of the results of the RPV, subject to an $Svsm^c$ process is that when jumps are added to a class of models described in (4) above, the RV can no longer estimate the IV, but instead it gives a result of the sum of the IV and the QV of the jump component. Hence, the need for a robust process that cannot be affected when jumps are incorporated in the process.

The realized multi-power variation process defined on a 1-dimensional semi martingale process in its generalized form is given as:

$$\{X\}_{\Delta,t}^{[r_1, \dots, r_m]} = \Delta^{1-\delta(r_1, \dots, r_m)} \sum_{j=1}^{c(t,m,\Delta)} f(x_j, r_j). \tag{1.5}$$

as defined in [14], with $\delta(r_1, \dots, r_m) = \frac{1}{2} \sum_{i=1}^m r_i$, $c(t, m, \Delta) = [t/\Delta] - (m-1)$ and $f(x_j, r_i) = \prod_{i=0}^{m-1} |x_{j+i}|^{r_{i+1}}$ for $n > m$. The asymptotic properties of (1.5) above, has been extensively given in [11, 15]. Particular cases of (1.5) are the Bipower, Tripower and the Quadpower processes which can be found in [16, 17, 18].

The BNS method for jump detection named after "Barndorff-Nielsen and Shephard" was established in [17] for $X_t \in Svsm^c$ subject to the above stated assumptions for the processes σ_t^2 , α_t and W_t . This method was basically derived from the asymptotic distribution of the difference of a particular realized bipower variation process: $\{X\}_{\Delta,t}^{[1,1]}$ and the realized variance process $[X]_{\Delta,t}^{[2]}$. That is, for $m = 2$, and $r_1 = r_2 = 1$ in (1.5) above, then,

$$\frac{\Delta^{-\frac{1}{2}} \left(\mu_1^{-2} \{X\}_{\Delta,t}^{[1,1]} - [X]_{\Delta,t}^{[2]} \right)}{\sqrt{\int_0^t \sigma_s^4 ds}} \xrightarrow{L} N\left(0, \varphi_{BPV}\right) \tag{1.6}$$

where φ_{BPV} is the asymptotic variance of the convergence in law result given in (1.6) above

$$\varphi_{BPV} = \mu_1^{-4} + 2\mu_1^{-2} - 5 \simeq 0.6091, \mu_1 = \frac{\sqrt{2}}{\sqrt{\pi}} \tag{1.7}$$

The main contribution of this work to the description in (1.5), (1.6), and (1.7), as well as to the work given in [17] entails a derivation of the asymptotic theories for particular cases of the realized multi-power variation ($RMPV$) process by considering the convergence in distribution (law) of the difference of the realized variance RV and particular cases of the $RMPV$ process. Hence, we obtain the asymptotic variances of the particular cases that is, φ_{RBV} , φ_{RTV} , φ_{QPV} , φ_{PPV} , φ_{HPV} , φ_{H_PPV} , φ_{OpV} , φ_{NPV} , φ_{DPV} , and φ_{UPV} respectively for the realized Bipower,

Tripower, Quadpower, Pentpower, Hexpower, Heptpower, Octpower, Nonpower, Decpower and Unopower variation processes. Based on the results obtained, we develop jump detection models from the asymptotic properties of the particular cases of the *RMPV* process; these results are extensions of the results in [17, 19, 20].

1.1 Data Realization

The empirical data set to be tested in this work is the data of the Nigerian All Share Index (NASI) obtained from the Nigerian Stock Exchange via www.nse.com.ng/market-data/indices, accessed on the 13th August 2018; these are closing market indices comprising the general indices of 5, 522 daily observations.

The remaining part of the paper is structured as follows: basic definitions of the concepts are given in Section 2, the asymptotic results of the *RMPV* process are spelled out in Section 3. The *RMPV* jump test models derived from the asymptotic properties in particular cases and an empirical application of models to the NASI data are given in Section 4, discussions and a concluding remark are in Section 5.

2 Basic definitions and Methods

We give some basic definitions and the jump detection method based on the bipower variation process as can be found in [17] in this section.

2.1 Basic definitions

The following definitions [9, 10, 21] are based on the assumption that the process X_t is defined on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})$

Definition 1. A Martingale: A process $\{M_t\}$ which is adapted to the filtration $(\mathcal{F}_t)_{t \geq 0}$ and $E(|M_t|^2) < \infty$, is called a martingale if $E(M_t/\mathcal{F}_s) = M_s$ for all $t \geq s$, where $0 \leq s \leq t$.

