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ON THE DEGREE OF APPROXIMATION OF CONJUGATE OF A FUNCTION BELONGING TO WEIGHTED $W(L^p, \xi(t))$ CLASS BY MATRIX SUMMABILITY MEANS OF CONJUGATE SERIES OF A FOURIER SERIES

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Abstract. In this paper a new theorem on the degree of approximation of conjugate of a function belonging to weighted $W(L^p, \xi(t))$ class by Matrix summability means of conjugate series of a Fourier series has been established. The main theorem is a generalization of serveral known and unknown results.

1. Introduction

Bernstein [14], Alexits [5], Sahney [3] and Chandra [13] have determined the degree of approximation of a function belonging to Lip α by $(C, 1)$, (C, δ), (N, p_n) and $(\overline{N}, p_n)$ means of its Fourier Series. Working in the same direction Sahney [11] and Khan [6] have studied the degree of approximation of functions belonging to $\mathrm{Lip}(\alpha, p)$ by (N, p_n) & (N, p, q) means respectively. (N, p, q) summability reduces to (N, p_n) summability for $q_n = 1 \forall_n$ and to $(\overline{N}, q_n)$ means when $p_n = 1 \forall_n$. After quite a good amount of work on degree of approximation of function by different summability means of its Fourier Series, for the first time in 1981, Quereshi ([7], [8]) discussed the degree of approximation of conjugate of a function belonging to Lip α and $\mathrm{Lip}(\alpha, p)$ by (N, p_n) means of conjugate series of a Fourier series. But till now no work seems to have been done to obtain the degree of apprxoimation of conjugate of function belonging to weighted class $W(L^p, \xi(t))$ by matrix means of conjugate series of a Fourier Series. The weighted $W(L^p, \xi(t))$ class is the generalization of Lip α, $\mathrm{Lip}(\alpha, p)$ and $\mathrm{Lip}(\xi(t), p)$. Matrix means includes $(C, 1)$, (C, δ), (N, p_n) and (N, p, q) means as special classes. In an attempt to make an advance study in this direction, in present paper, one theorem on degree of approximation of conjugate of a function of $W(L^p, \xi(t))$ class by matrix means of conjugate series of a Fourier series has been established.

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2. Definitions and Notations

Let f by periodic with period 2π and integrable in the Lebesgue senses. Let its Fourier Series be given by

$$f(x) \approx \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \tag{2.1}$$

The conjugate series of the Fourier series (2.1) is given by

$$\sum_{n=1}^{\infty}(a_n \sin nx - b_n \cos nx) \tag{2.2}$$

Let $T = (a_{n,k})$ be an infinite triangular matrix satisfying the Silverman-Toeplitz [12] condition of regularity i.e.

$$\sum_{k=0}^{n} a_{n,k} \to 1, \quad \text{as } n \to \infty,\ a_{n,k} = 0 \text{ for } k > n$$

and $\sum_{k=0}^{n} |a_{n,k}| \leq M$, a finite constant Let $\sum_{m=0}^{\infty} u_m$ be an infinite series such that $S_k = \sum_{v=0}^{k} u_v$ The sequence-to-sequence transformation

$$t_n = \sum_{k=0}^{n} a_{n,k} S_k = \sum_{k=0}^{n} a_{n,n-k} S_{n-k}$$

defines the sequence $\{t_n\}$ of matrix means of the sequence $\{S_n\}$ generated by the sequence of coefficients $(a_{n,k})$. The series $\sum u_n$ is said to be summable to the sum S by matrix method if $\mathrm{Lim}_{n\to\infty} t_n$ exists and equal to S (Zygmund [1]) and we write:

$$t_n \to S(T), \text{ as } n \to \infty.$$

Seven important particular cases of matrix means are:

(i) $(C, 1)$ means when $a_{n,k} = \frac{1}{n+1}\ \forall k$

(ii) Harmonic means when $a_{n,k} = \frac{1}{(n-k+1)\log n}$

(iii) (C, δ) means when $a_{n,k} = \frac{\binom{n-k+\delta-1}{\delta-1}}{\binom{n+\delta}{\delta}}$

(iv) (H, p) means when $a_{n,k} = \frac{1}{(\log)^{p-1}_{(n+1)}} \prod_{q=0}^{p-1} \log^q(k+1)$

(v) Nörlund means [10] when $a_{n,k} = \frac{p_{n-k}}{P_n}$, where $P_n = \sum_{k=0}^{n} p_k$

(vi) Riez means $(\overline{N}, p_n)$ when $a_{n,k} = p_k/P_n$

(vii) Generalized Nölund mean (N, p, q) [4] when

$$a_{n,k} = \frac{p_{n-k}q_k}{R_n} \text{ where } R_n = \sum_{k=0}^{n} p_k q_{n-k}$$

We define norm by

$$\|f\|_p = \Big(\int_0^{2\pi} |f(x)|^p dx\Big)^{\frac{1}{p}}; \qquad p \geq 1$$

And let the degree of approximation be given by (Zygmund [1])

$$E_n(f) = \underset{Tn}{\mathrm{Min}} \|f - T_n\|_p$$

where $T_n(x)$ is some n^{th} degree trigonmetric polynomial.

