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ON THE BLUE OF THE SCALE PARAMETER OF THE GENERALIZED PARETO DISTRIBUTION

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Abstract. In this article, we will study the linear estimation of the scale parameter of the generalized Pareto distribution (GPD) which has the probability density function (p.d.f.)

$$f(x)=\begin{cases}\sigma^{-1}(1-rx/\sigma)^{1/r-1}, & r\neq 0\\ \sigma^{-1}\exp(-x/\sigma), & r=0.\end{cases}$$

We first derive the expected value, variances and covariances of the order statistics from the GPD. Then proceed to find the best linear unbiased estimates of the scale parameter σ based on a few order statistics selected from a complete sample or a type-II censored sample. Results of some chosen cases were tabulated.

Introduction

We consider the random variable X having the generalized Pareto distribution (GPD) with a probability density function (p.d.f.)

$$f(x)=\begin{cases}\sigma^{-1}(1-rx/\sigma)^{1/r-1}, & r\neq 0\\ \sigma^{-1}\exp(-x/\sigma), & r=0.\end{cases}$$

and the cumulative distribution function (c.d.f)

$$F(x)=\begin{cases}1-(1-rx/\sigma)^{1/r}, & r\neq 0\\ 1-\exp(-x/\sigma), & r=0.\end{cases}$$

where σ and r are, respectively, the scale and shape parameter. The range of x is $0\le x<\infty$ for $r\le 0$ and $0\le x\le \sigma/r$ for $r>0$.

Clearly, the standardized random variable $U=X/\sigma$ has the p.d.f.

$$f(u)=\begin{cases}(1-ru)^{1/r-1}, & r\neq 0\\ \exp(-u), & r=0.\end{cases}$$

and the c.d.f.

$$F(u)=\begin{cases}1-(1-ru)^{1/r}, & r\neq 0\\ 1-\exp(-u), & r=0.\end{cases}$$

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The range of u is $0 \leq u \leq \infty$ for $r \leq 0$ and $0 \leq u \leq 1/r$ for $r > 0$.

The Pareto distribution was first proposed by Pareto (1897). The generalized Pareto distribution was introduced by Pickands (1975). Maximum likelihood estimation of generalized Pareto was discussed by Davison (1984) and Smith (1984, 1985). Hosking and Wallis (1987) derived estimators of parameters and quantiles by the method of moments and the method of probability weighted moments and restrict attention to the case $-1/2 < r < 1/2$, for both practical and theoretical reasons. Also, they point out a close connection between generalized Pareto and generalized extreme-value distributions (GEV) with equal value for their shape parameters, and, as Hosking, Wallis, and Wood (1985) remarked, applications of the GEV distributions, particularly in hydrology usually involve the case $-1/2 < r < 1/2$.

For $r < 0$ the distribution has a heavy Pareto-type upper tail. The case $r = 0$ is the exponential distribution for which many statistical techniques are available. When $r > 0$ the distribution has an upper endpoint at σ/r. For $r = 0.5$ and $r = 1$ the distribution is triangular and uniform respectively.

The applications of the GPD include the use in the analysis of extreme events, in the modeling of large insurance claims, as a failure-time distribution in reliability studies.

Consider a sample of n order statistics

$$X_{(1)} < X_{(2)} < \cdots < X_{(n)} \tag{1.1}$$

from a continuous distribution with the p.d.f. $f[(x-\mu)/\sigma]/\sigma$ and the c.d.f. $F[(x-\mu)/\sigma]$, where μ and σ are the location and scale parameters respectively.

Lloyd (1952) obtained the generalized least-square estimators of the location and scale parameters using order statistics. We would like to find the best linear unbiased estimators (BLUE's) of the parameters based on $k(\leq n)$ order statistics selected from (1.1).

Let E' and V denote the mean vector and the covariance matrix of the standardized order statistics $(X_{(n_i)} - \mu)/\sigma$ in X^*, where

$$X^* = (X_{(n_1)} < X_{(n_2)} < \cdots < X_{(n_k)}), \quad 1 \leq n_1 \leq \cdots \leq n_k \leq n \tag{1.2}$$

and $1'$ denotes the unit vector with k components.