Definition 2. A Semimartingale: A process $\{X_t\}$ is called a total semimartingale if X is a càdlàg, adapted and the stochastic integral of X is continuous. Moreso, it is called a classical semimartingale if it can be decomposed into two adapted, càdlàg processes: $\{M_t\}$ and $\{A_t\}$, such that,

$$X_t = M_t + A_t \tag{2.1}$$

where A_t is a locally finite variation process and M_t is a local martingale where $A(0) = M(0) = 0$.

Definition 3. $Svsm^c$ and $Svsm^j$ processes: Given that the log-returns is a semimartingale, as given in definition 2 above, such that the processes $M_t, \sigma_t > 0$ and A_t satisfy the conditions given in (1.4) above, then X_t is said to be in a class of continuous stochastic volatility semimartingale process ($Svsm^c$). Given also that X_t is with a continuous part $X_t^c \in Svsm^c$ and a discontinuous part X_t^j ,

Such that:

$$X_t = X_t^c + X_t^j \tag{2.2}$$

where, X_t^j is the discontinuous (jump) process defined as:

$$X_t^j = \sum_{i=1}^{N(t)} \gamma_i \tag{2.3}$$

where $N(t)$ is a simple counting process, which stands for the number of jumps at time t and γ_i is a non- zero stochastic process. Then X_t is said to belong to a class of stochastic volatility semimartingale with added jumps ($X_t \in Svsm^j$).

Thus, we have

$$X_t = X_0 + \int_0^t \alpha_s ds + \int_0^t \sigma_s dW_s + \sum_{i=1}^{N(t)} \gamma_i \tag{2.4}$$

with processes $\alpha_t, \sigma_t, W_t, \gamma_i$ and $N(t)$ as defined above. .

2.2 The BNS jump detection method

The BNS jump-test ([17, 19, 20]) is classified into the linear jump test and the ratio-jump test comprising:the feasible linear jump test, ratio test and adjusted ratio test given respectively as:

$$\hat{P} = \frac{\Delta^{-\frac{1}{2}} \left(\mu_1^2 \{X\}_{\Delta,t}^{[1,1]} - [X]_{\Delta,t}^{[2]} \right)}{\sqrt{\mu_1^4 \{X\}_{\Delta,t}^{[1,1,1,1]}}} \tag{2.5}$$

$$\hat{Q} = \frac{\Delta^{-\frac{1}{2}} \left(\mu_1^2 \{X\}_{\Delta,t}^{[1,1]} / [X]_{\Delta,t}^{[2]} - 1 \right)}{\sqrt{\{X\}_{\Delta,t}^{[1,1,1,1]} / \left(\{X\}_{\Delta,t}^{[1,1]} \right)^2}} \tag{2.6}$$

$$\hat{R} = \frac{\Delta^{-\frac{1}{2}} \left(\mu_1^2 \{X\}_{\Delta,t}^{[1,1]} / [X]_{\Delta,t}^{[2]} - 1 \right)}{\max \left(\frac{1}{t}, \sqrt{\{X\}_{\Delta,t}^{[1,1,1,1]} / \left(\{X\}_{\Delta,t}^{[1,1]} \right)^2} \right)} \tag{2.7}$$

where, \hat{P} , \hat{Q} and \hat{R} converges in law to $N(0, \varphi_{BPV})$.

Given that,

$$\{X\}_{\Delta,t}^{[1,1,1,1]} = \sum_{j=4}^{\lfloor \frac{t}{\Delta} \rfloor} |x_j||x_{j-1}||x_{j-2}||x_{j-3}| \tag{2.8}$$

and

$$\{X\}_{\Delta,t}^{[1,1]} = \sum_{j=2}^{\lfloor \frac{t}{\Delta} \rfloor} |x_j||x_{j-1}| \tag{2.9}$$

The hypothesis for the BNS jump test is as follows:

$$H_0 : X_t \in Svs m^c \tag{2.10}$$

$$H_1 : X_t \in Svs m^j$$

3 Asymptotic results for the realised multipower variation process

We give here some asymptotic properties of the RMPV process.

