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SPECIAL RADICALS FOR NEAR-RING

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Abstract. The concept of a special radical for near-rings has been treated in several nonequivalent, but related, ways in the recent literature. We use the version due to K. Kaarli to establish that various prime radicals and the nil radical are special radicals on the class $\mathcal{A}$ of all near-rings which satisfy an extended version of the Andrunakievich Lemma. Since $\mathcal{A}$ includes all d.g. near-rings—and much more—these results significantly extend results previously obtained by Kaarli and by Groenewald. We also obtain special radical results for the Jacobson type radicals $\mathfrak{J}_0$ and $\mathfrak{J}_1$, albeit on less extensive classes. Examples are given which illustrate and delimit the theory developed.

Introduction

Let $\mathfrak{p}$ be a radical map (a Hoehnke radical) on the class $\mathcal{R}_0$ of all zero symmetric left near-rings and let $\mathcal{T}$ be a nonempty subclass of $\mathcal{R}_0$ which is homomorphically closed (i.e., if $R \in \mathcal{T}$ and S is a homomorphic image of R, then $S \in \mathcal{T}$). Following Kaarli, [17], we say $\mathfrak{p}$ is a *special* $\mathcal{T}$ *- radical* if there exists a nonempty class $\mathcal{C}$ of prime near-rings such that the following hold:
(1) if $R \in \mathcal{C} \cap \mathcal{T}$ and $I \triangleleft R$, then $I \in \mathcal{C}$;
(2) if $R \in \mathcal{T}$ and $K \triangleleft I \triangleleft R$,with $I/K \in \mathcal{C}$, then $K \triangleleft R$ and $R/(K : I) \in \mathcal{C}$;
(3) for each $R \in \mathcal{R}_0$, $\mathfrak{p}(R) = \cap\{I : I \triangleleft R, R/I \in \mathcal{C}\}$.
(Here $X \triangleleft A$ means X is an ideal of the near-ring A and $(K : I) = \{r \in R : Ir \subseteq K\}$. Recall that a radical map on $\mathcal{R}_0$ is a function $\mathfrak{p}$: $\mathcal{R}_0 \to \mathcal{R}_0$ such that for each $R \in \mathcal{R}_0$: (i) $\mathfrak{p}(R) \triangleleft R$;(ii) $\mathfrak{p}(R/\mathfrak{p}(R)) = 0$; and (iii) $\theta(\mathfrak{p}(R)) \subseteq \mathfrak{p}(\theta(R))$, for each homomorphism θ of R).

In this paper we show that many of the standard radicals for near-rings are special $\mathcal{A}$-radicals, where $\mathcal{A}$ is the subclass of $\mathcal{R}_0$ composed of all R such that each ideal of R is an $\mathcal{A}$-ideal. An ideal I of R is an $\mathcal{A}$*-ideal* if for each ideal K of the near-ring I there is

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some $n \geq 1$, perhaps depending on K, such that $(\langle K\rangle_R)^n \subseteq K$. (Here $\langle X\rangle_R$ is the ideal of R generated by the nonempty set X. If no ambiguity will arise we drop the subscript). The class $\mathcal{A}$ is wide and varied, including all d.g. near-rings and all near-rings which are either nilpotent or strongly regular. These and many other examples and the basic properties of near-rings in class $\mathcal{A}$ and $\mathcal{A}$-ideals are given in [7].

For convenience of the reader we list below the radicals which will be encountered in the sequel, together with a brief description of each.

Radical	Description
$\mathfrak{J}_\nu(R), \nu = 0, 1, 2$	intersection of all ν-primitive ideals
$\mathfrak{B}_0(R)$	intersection of all prime ideals
$\mathfrak{B}_3(R)$	intersection of all 3-prime ideals
$\mathfrak{C}(R)$	intersection of all completely prime ideals
$\mathfrak{N}(R)$	sum of all nil ideals, or intersection of all s-prime ideals

The radicals $\mathfrak{B}_0$, $\mathfrak{N}$, and $\mathfrak{J}_\nu$, $\nu = 0, 1, 2$, are well-known and are covered at length in [20], [21]. For a thorough discussion of $\mathfrak{B}_3$ and $\mathfrak{C}$ see [4], [12] and [5], [11], respectively. (Be advised that various other notations have been used for these two radicals. In particular, in [4] and [5] we used $\mathbf{P}_1$ and $\mathbf{P}_2$, respectively.)