A function $f \in \mathrm{Lip}\ \alpha$ if

$$f(x+t) - f(x) = 0(|t|^\alpha) \text{ for } 0 < \alpha \leq 1$$
$$f \in Lip(\alpha, p) \text{ if}$$
$$\Big(\int_0^{2\pi} |f(x+t) - f(x)|^p dx\Big)^{1/p} = 0(t^\alpha), \quad 0 < \alpha \leq 1, p \geq 1$$

(def. 5.38 of Mc Fadden [9])

Given a positive increasing function $\xi(t)$ and an integer $p > 1$, we find (Siddiqui [2]) that

$$f(x) \in Lip(\xi(t), p) \text{ if } \quad \Big(\int_0^{2\pi} |f(x+t) - f(x)|^p dx\Big)^{1/p} = 0(\xi(t))$$

and that

$$f(x) \in W(L^p, \xi(t)).\text{(Lal [15]) if } \Big(\int_0^{2\pi} |f(x+t) - f(x)|^p \sin^{\beta p} dx\Big)^{1/p} = 0(\xi(t)), (\beta \geq 0).$$

In case $\beta = 0$, we notice that $W(L^p, \xi(t))$ class concides with the class Lip $(\xi(t), p)$.

We shall use following notations:

$$\Psi(t) = f(x+t) - f(x-t)$$
$$A_{n,\tau} = \sum_{k=0}^{\tau} a_{n,n-k},$$
$$\tau = \text{Integral part of } \frac{1}{t} = [\frac{1}{t}]$$
$$\overline{K}_n(t) = \frac{1}{2\pi} \sum_{k=0}^{n} a_{n,n-k} \frac{\cos(n-k-\frac{1}{2})t}{\sin\frac{t}{2}}$$

3. Main Theorem

Our object of this paper is to prove the following theorem:

Theorem. *If $T = (a_{n,k})$ is an infinite regular triangular matrix such that the element $a_{n,k}$ is non-negative and non-decreasing with k then the degree of approximation of a function $\overline{f(x)}$, conjugate to a 2π periodic function f belonging to weighted $W(L^p, \xi(t))$ class by Matrix summability means of its conjugate series, is givey by*

$$\left\|\overline{t}_n(x) - \overline{f}(x)\right\|_p = 0(\xi(\frac{1}{n})_n^{\beta+\frac{1}{p}})$$

provided $\xi(t)$ satisfies the following conditons:

$$\Big(\int_0^{1/n} (\frac{t|\psi(t)|}{\xi(t)})^p \sin^{\beta p} tdt\Big)^{1/p} = 0(\frac{1}{n}) \tag{3.1}$$

$$\Big(\int_{\frac{1}{n}}^{\pi} (\frac{t^{-\delta}|\psi(t)|}{\xi(t)})^p dt\Big)^{1/p} = 0(n^{\delta}) \tag{3.2}$$

where δ is an arbitrary number such that $q(1-\delta) - 1 > 0$, conditions (3.1) & (3.2) hold uniformly in x,

$$\overline{t}_n(x) = \sum_{k=0}^{n} a_{n,n-k}\overline{S_{n-k}}$$

i.e. matrix means of conjugate series of a Fourier Series (2.2),

$$1/p + 1/q = 1 \text{ such that } 1 \le p \le \infty \text{ and}$$
$$\overline{f}(x) = -\frac{1}{2\pi}\int_0^{\pi} \psi(t)\cot\frac{1}{2}tdt$$

4. Lemmas

For the proof of our theorem following lemmas are required.