3.1 Convergence in probability of the realised multipower variation process

Theorem 3.1. ([19, 20]) *Let the process $X_t \in Svs m^c$, defined on $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})$ which can be expressed as given in (1.4) above. Then for $\alpha_t = 0$, the multipower variation process $\{X\}_t^{[r_1, \dots, r_m]}$ satisfies the following:*

$$\{X\}_t^{[r_1, \dots, r_m]} \xrightarrow{\mathbb{P}} \mu_{r_1} \dots \mu_{r_m} \int_0^t |\sigma_s|^{2(\delta(r_1, \dots, r_m))} ds \tag{3.1}$$

where,

$$2\delta(r_1, \dots, r_m) = \sum_{i=1}^m r_i, \quad \mu_{r_i} = \mathbb{E}(|\nu|^{r_i}), \quad \nu \sim N(0, 1)$$

3.2 Convergence in Distribution result for the difference of the RV and the RMPV process

The methods adopted by [17, 22, 23, 24] to test for jumps in the log price process were built on the idea of the theory of convergence in distribution for the difference of the RV and a special case of the realised bipower variation process. The RMPV version of the convergence result in [17, 22, 23, 24] is given in the following theorem:

Theorem 3.2. ([20]) Let $X_t \in Svs m^c$, then as $\Delta \rightarrow 0$, and $A_t = 0$

$$\frac{1}{\sqrt{\Delta \int_0^t \sigma^4 ds}} \left(\mu_{\frac{2}{m}}^{-m} \{X\}_{\Delta,t}^{[r_1, \dots, r_m]} - [X]_{\Delta,t}^{[2]} \right) \xrightarrow{L} N(0, \varphi_{MPV}) \tag{3.2}$$

where φ_{RMV} is the asymptotic variance of the convergence in distribution of the difference of the RV and the RMPV process, given as:

$$\varphi_{MPV}(m, v_i) = Var(|\nu|^2) + \mu_{\frac{2}{m}}^{-2m} \omega_m^2 - \mu_{\frac{2}{m}}^{-m} m Cov \left(|\nu|^2, \prod_{i=1}^m |\nu_i|^{\frac{2}{m}} \right) \tag{3.3}$$

where

$$\mu_{\frac{2}{m}} = E(|\nu|^{\frac{2}{m}}) = \frac{2^{\frac{1}{m}} \Gamma(m + \frac{1}{2})}{\sqrt{\pi}}, \tag{3.4}$$

$$\omega_m^2 = Var \left(\prod_{i=1}^m |\nu_i|^{\frac{2}{m}} \right) + 2 \sum_{j=1}^{m-1} Cov \left(\prod_{i=1}^m |\nu_i|^{\frac{2}{m}}, \prod_{i=1}^m |\nu_{i+j}|^{\frac{2}{m}} \right) \tag{3.5}$$

4 The Jump Detection Models

In this section, we shall derive the jump test models for the particular cases of the RMPV process from the asymptotic results obtained in section 3 of this work and then apply the models to the Nigeria All Share Index (NASI) data.

4.1 The RMPV Jump test Models

With reference to [17], we observe basically three types of test for jumps based on the first order RMPV process, namely: the linear jump test, the ratio jump test and adjusted ratio jump test.

For the RMPV process $\{X\}_{\Delta,t}^{[r_1, r_2, \dots, r_m]}$, the linear jump test is based on the asymptotic result given in (3.2).

Given that $\{X\}_{\Delta,t}^{[2]} \xrightarrow{L} \int_0^t \sigma^2 ds$ in [8], then we can obtain the ratio jump test from (3.2) by dividing through by $\{X\}_{\Delta,t}^{[2]} = \int_0^t \sigma^2 ds$ to obtain

$$\frac{\left(\frac{\mu_{2/m}^{-m} \{X\}_{\Delta,t}^{[r_1, \dots, r_m]}}{\{X\}_{\Delta,t}^{[2]}} - 1 \right)}{\Delta^{1/2} \varphi_{RMV} \sqrt{\frac{\int_0^t \sigma^4 ds}{(\int_0^t \sigma^2 ds)^2}}} \xrightarrow{L} N(0, 1) \tag{4.1}$$