The BLUE σ^+ of σ while μ is known is

$$\begin{aligned} \sigma^+ &= \frac{E'V^{-1}X^*}{E'V^{-1}E} - \frac{E'V^{-1}1}{E'V^{-1}E} \cdot \mu = \sum_{i=1}^{k} b_i X_{(n_i)} - b_0\mu \\ \mathrm{Var}\,(\sigma^+) &= \frac{\sigma^2}{E'V^{-1}E}, \\ \mathrm{RE}(\sigma^+) &= \frac{\mathrm{VAR}[\text{BLUE based on (1.1)}]}{\mathrm{VAR}(\sigma^+)}. \end{aligned}$$

Chan and Cheng (1982) obtained an algorithm to compute the coefficients, variance, and relative efficiency of the best linear unbiased estimators of the location, μ, and/or scale, σ, parameters based on k optimally chosen order statistics from a sample which could be a censored sample.

In estimating a parameter by a linear combination of $k(< n)$ chosen order statistics from a sample, the set of k order statistics which gives the minimum variance among all possible choices of k order statistics is the preferred one.

The BLUE

In order to find the BLUE, we have to obtain the expected values, the variances and the covariances of the order statistics. Hence we obtain the p.d.f. of order statistics and the joint p.d.f. of any two order statistics and then derive the moments of order statistics.

Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution of continuous type having a p.d.f. $f(x)$ that is positive, provided $a < x < b$. The from of (1.1) denotes the order statistics from this sample. For convenience, we let $X_{(1)} = Y_1$, $X_{(2)} = Y_2, \ldots, X_{(n)} = Y_n$.

The p.d.f of Y_i, the ith order statistics is given by

$$g_i(y_i) = C \cdot [F(y_i)]^{i-1}[1 - F(y_i)]^{n-1} f(y_i), \quad a < y_i < b, \tag{2.1}$$

where $C = \frac{n!}{(i-1)!(n-i)!}$.

The joint p.d.f. of any two order statistics Y_i and Y_j, $(Y_i < Y_j)$ is given by

$$g_{ij}(y_i, y_j) = D[F(y_i)]^{i-1}[F(y_j) - F(y_i)]^{j-i-1} \cdot [1 - F(y_j)]^{n-j} \cdot f(y_i) f(y_j), \quad a < y_i < y_j < b, \tag{2.2}$$

where $D = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}$.

$$E(Y_i^s) = \int_a^b y_i^s g_i(y_i) dy_i, \quad 1 \le i \le n. \tag{2.3}$$

The expected value of the cross product of any two order statistics Y_i and Y_j is

$$E(Y_i^s Y_j^t) = \int_a^b \int_{y_i}^b y_i^s y_j^t g_{ij}(y_i, y_j) dy_j dy_i, \tag{2.4}$$

for $a < y_i < y_j < b$, and $1 \le i \le j \le n$.

When the shape parameter $r > 0$, the range of x is not the same as that of the shape parameter $r < 0$. First, we consider the case $r > 0$. The range of x is $0 \le x \le \sigma/r$. Thus, the p.d.f. of Y_i, from the GPD, is given by

$$g_i(y) = C \cdot [1 - (1 - ry/\sigma)^{1/r}]^{i-1}[1 - ry/\sigma]^{(n-i)/r} \cdot (1/\sigma)[1 - ry/\sigma]^{(1/r)-1},$$

Therefore,

$$\begin{aligned} E(Y_i^a) &= \int_0^{\sigma/r} y^a g_i(y) dy \\ &= (C/\sigma) \cdot \int_0^{\sigma/r} y^a [1 - (1 - ry/\sigma)^{i/r}]^{i-1} (1 - ry/\sigma)^{(n-i-r+1)/r} dy. \end{aligned}$$

Using transformation and binomial expansion to express the form $(1-t^r)^a$, we then have

$$E(Y_i^a) = \sum_{k=0}^{a} (-1)^k (a_k) \cdot C \cdot (\sigma/r)^a \cdot B[n-i+rk+1, i]$$

where $B[a,b]$ stands for a beta function.