Lemma 4.1. *If $a_{n,k}$ is non-negative and non-dcreasing with k then for $0 \le a \le b \le \infty$, $0 \le t \le \pi$ and for any n,*

$$\left|\sum_{k=a}^{b} a_{n,n-k}e^{i(n-k)t}\right| \le 0(A_{n,\tau}) \qquad \text{where } \tau = \text{Integral part of } \frac{1}{t} = [\frac{1}{t}].$$

Proof. Let $\tau = [\frac{1}{t}]$, then

$$\begin{aligned}\left|\sum_{k=a}^{b} a_{n,n-k}e^{i(n-k)t}\right| &= \left|e^{int}\sum_{k=0}^{b} a_{n,n-k}e^{-ikt}\right| \\ &\le \left|\sum_{k=a}^{\tau-1} a_{n,n-k}e^{-ikt}\right| + \left|\sum_{k=\tau}^{b} a_{n,n-k}e^{-ikt}\right|\end{aligned}$$

$$\leq \left|\sum_{k=a}^{\tau-1} a_{n,n-k}\right| + a_{n,n-\tau}\left|\sum_{\tau\leq k\leq r\leq b}^{r} e^{-ikt}\right|, \quad \text{by Abel's Lemma}$$
$$\leq A_{n,\tau-1} + a_{n,n-\tau}\left|\frac{e^{-i\tau t}\{1-(e^{-it})^{r-\tau+1}\}}{(1-e^{-it})}\right|$$
$$\leq A_{n,\tau} + a_{n,n-\tau}\left|\frac{e^{-i\tau t}}{e^{-it/2}}\right|\left|\frac{1-(e^{-i(r-\tau+1)t}}{e^{it/2}-e^{-it/2}}\right|$$

or,

$$\left|\sum_{k=a}^{b} a_{n,n-k}e^{i(n-k)t}\right| \leq A_{n,\tau} + \frac{2a_{n,n-\tau}}{\sin\frac{t}{2}}$$
$$= A_{n,\tau} + 0(\frac{a_{n,n-\tau}}{t}) \tag{4.1}$$

Now

$$\begin{aligned} A_{n,\tau} &= \sum_{k=0}^{\tau} a_{n,n-k} \\ &= a_{n,n} + a_{n,n-1} + \cdots + a_{n,n-\tau} \\ &\geq a_{n,n-\tau} + a_{n,n-\tau} + \cdots + a_{n,n-\tau} \\ &= (\tau+1)a_{n,n-\tau} \\ &\geq \frac{a_{n,n-\tau}}{t} \ (\because \tau = [\frac{1}{t}]). \quad \text{Thus we obtain that} \end{aligned}$$
$$\frac{a_{n,n-\tau}}{t} = 0(A_{n,\tau}) \tag{4.2}$$

By (4.1) & (4.2) we have

$$\left|\sum_{k=a}^{b} a_{n,n-k}e^{i(n-k)t}\right| \leq A_{n,\tau} + 0(A_{n,\tau})$$
$$= 0(A_{n,\tau}).$$

In this way Lemma is proved.

Lemma 4.2. *Under the condition of our theorem on* $(a_{n,k})$, *for* $0 < 1/n \leq t \leq \pi$,

$$\overline{K}_n(t) = 0(\frac{A_{n,\tau}}{t})$$

Proof. Since for $0 \leq 1/n \leq t \leq \pi$, $\sin(t/2)\ \frac{t}{\pi}$ therefore for $t > 0$ and $\tau \leq n$, we have

$$|\overline{N}_n(t)| = \left|\frac{1}{2\pi}\sum_{k=0}^{n} a_{n,n-k}\frac{\cos(n-k-\frac{1}{2})t}{\sin\frac{t}{2}}\right|$$

$$\leq \left| \frac{1}{2\pi} \text{ Real part of } \sum_{k=0}^{n} \frac{a_{n,n-k} e^{i(n-k-\frac{1}{2})t}}{\sin\frac{t}{2}} \right|$$
$$= 0\{\frac{1}{t} \left| \sum_{k=0}^{n} a_{n,n-k} e^{i(n-k)t} \right| \left| e^{-it/2} \right|$$
$$= 0(\frac{1}{t} \left| \sum_{k=0}^{n} a_{n,n-k} e^{i(n-k)t} \right|)$$
$$= 0(\frac{A_{n,\tau}}{t}) \quad \text{by Lemma (4.1).}$$

5. Proof of Main Theorem

Let $\overline{S_n}(x)$ denote nth partial sum of the series (2.2). Then following Lal [16], we have

$$\overline{S_n}(x) - (-\frac{1}{2\pi}\int_0^\pi \psi(t)\cot\frac{1}{2}tdt) = \frac{1}{2\pi}\int_0^\pi \psi(t)\frac{\cos(n+\frac{1}{2})}{\sin t/2}dt.$$