In this work, we shall use the feasible adjusted ratio jump test for the particular cases of the model in (4.1) by the following steps below:

i. Since the values of the component $\int_0^t \sigma_s^4 ds$ and $\int_0^t \sigma_s^2 ds$ cannot be observed when working with the NASI discrete data, we shall need estimators of the quantities. Thus, using (3.1) to obtain these estimators, let \hat{q} be the estimator for $\int_0^t \sigma_s^4 ds$ and \hat{p} be the estimator for $\int_0^t \sigma_s^2 ds$ where, \hat{p} is the realized quad power variation $\{X\}_{\Delta,t}^{[1,1,1,1]}$ process given as:

$$\hat{q} = \mu_1^{-4} \sum_{j=4}^n |x_j||x_{j+1}||x_{j+2}||x_{j+3}| = \mu_1^{-4} \sum_{j=4}^n \prod_{i=0}^3 |x_{j+i}| \tag{4.2}$$

and the estimator for $\int_0^t \sigma_s^2 ds$ is the realized bipower variation process $\{X\}_{\Delta,t}^{[1,1]}$, \hat{p} given as;

$$\hat{p} = \mu_1^{-2} \sum_{j=2}^n |x_j||x_{j+1}| = \mu_1^{-2} \{X\}_{\Delta,t}^{[1,1]} \tag{4.3}$$

Therefore, from the above we obtain a feasible ratio jump test for the RMPV model given as:

$$\frac{\left(\frac{\mu_{2/m}^{-m} \{X\}_{\Delta,t}^{[r_1, \dots, r_m]}}{\{X\}_{\Delta,t}^{[2]}} - 1 \right)}{\varphi_{RMV} \Delta^{1/2} \sqrt{\frac{\hat{q}}{\hat{p}^2}}} = \hat{Z}_m \tag{4.4}$$

where $\hat{Z}_m \sim N(0, 1)$.

ii. By Jensen’s inequality, for small samples, in the feasible test [25];

$$\frac{\hat{q}}{\hat{p}^2} \geq 1$$

Hence, we employ the adjusted ratio for the estimators $\frac{\hat{q}}{\hat{p}^2}$ to get

$$\max \left(1, \frac{\hat{q}}{\hat{p}^2} \right)$$

. Thus, we obtain the feasible adjusted ratio jump test for the RMPV model as follows:

$$\hat{Z}_m = \frac{\frac{\mu_{2/m}^{-m} \{X\}_{\Delta,t}^{[r_1, \dots, r_m]}}{\{X\}_{\Delta,t}^{[2]}} - 1}{\varphi_{RMV} \Delta^{1/2} \sqrt{\max \left(1, \frac{\hat{q}}{\hat{p}^2} \right)}} \tag{4.5}$$

(4.5) is the jump test model that will be used in this work for particular cases, subject to the following hypothesis:

$$H_0 : X_t \in Svsm^c.$$

$$H_1 : X_t \in Svsm^j$$

Thus for the BP case, we have for $m = 2$, and $r_1 = r_2 = 1$ in (4.5) that,

$$\hat{Z}_2 = \frac{\frac{\mu_1^{-2}\{X\}_{\Delta,t}^{[1,1]}}{\{X\}_{\Delta,t}^{[2]}} - 1}{\varphi_{BP}\Delta^{1/2}\sqrt{\max\left(1, \frac{\hat{q}}{\hat{p}^2}\right)}} \tag{4.6}$$

where, $\varphi_{BP} = 0.6090$ and $\hat{Z}_2 \sim N(0, 1)$.

For the realized tripower variation model, we have for $m = 3$, and $r_1 = r_2 = r_3 = \frac{2}{3}$ in (4.5) that,

$$\hat{Z}_3 = \frac{\frac{\mu_{2/3}^{-3}\{X\}_{\Delta,t}^{[2/3,2/3,2/3]}}{\{X\}_{\Delta,t}^{[2]}} - 1}{\varphi_{TP}\Delta^{1/2}\sqrt{\max\left(1, \frac{\hat{q}}{\hat{p}^2}\right)}} \tag{4.7}$$

where, $\varphi_{RTP} \approx 1.0613$

The RQV jump test model, we have for $m = 4$, and $r_1 = \dots = r_4 = \frac{1}{4}$ in (4.5) that,

$$\hat{Z}_4 = \frac{\frac{\mu_{1/2}^{-4}\{X\}_{\Delta,t}^{[1/4,1/4,1/4,1/4]}}{\{X\}_{\Delta,t}^{[2]}} - 1}{\varphi_{RQV}\Delta^{1/2}\sqrt{\max\left(1, \frac{\hat{q}}{\hat{p}^2}\right)}} \tag{4.8}$$