Hence, the first moment and the second moment of Y_i are

$$E(Y_i) = \frac{n!}{(i-1)!(n-i)!} \cdot \frac{\sigma}{r} \cdot (B[n-i+1,i] - B[n-i+r+1,i]), \tag{2.5}$$

and

$$E(Y_i^2) = \frac{n!}{(i-1)!(n-i)!} \cdot \frac{\sigma^2}{r^2} \cdot (B[n-i+1,i] - 2 \cdot B[n-i+r+1,i] + B[n-i+2r+1,i]). \tag{2.6}$$

The joint p.d.f. of Y_i, Y_j is

$$g_{ij}(x,y) = D[1-(1-rx/\sigma)^{1/r}]^{i-1}[(1-rx/\sigma)^{1/r} - (1-ry/\sigma)^{1/r}]^{j-i-1} \cdot (1-ry/\sigma)^{n-j} \\ \cdot (1/\sigma)(1-rx/\sigma)^{(1/r)-1} \cdot (1/\sigma)(1-ry/\sigma)^{(1-r)/r},$$

then

$$E(Y_i^s Y_j^t) = \int_0^{\sigma/r} D \cdot x^s [1-(1-rx/\sigma)^{1/r}]^{i-1} \cdot (1/\sigma^2)(1-rx/\sigma)^{(1/r)-1} \cdot Q dx, \tag{2.7}$$

where

$$Q = \int_x^{\sigma/r} y^t [(1-rx/\sigma)^{1/r} - (1-ry/\sigma)^{1/r}]^{j-i-1} \cdot (1-ry/\sigma)^{(n-j-r+1)/r} dy.$$

After the substitutions, then we can obtain

$$E(Y_i^s Y_j^t) = D \cdot (\sigma/r)^{t+s} \sum_{a=0}^{s} \cdot \sum_{b=0}^{t} (-1)^{a+b} (s_a)(t_b) \cdot B[n-j+br+1, j-i] \cdot B[n-i+(a+b)r+1, i]. \tag{2.8}$$

When $s = t = 1$, the

$$E(Y_i Y_j) = \frac{D \cdot \sigma^2}{r^2} \cdot \Big\{ B[n-j+1, j-i] \Big[B[n-i+1,i] - B[n-i+r+1,i \Big] \\ - B[n-j+r+1, j-i] \Big[B[n-i+r+1,i] - B[n-i+2r+1,i] \Big] \Big\}. \tag{2.9}$$

Secondly, we consider the case $r < 0$. Then $0 \le x < \infty$. Thus,

$$E(Y_i^a) = \int_0^\infty y^a g_i(y) dy \\ = \frac{C}{\sigma} \cdot \int_0^\infty y^a [1-(1-ry/\sigma)^{1/r}]^{i-1} (1-ry/\sigma)^{(n-i-r+1)/r} dy,$$

After the substitutions, we can rewrite

$$E(Y_i^a) = C \cdot (\sigma/r)^a \sum_{k=0}^{a} (-1)^a (a_k) \cdot B[n-i+kr+1, i].$$

When $a = 1$, then

$$E(Y_i) = C \cdot (\sigma/r) \cdot \Big\{ B[n-i+1, i] - B[n-i+r+1, i] \Big\}. \tag{2.10}$$

When $a = 2$, then

$$E(Y_i^2) = C \cdot (\sigma/r)^2 \cdot \Big\{ B[n-i+1, i] - 2 \cdot B[n-i+r+1, i] + B[n-i+2r+1, i] \Big\}. \tag{2.11}$$

And then,

$$E(Y_i^s Y_j^t) = \int_x^\infty D \cdot x^s [1-(1-rx/\sigma)^{1/r}]^{i-1} \cdot (1/\sigma) \cdot (1-ry/\sigma)^{(1/r)-1} \cdot F \cdot dx \tag{2.12}$$

where

$$F = \int_x^\infty (\sigma/r)^{t+1} [(1-rx/\sigma)^{1/r} - (1-ry/\sigma)^{1/r}]^{j-i-1} \cdot (1-ry/\sigma)^{(n-j-r+1)/r} dy$$

After the substitutions, then we can also obtain

$$E(Y_i^s Y_j^t) = D \cdot (\sigma/r)^{t+s} \sum_{a=0}^{s} \cdot \sum_{b=0}^{t} (-1)^{a+b} (s_a)(t_b) \cdot B[n-j+br+1, j-i] \cdot B[n-i+(a+b)r+1, i]. \tag{2.13}$$

When $s = t = 1$, then $E(Y_i Y_j)$ is equal to the form of (2.9) for the shape parameter $r < 0$.