Now

$$\sum_{k=0}^{n} a_{n,n-k}[\overline{S}_{n-k} - (-\frac{1}{2\pi}\int_0^\pi \psi(t)\cot\frac{1}{2}tdt)] = \frac{1}{2\pi}\int_0^\pi \psi(t)\sum_{k=0}^{n} a_{n,n-k}\frac{\cos(n-k-\frac{1}{2})t}{\sin\frac{t}{2}}dt$$

or

$$\begin{aligned}\overline{t}_n(x) - \overline{f}(x) &= \frac{1}{2\pi}\int_0^\pi \psi(t)\sum_{k=0}^{n} a_{n,n-k}\frac{\cos(n-k-1/2)t}{\sin\frac{t}{2}}dt \\ &= \int_0^\pi \psi(t)\overline{K_n}(t)dt \\ &= \int_0^{1/n} \psi(t)\overline{K_n}(t)dt + \int_{1/n}^\pi \psi(t)\overline{K_n}(t)dt \\ &= I_1 + I_2, \qquad \text{say} \end{aligned} \tag{5.1}$$

Applying Hölder's inequality and the fact that:

$\psi(t) \in W(l^p, \xi(t),$ we get

$$\begin{aligned}|I_1| &= \int_0^{1/n} |\psi(t)||\overline{K}_n(t)|dt \\ &\leq 0\Big[\int_0^{1/n}\Big(\frac{t(\psi(t)}{\xi(t)}\sin^\beta t\Big)^p dt\Big]^{1/p}\Big[\int_0^{1/n}\Big(\frac{\overline{K_n}(t)\xi(t)}{t\sin^\beta t}\Big)^q dt\Big]^{1/q} \\ &= 0(\frac{1}{n})\Big[\int_0^{1/n}\Big(\frac{\xi(t)}{t\sin^\beta t}\frac{1}{2\pi}\left|\sum_{k=0}^{n} a_{n,n-k}\right| \frac{\cos(n-k-\frac{1}{2})t}{\sin t/2}\Big)^q dt\Big]^{1/q}, \text{ by condition (3.1)}\end{aligned}$$

$$= 0(\frac{1}{n})\Big[\int_0^{1/n}\Big(\frac{\xi(t)}{t^{1+\beta}}\sum_{k=0}^{n}\frac{a_{n,n-k}}{t}\Big)^q dt\Big]^{1/q}$$
$$= 0(\frac{1}{n})\Big[\int_0^{1/n}\Big(\frac{\xi(t)}{t^{2+\beta}}\Big)^q dt\Big]^{1/q}$$
$$= 0(\frac{1}{n})0\Big(\xi(\frac{1}{n})\Big)\Big[\int_1^{1/n}\Big[\frac{dt}{t^{q(\beta+2)}}\Big]^{1/q}\Big], \qquad \text{by mean value theorem}$$
$$= 0(\frac{1}{n})0\Big(\xi(\frac{1}{n})\Big)\Big[\Big\{\frac{t^{-q(\beta+2)+1}}{-q(\beta+2)^{+1}}\Big\}^{1/n}\Big]^q$$
$$= 0(\frac{\xi(\frac{1}{n})}{n}) \quad 0(n^{\beta+2-\frac{1}{q}})$$
$$= 0(\xi(\frac{1}{n})n^{\beta+1-\frac{1}{q}})$$
$$I_1 = 0(\xi(\frac{1}{n})n^{\beta+\frac{1}{p}}) \quad (\because \frac{1}{p}+\frac{1}{q}=1). \tag{5.2}$$