where, $\varphi_{RQP} \approx 1.3770$

The RPV jump test model for $m = 5$, and $r_1 = \dots = r_5 = \frac{2}{5}$ in (4.5) that,

$$\hat{Z}_5 = \frac{\frac{\mu_{2/5}^{-5}\{X\}_{\Delta,t}^{[2/5,2/5,2/5,2/5,2/5]}}{\{X\}_{\Delta,t}^{[2]}} - 1}{\varphi_{RPV}\Delta^{1/2}\sqrt{\max\left(1, \frac{\hat{q}}{\hat{p}^2}\right)}} \tag{4.9}$$

where, $\varphi_{RQP} \approx 1.6053$

The RH_XV jump test model for $m = 6$, and $r_1 = \dots = r_6 = \frac{1}{3}$ in (4.5) that,

$$\hat{Z}_6 = \frac{\frac{\mu_{1/3}^{-6}\{X\}_{\Delta,t}^{[1/3,1/3,1/3,1/3,1/3,1/3]}}{\{X\}_{\Delta,t}^{[2]}} - 1}{\varphi_{RH_XV}\Delta^{1/2}\sqrt{\max\left(1, \frac{\hat{q}}{\hat{p}^2}\right)}} \tag{4.10}$$

where, $\varphi_{RH_XV} \approx 1.7769$

The RH_pV jump test model, for $m = 7$, and $r_1 = \dots = r_7 = \frac{2}{7}$ in (4.5) that,

$$\hat{Z}_7 = \frac{\frac{\mu_{2/7}^{-7} \{X\}_{\Delta,t}^{[2/7,2/7,2/7,2/7,2/7,2/7,2/7]} - 1}{\{X\}_{\Delta,t}^{[2]}}}{\varphi_{RH_pV} \Delta^{1/2} \sqrt{\max\left(1, \frac{\hat{q}}{\hat{p}^2}\right)}} \quad (4.11)$$

where, $\varphi_{RH_pV} \approx 1.9100$

The RO_pV jump test model, for $m = 8$, and $r_1 = \dots = r_8 = \frac{1}{4}$ in (4.5) that,

$$\hat{Z}_8 = \frac{\frac{\mu_{1/4}^{-8} \{X\}_{\Delta,t}^{[1/4,1/4,1/4,1/4,1/4,1/4,1/4,1/4]} - 1}{\{X\}_{\Delta,t}^{[2]}}}{\varphi_{RO_pV} \Delta^{1/2} \sqrt{\max\left(1, \frac{\hat{q}}{\hat{p}^2}\right)}} \quad (4.12)$$

where, $\varphi_{RO_pV} \approx 2.0161$

The RNV jump test model, for $m = 9$, and $r_1 = \dots = r_9 = \frac{2}{9}$ in (4.5) that,

$$\hat{Z}_9 = \frac{\frac{\mu_{2/9}^{-9} \{X\}_{\Delta,t}^{[2/9,2/9,2/9,2/9,2/9,2/9,2/9,2/9,2/9]} - 1}{\{X\}_{\Delta,t}^{[2]}}}{\varphi_{RNV} \Delta^{1/2} \sqrt{\max\left(1, \frac{\hat{q}}{\hat{p}^2}\right)}} \quad (4.13)$$

where, $\varphi_{RNV} \approx 2.1026$

The RDV jump test model, for $m = 10$, and $r_1 = \dots = r_{10} = \frac{1}{5}$ in (4.5) that,

$$\hat{Z}_{10} = \frac{\frac{\mu_{1/5}^{-10} \{X\}_{\Delta,t}^{[1/5,1/5,1/5,1/5,1/5,1/5,1/5,1/5,1/5,1/5]} - 1}{\{X\}_{\Delta,t}^{[2]}}}{\varphi_{RDV} \Delta^{1/2} \sqrt{\max\left(1, \frac{\hat{q}}{\hat{p}^2}\right)}} \quad (4.14)$$

where, $\varphi_{RDV} \approx 2.1744$

THE RUV jump test model for $m = 11$, and $r_1 = \dots = r_{11} = \frac{2}{11}$ in (4.5) that,

$$\hat{Z}_{11} = \frac{\frac{\mu_{2/11}^{-11} \{X\}_{\Delta,t}^{[2/11,2/11,2/11,2/11,2/11,2/11,2/11,2/11,2/11,2/11,2/11]} - 1}{\{X\}_{\Delta,t}^{[2]}}}{\varphi_{RUV} \Delta^{1/2} \sqrt{\max\left(1, \frac{\hat{q}}{\hat{p}^2}\right)}} \quad (4.15)$$