We standardize the random variable of order statistics and express the beta function in terms of gamma functions, then (2.5), (2.6), and (2.9) can be rewritten as follows:

$$E(Y_i/\sigma) = \frac{1}{r} \cdot \left\{ 1 - \frac{\Gamma(n+1)\Gamma(n-i+r+1)}{\Gamma(n-i+1)\Gamma(n+r+1)} \right\}. \tag{2.14}$$

$$E(Y_i^2/\sigma^2) = \frac{1}{r^2} \cdot \left\{ \frac{\Gamma(n+1) \cdot \Gamma(n-i+2r+1)}{\Gamma(n-i+1) \cdot \Gamma(n+2r+1)} \right\}, \tag{2.15}$$

$$E(Y_i Y_j/\sigma^2) = \frac{1}{r^2} \cdot \left\{ \frac{\Gamma(n+1) \cdot \Gamma(n-j+r+1) \cdot \Gamma(n-i+2r+1)}{\Gamma(n-i+1) \cdot \Gamma(n-i)+r+1) \cdot \Gamma(n+2r+1)} \right\}, \tag{2.16}$$

where $\Gamma(a)$ stands for the gamma function. And the covariance of Y_i and Y_j is given by

$$\mathrm{COV}(Y_i/\sigma, Y_j/\sigma) = \frac{1}{r^2} \cdot \left\{ \frac{\Gamma(n+1) \cdot \Gamma(n-j+r+1) \cdot \Gamma(n-i+2r+1)}{\Gamma(n-j+1) \cdot \Gamma(n-i+r+1) \cdot \Gamma(n+2r+1)}, \right. \\ \left. - \frac{\Gamma^2(n+1) \cdot \Gamma(n-j+r+1) \cdot \Gamma(n-i+r+1)}{\Gamma^2(n+r+1) \cdot \Gamma(n-i+1) \cdot \Gamma(n-r+1)} \right\}, \tag{2.17}$$

And (2.14) is the ith element component of the mean vector E' and (2.17) is the (i, j)th element of the convariance matrix V.

Let

$$E' = (E_i), \quad i = 1, \ldots, n. \qquad V = \frac{1}{r^2}(V_{ij}), \quad i, j = 1, \ldots, n,$$

denote the means and the covariacnes of the order statistics of the standardized $r.v.$, say, $U_{(1)} < U_{(2)} < \cdots U_{(n)}$.

We rewrite the mean vector and the covariance matrix as

$$E[U_{(i)}] = (1/r) \cdot [1 - A_{(i)}], \quad 1, \ldots, n,$$

and

$$\mathrm{COV}[U_{(i)}, U_{(j)}] = (V_{ij}) \cdot (1/r^2) = A_j D_i \cdot (1/r^2), \quad i \leq j;\ i, j = 1, \ldots, n,$$

where

$$D_i = 1/[A_i B_i] - A_i,$$
$$A_i = \frac{\Gamma(n+1)\Gamma(n-i+r+1)}{\Gamma(n-i+1)\Gamma(n+r+1)}, \qquad B_i = \frac{\Gamma(n-i+1)\Gamma(n+2r+1)}{\Gamma(n+1)\Gamma(n-i+2r+1)}.$$

We apply the theorem in Graybill (1983) [p.198-199], then the inverse of the matrix V, denoted by $\Omega = (\Omega_{ij})$, is a diagonal matrix of the second type having elements:

$$\Omega_{i1} = \frac{1}{r^2} \frac{-(D_{i-1}A_{i+1} - D_{i+1}A_{i-1})}{(D_{i-1}A_i - D_i A_{i-1})(D_i A_{i+1} - D_{i+1}A_i)},$$
$$\Omega_{i,i-1} = \Omega_{i-1,i} = \frac{1}{r^2} \frac{1}{(D_{i-1}A_i - D_i A_{i-1})},$$
$$= \frac{(n-i+1) \cdot (n-i+r+1) \cdot B_i}{-r^2}, \quad i = 1, \ldots, n,$$
$$\Omega_{ij} = 0, \quad \text{for } |i-j| > 1,$$

where $A_0 = D_{n+1} = 1,\ A_{n+1} = D_0 = 0.$

Applying the generalized last-square method and the results in the above Section, we can obtain the BLUE of σ based on k order statistics. When μ is known, the BLUE σ^+ of σ is given by

$$\sigma^+ = \frac{E'V^{-1}X^*}{E'V^{-1}E}$$

and,

$$\mathrm{VAR}(\sigma^+) = \frac{\sigma^2}{E'V^{-1}E},$$

and, $\mathrm{RE}(\sigma^+) = \dfrac{\mathrm{VAR}[\text{BLUE based on (1.1)}]}{\mathrm{VAR}(\sigma^+)}$.