Let us consider I_2

$$|I_2| = \Big[\int_{1/n}^{\pi}\Big(\frac{|t^{-\delta}\sin^\beta t\psi(t)|}{\xi(t)}\Big)^p dt\Big]^{1/p}\Big[\int_{1/n}^{\pi}\Big\{\frac{|\overline{k_n(t)}\xi(t)|}{\sin^\beta t\cdot t^{-\delta}}\Big\}^q dt\Big]^{1/q}$$
$$= 0\Big[\int_{1/n}^{\pi}\Big(\frac{|t^{-\delta}(\psi(t))|}{\xi(t)}\Big)^p dt\Big]^{1/p}0\Big[\int_{1/n}^{\pi}\Big\{\frac{\xi(t)An,\tau}{t^{-\delta+1}\sin^\beta t}\Big\}^q dt\Big]^{1/q} \quad \text{by Lemma (4.2)}$$
$$= 0(n^\delta)0\Big[\int_{1/n}^{\pi}\Big(\frac{\xi(t)An,\tau}{t^{-\delta+1+\beta}}\Big)^q dt\Big]^{1/q}, \qquad \text{by condition (3.2)}$$
$$= 0(n^\delta)0\Big[\int_{1/\pi}^{n}\Big(\frac{\xi(\frac{1}{y})An,[y]}{y^{\delta-1-\beta}}\Big)^q\frac{dy}{y^2}\Big]^{1/q}$$
$$= 0(n^\delta)0(\xi(\frac{1}{n})An,n)\Big[\int_1^n\frac{dy}{y^{q(\delta-1-\beta)+2}}\Big]^{1/q}, \quad \text{by mean value theorem}$$
$$= 0(n^\delta\xi(\frac{1}{n}))\Big(\Big[\frac{y^{-q(\delta-1-\beta)-1}}{-q(\delta-1-\beta)-1}\Big]_1^n\Big)^{1/q}$$
$$= 0(n^\delta\xi(\frac{1}{n}))0(n^{-\delta+1+\beta-\frac{1}{q}})$$
$$= 0(\xi(\frac{1}{n})n^{\beta+1-\frac{1}{q}})$$
$$I_2 = 0(\xi(\frac{1}{n})n^{\beta+\frac{1}{p}}) \quad (\because \frac{1}{p}+\frac{1}{q}=1) \tag{5.3}$$

By (5.1), (5.2) & (5.3) we get

$$|\overline{t_n}(-x)-\overline{f}(x)| = 0(\xi(\frac{1}{n})n^{\beta+\frac{1}{p}}).$$

Therefore

$$\|\overline{t_n}(x)-\overline{f}(x)\|_p = 0\Big(\big(\int_0^{2\pi}\{\xi(\frac{1}{n})n^{\beta+1/p}\}^p dx)^{1/p}\Big)$$
$$= 0[(\xi(\frac{1}{n})n^{\beta+1/p})]\Big[\Big(\int_0^{2\pi} dx\Big)^{1/p}\Big].$$

Thus

$$\|\overline{t_n}(x)-\overline{f}(x)\|_p = 0[(\xi(\frac{1}{n})n^{\beta+\frac{1}{p}})].$$

This complutes the proof of the theorem.

6. Corollaries

Following corollaries can be devived from the main theorem.

Corollary 1. *If $\beta = 0$ and $\xi(t) = t^\alpha$, $0 < \alpha \leq 1$ then degree of approximation of a function $\overline{f(x)}$, conjugate to 2π periodic function f belonging to the class $Lip(\alpha, p)$, is given by*

$$|\overline{t_n}(x)-\overline{f}(x)| = 0(\frac{1}{n^{\alpha-1/p}})$$

Proof.

$$\|\overline{t_n}(x)-\overline{f}(x)\|_p = \Big(\int_0^{2\pi}|\overline{t}_n(x)-\overline{f}(x)|^p dx\Big)^{1/p}$$

or,

$$0(\xi(\frac{1}{n})n^{\beta+1/p}) = \Big(\int_0^{2\pi}|\overline{t_n}(x)-\overline{f}(x)|^p dx\Big)^{1/p}$$

or,

$$0(1) = \Big(\int_0^{2\pi}|\overline{t}_n(x)-\overline{f}(x)|^p dx\Big)^{1/p} 0(\frac{1}{\xi(\frac{1}{n})n^{\beta+1/p}})$$

Hence

$$|\overline{t}_n(x)-\overline{f}(x)| = 0(\xi(\frac{1}{n})n^{\beta+1/p})$$

for if not the right hand side will not be 0(1), therefore

$$|\overline{t}_n(x)-\overline{f}(x)| = 0((\frac{1}{n})^\alpha n^{1/p}).$$
$$= 0(\frac{1}{n^{\alpha-1/p}}).$$

This completes the proof.

Corollary 2. *If $p \to \infty$ in corollary 1 then we have, for $0 < \alpha < 1$,*

$$|\overline{t}_n(x)-\overline{f}(x)| = 0(\frac{1}{n^\alpha})$$

Remark. An independent proof of corollary 1 can be derived along the same lines as the theorem.

7. Particular Cases

(a) If $a_{n,k} = (p_{n-k}/P_n)$, $\beta = 0$, $\xi(t) = t^\alpha$, $0 < \alpha < 1$ and $p \to \infty$ then result of Qureshi [7] becomes the particular case of main theorem.
(b) Result of Qureshi [8] becomes the particular case of our theorem if $a_{n,k}$ and β are defined as in (a) and $\xi(t) = t^\alpha$, $0 < \alpha \leq 1$

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