where, $\varphi_{RUV} \approx 2.2348$

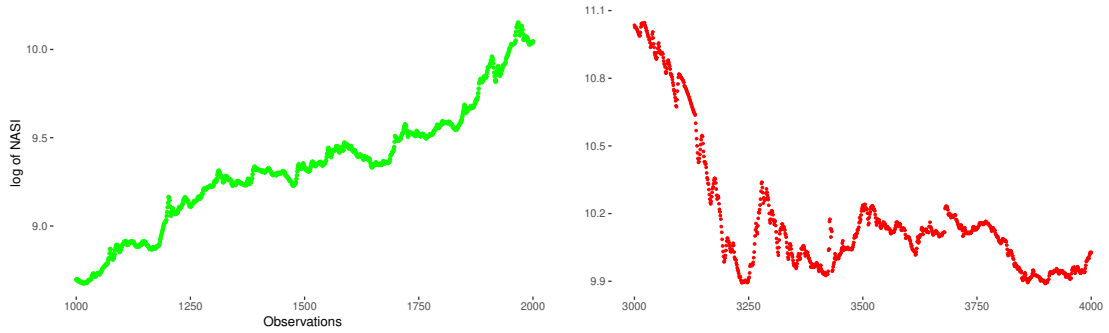


Figure 1: Evidence of jumps in the NASI price process for 1000 to 4000 observations

Remark 1. The values of the asymptotic variances φ_{RBV} , φ_{RTV} , φ_{QPV} , φ_{PPV} , φ_{HPV} , φ_{H_PPV} , φ_{OpV} , φ_{NPV} , φ_{DPV} and φ_{UPV} above, were obtained from (3.3)-(3.5) for $m = 2, \dots, 11$ respectively.

4.2 Empirical Application

An empirical Application of the models in Eqns. (4.6)-(4.15) was carried out on the Nigerian All Share Index comprising of 5, 522 daily market indices. We set the null hypothesis H_0 to be “No jump” in the log-return of the NASI process. That is,

H_0 : No jumps in the log-returns of the NASI data.

H_1 : Jumps are in the log-return of the NASI data.

The test was carried out on a significant level of 5%(0.05). In the analysis, we used the R-program to compute asymptotic variance φ_{PMV} for the particular cases, that is we computed; φ_{RBV} , φ_{RTV} , φ_{RQV} , φ_{RPV} , φ_{RH_XV} , φ_{RH_pV} , φ_{ROV} , φ_{ROV} , φ_{RNV} , φ_{RDV} , and φ_{RUV} . The different values of \hat{Z}_m for values of $m = 2, \dots, 11$ as given in (4.6)- (4.15) were also computed. The p-values (estimated probability value) and the test-Statistics values Z_m for $m = 1 \dots 11$ were obtained via the R-Program. Computationally, we calculated the realized variance $\{X\}_{\Delta,t}^{[2]}$ of the NASI data and the respective r^{th} -power variation process. Since we are dealing with a two-tailed test, we reject H_0 if $\hat{Z}_m > \hat{Z}_{tab}$.

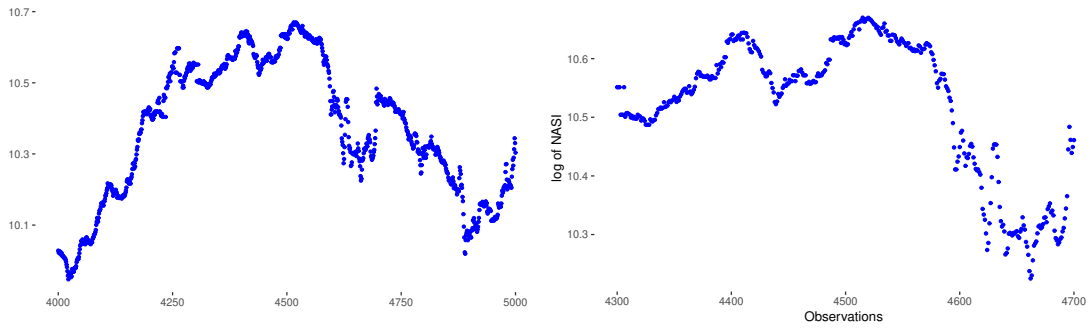


Figure 2: Evidence of jumps in the NASI price process for 4000 and above observations