Using the algorithm of Chan and Cheng (1982), we obtain the estimate for the finite sample cases.

The BLUE is the linear unbiased estimate which has the minimum variance of the estimator among the set of all ${}_nC_k$ estimators based on the k selected order statistics in (1.1). When n and/or k is large, this algorithm for finding the minimum variance of the estimator is time-consuming even with the use of a computer.

Here, the results for the complete sample and censored sample are summarized in tables. The sample size $n = 12$; $k = 2$; number of left censored observations $= IL$; number of right censored observations $= IR$; the optimum ranks n_i; the coefficient, the variance, and the relative efficiencies for the BLUE are all given in Table 1 for $r = 0.2$.

Table 1. The BLUE of σ of GDP for $k = 2$ with $r = 0.2$

n	k	IL	IR	n_1	n_2	b_1	b_2	VAR	RE(σ^+)
12	2	0	0	9	12	0.367805	0.265505	0.058199	0.924178
12	2	0	1	7	11	0.399065	0.417331	0.067634	0.955334
12	2	0	2	6	10	0.398564	0.562122	0.077634	0.972057
12	2	0	3	6	9	0.379265	0.696398	0.089361	0.981886
12	2	0	4	5	8	0.385610	0.894100	0.103631	0.988675
12	2	0	5	4	7	0.391935	1.144834	0.121751	0.992937
12	2	0	6	3	6	0.395748	1.477544	0.145740	0.995342
12	2	0	7	3	5	0.384193	1.870026	0.178896	0.997612
12	2	0	8	2	4	0.387486	2.542592	0.228423	0.999162
12	2	0	9	2	3	0.380764	3.536545	0.311076	0.999964
12	2	0	10	1	2	-0.082129	5.929866	0.476121	1.000000
12	2	1	1	7	11	0.399065	0.417331	0.067634	0.955341
12	2	1	2	6	10	0.398564	0.562122	0.077634	0.972065
12	2	1	3	6	9	0.379265	0.696398	0.089361	0.981897
12	2	1	4	5	8	0.385610	0.894100	0.103631	0.988687
12	2	1	5	4	7	0.391935	1.144834	0.121751	0.992954
12	2	1	6	3	6	0.395748	1.477544	0.145740	0.995359
12	2	1	7	3	5	0.384193	1.870026	0.178896	0.997632
12	2	1	8	2	4	0.387486	2.542592	0.228423	0.999192
12	2	1	9	2	3	0.380764	3.536545	0.311076	1.000000
12	2	2	2	6	10	0.398564	0.562122	0.077634	0.972664
12	2	2	3	6	9	0.379265	0.696398	0.089361	0.982607
12	2	2	4	5	8	0.385610	0.894100	0.103631	0.989518
12	2	2	5	4	7	0.391935	1.144834	0.121751	0.993941
12	2	2	6	3	6	0.395748	1.477544	0.145740	0.996543
12	2	2	7	3	5	0.384193	1.870026	0.178896	0.999080
12	2	2	8	3	4	0.371073	2.454526	0.228666	1.000000
12	2	3	3	6	9	0.379265	0.696398	0.089361	0.984229
12	2	3	4	5	8	0.385610	0.894100	0.103631	0.991423
12	2	3	5	4	7	0.391935	1.144834	0.121751	0.996211
12	2	3	6	4	6	0.381847	1.418741	0.145777	0.999012
12	2	3	7	4	5	0.372062	1.796979	0.179336	1.000000
12	2	4	4	5	8	0.385610	0.894100	0.103631	0.995404
12	2	4	5	5	7	0.373716	1.096313	0.121999	0.998902
12	2	4	6	5	6	0.365597	1.360425	0.146461	1.000000
12	2	5	5	6	7	0.354217	1.050366	0.122932	1.000000

We will now give an example below:

Example: Let $n = 12$, $k = 2$, $IL = 1$, $IR = 3$. The optimum ranks and the coefficients can easily be found from Table 1. The BULE of σ is

$$\sigma^{+} = 0.379265 \cdot x_{(6)} + 0.696398 \cdot x_{(9)}.$$

And the RE(σ^{+}) is 98.1897%.