$\{X\}_{\Delta,t}^{[r_1, \dots, r_m]}$	$\varphi_{RMPV}(m, \mu_r)$	\hat{Z}_m	p-value	$\{X\}_{\Delta,t}^{[2]}$	r^{th} RPV
m=2, (RBP)	0.6090	3.8995	$9.64E - 05$	0.7251	0.6715
m=3, (RTP)	1.0613	8.0285	$9.87E - 15$	0.7251	0.5794
m=4 (RQP)	1.3770	8.7791	$1.65 E - 18$	0.7251	0.5436
m=5 (RPP)	1.6053	9.3854	$6.26 E - 21$	0.7251	0.5156
m=6 ($RH_X P$)	1.7769	9.8718	$5.52 E - 23$	0.7251	0.4932
m=7 ($RH_p P$)	1.9100	10.2100	$1.79 E - 24$	0.7251	0.4765
m=8 (ROP)	2.0161	10.6183	$2.45 E - 26$	0.7251	0.4594
m=9 (RNP)	2.1026	10.9508	$6.59 E - 28$	0.7251	0.4453
m=10 (RDP)	2.1744	11.2778	$1.69 E - 29$	0.7251	0.4321
m=11 (RUP)	2.2348	11.5801	$5.20 E - 31$	0.7251	0.4201

Table 1: Results of the Jump detection in the NASI data via the RMPV models

4.3 Distributions of the RV and the $RMVP$ models of the NASI Data

We shall take a look at the behavior of the difference of the realized variance and each of the particular realized multipower variation processes by plotting their graphs in the case of the daily observed NASI data.

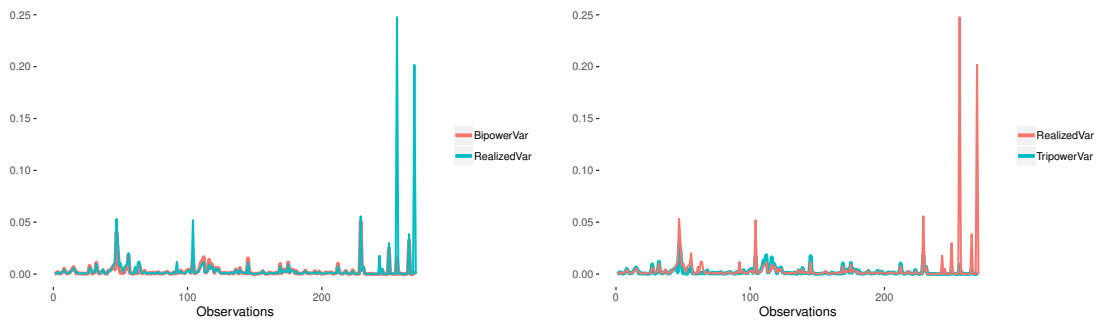


Figure 3: Distribution of RV and RPV; RV and RTPV

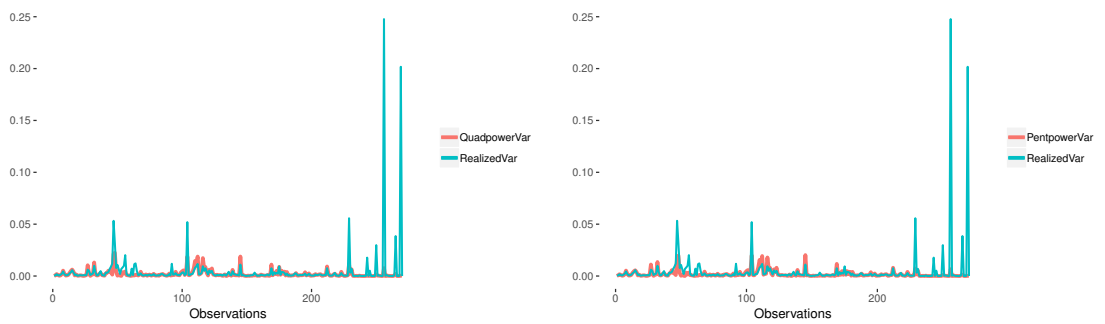


Figure 4: Distribution of RV and RQPV; RV and RPPV

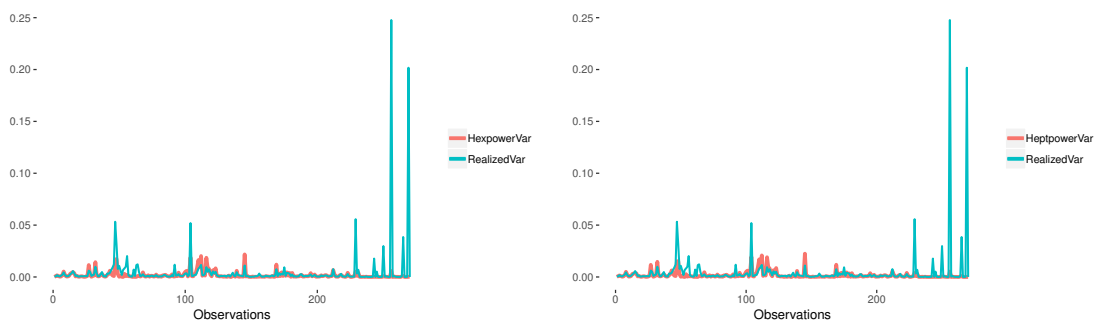


Figure 5: Distribution of RV and RH_X PV; RV and RH_p PV

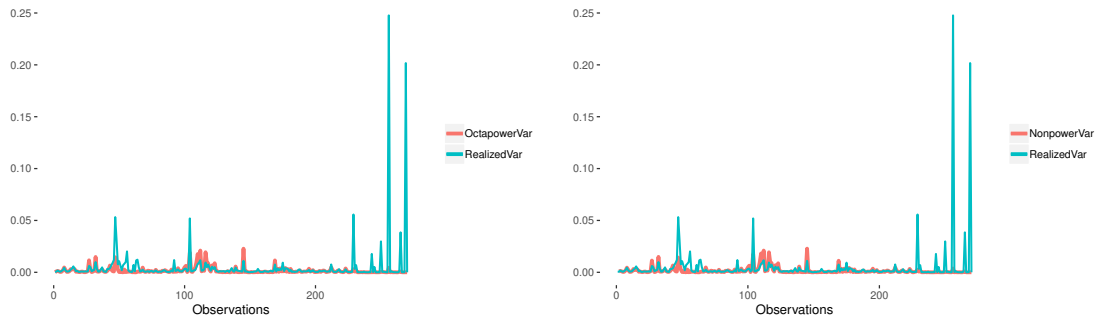


Figure 6: Distribution of RV and OPV; RV and NPV

5 Discussions and Conclusion

If $X_t \in Svs_m^j$, then the quadratic variation of the jump part of X_t can be obtained from the difference of $X_{\Delta,t}^{[1,1]}$ and $[X]_{\Delta,t}^{[2]}$; which establishes a method for jump detection as can be seen in [16, 17]. However, an extension of the above-mentioned concept has been carried out in this work. The jump test models (Eqns. (4.7) – Eqns. (4.14)) for higher-order particular cases of the realised multipower variation process restricted to $\sum_{i=1}^m r_i = 2$, have been shown to be better estimators of jumps than the bipower variation case. The distribution of the particular cases of the *RMPV* models as shown in figs. (3)-(6) presents slight deviants between the distributions of the particular cases of the realised multipower variation. However, the asymptotic variances obtained in particular cases, satisfy the following inequality:

$$\varphi_{RBV} < \varphi_{RTV} < \varphi_{QPV} < \varphi_{PPV} < \varphi_{HPV} < \varphi_{H_PPV} < \varphi_{OpV} < \varphi_{NPV} < \varphi_{DPV} < \varphi_{UPV}$$

Based on the jump test analysis result on the NASI data given in table 1 above, with a significant level of 5%, we reject the null hypothesis since all $\hat{Z}_m > \hat{Z}_{tab}$ and conclude that jumps are present in the NASI price process. Observations made on the sizes of the jumps present in the path of the NASI price process, as given in Figs 1 and 2, show that the jump sizes vary at different times of occurrence and large jumps are more prominent in the later paths of the process. Hence, we suggest that stochastic models with imputed jump components will be better representations of the NASI price process.

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Mabel Eruore Adeosun Department of Mathematics & Statistics, Osun State College of Technology. Esa-Oke. Osun State. Nigeria.

E-mail: maberuore74@gmail.com

Olabisi Oreofe Ugbebor Department of Mathematics, University of Ibadan, Ibadan, Nigeria.

E-mail: ugbebor1@yahoo.com