Table 2. The BLUE of σ of the GPD of the censored case The Sample Size $n = 11, 12$; $k = 3$; Number of Left Censored Observations = IL; Number of the Right Censored Observation = IR; The Optimum Ranks n_i; The Coefficients b_i

n	IL	Ik	n_1	n_2	n_3	b_1	b_2	b_3	VAR(σ^*)	RE(σ^+)
11	0	0	4	7	10	0.640658	0.372749	0.075351	0.188997	0.931421
11	0	1	4	7	10	0.640658	0.372749	0.075351	0.188997	0.936657
11	0	2	4	7	9	0.642020	0.291606	0.158300	0.189399	0.961698
11	0	3	3	6	8	0.646221	0.359718	0.296621	0.195992	0.976408
11	1	1	4	7	10	0.640658	0.372749	0.075351	0.188997	0.938145
11	1	2	4	7	9	0.642020	0.291606	0.158300	0.189399	0.963269
11	1	3	3	6	8	0.646221	0.359718	0.296621	0.195992	0.978085
11	1	4	3	5	7	0.557421	0.364114	0.490796	0.208627	0.987137
11	2	2	4	7	9	0.642020	0.291606	0.158300	0.189399	0.968372
11	2	3	3	6	8	0.646221	0.359718	0.296621	0.195992	0.983531
11	2	4	3	5	7	0.557421	0.364114	0.490796	0.208627	0.993047
11	2	5	3	5	6	0.612454	0.289550	0.689382	0.229243	0.996917
11	3	3	4	6	8	0.553698	0.298605	0.297820	0.196787	0.991748
11	3	4	4	6	7	0.592740	0.229569	0.437154	0.210668	0.996589
11	3	5	4	5	6	0.528182	0.201230	0.697418	0.231914	1.000000
11	4	4	5	6	7	0.504062	0.161792	0.446666	0.215248	1.000000
12	0	0	4	7	10	0.606517	0.371919	0.155689	0.172239	0.930469
12	0	1	4	7	10	0.606517	0.371919	0.155689	0.172239	0.934909
12	0	2	4	7	10	0.606517	0.371919	0.155689	0.172239	0.957764
12	0	3	4	7	9	0.623247	0.303180	0.249995	0.176987	0.971656
12	1	1	4	6	10	0.606517	0.371919	0.155689	0.172239	0.936106
12	1	2	4	7	10	0.606517	0.371919	0.155689	0.172239	0.959020
12	1	3	4	7	9	0.623247	0.303180	0.249995	0.176987	0.972984
12	1	4	3	6	8	0.628210	0.368165	0.408112	0.185876	0.983489
12	2	2	4	7	10	0.606517	0.371919	0.155689	0.172239	0.962949
12	2	3	4	7	9	0.623247	0.303180	0.249995	0.176987	0.977140
12	2	4	3	6	8	0.628210	0.368165	0.408112	0.185876	0.987948
12	2	5	3	5	7	0.550189	0.367417	0.624609	0.200062	0.994821
12	3	3	4	7	9	0.623247	0.303180	0.249995	0.176987	0.986085
12	3	4	4	6	8	0.547885	0.303818	0.409504	0.186506	0.994191
12	3	5	4	6	7	0.592340	0.237694	0.565931	0.201636	0.997467
12	3	6	4	5	6	0.534399	0.204022	0.853785	0.223672	1.000000
12	4	4	5	7	8	0.568369	0.192515	0.366662	0.189314	0.997138
12	4	5	5	6	7	0.513054	0.166076	0.575564	0.205072	1.000000

Since too many censored cases are there, we cannot list all of them here. A few selected cases for $n = 11, 12$ and $k = 3$ are listed in Table 2. However the program for other different cases can be obtained from the author.

Conclusion

In this paper we can see from the tables that most of the relative efficiencies are quite high for the BLUE of σ. From the results it is easy to obtain the linear estimate of the scale parameter σ of the GPD for the various shape parameters. We must notice one point that the moments of order statistics may not exist if the argument of the gamma function is negative integer.

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