The condition of being a special $\mathcal{R}_0$-radical is a stringent one. It is known that $\mathfrak{J}_2$ is a special $\mathcal{R}_0$-radical, [23]; however, $\mathfrak{J}_0, \mathfrak{J}_1, \mathfrak{B}_0$, and $\mathfrak{N}$ are not. That these four radicals are not $\mathcal{R}_0$-special radicals is implicit in the list of radical properties given in Kaarli's survey article, [17]. Kaarli, [16], has shown that $\mathfrak{J}_0$, $\mathfrak{B}_0$ and $\mathfrak{N}$ are special $\mathcal{D}$-radicals, where $\mathcal{D}$ is the class of all d.g. near-rings. Groenewald proved that $\mathfrak{B}_3$ and $\mathfrak{C}$ are special $\mathcal{D}$-radicals; see [12, Theorem 4.8] and [11, Theorem 4.3], respectively. In Theorems 2 and 6 we extend the results obtained by Kaarli on $\mathfrak{N}$ and $\mathfrak{B}_0$ to the wider class $\mathcal{A}$; in Theorems 3 and 4 we extend the results of Groenewald on $\mathfrak{B}_3$ and $\mathfrak{C}$ to $\mathcal{A}$. We also give some special radical results for $\mathfrak{J}_0$ and $\mathfrak{J}_1$. (There are at least three nonequivalent definitions of a special radical for near-rings: Kaarli, [17]; Booth and Groenewald, [8]; and Veldsman, [23]. We use the version given by Kaarli. The other two are more restrictive and imply Kaarli's conditions.)

Throughout this paper R will be in $\mathcal{R}_0$. If G is an R-module, then $\mathbf{Ann}_R(G) = \{r \in R : Gr = 0\}$.

Main Results. We begin with a lemma which will be used frequently.

Lemma 1. *Let I be an $\mathcal{A}$-ideal of R.*

(i) *Each prime ideal of the near-ring I is an ideal of R.*

(ii) *If $h : R \to S$ is a subjective near-ring homomorphism then $h(I)$ is an $\mathcal{A}$-ideal of S.*

Proof. (i) Let K be a prime ideal of the near-ring I. From this and $(\langle K\rangle_R)^n \subseteq K$ we get $\langle K\rangle_R = K$.

(ii) Consider the restriction of h to the near-ring I. Then any ideal of the near-ring $h(I)$ has the form $h(X)$, where X is an ideal of the near-ring I. Since $(\langle X\rangle_R)^n \subseteq X$, apply h in its unrestricted mode to this containment to obtain: $(\langle h(X)\rangle_S)^n \subseteq h(X)$. So $h(X)$ is an $\mathcal{A}$-ideal of S.

Note. In part (i), semiprime can be used in place of prime. From (ii) we immediately have that the class $\mathcal{A}$ is homomorphically closed.

Theorem 2. *$\mathfrak{B}_0$ is a special $\mathcal{A}$-radical.*

Proof. Let $\mathcal{C}$ be the class of all prime near-rings in $\mathcal{R}_0$ and let I be an ideal of $R \in \mathcal{C} \cap \mathcal{A}$. Consider $X, Y \lhd I$ such that $XY = 0$. Then there exist m, n such that : $(\langle X\rangle_R)^m(\langle Y\rangle_R)^n \subseteq XY = 0$. Since R is prime we have $\langle X\rangle_R = 0$ or $\langle Y\rangle_R = 0$. So either $X = 0$ or $Y = 0$ and hence 0 is a prime ideal of the near-ring I; so $I \in \mathcal{C}$.

Next, take $R \in \mathcal{A}$ and $K \lhd I \lhd R$ with $K \neq I$, so that $I/K \in \mathcal{C}$. From Lemma 1 we have $K \lhd R$; hence $(K : I) \lhd R$. Consider $A, B \lhd R$ such that $AB \subseteq (K : I)$. Then $IAB \subseteq K$. Consequently, $I \cdot (I \cap A) \cdot (I \cap B) \subseteq IAB \subseteq K$. Since K is a proper prime ideal of the near-ring I, this yields either $I \cap A \subseteq K$ or $I \cap B \subseteq K$. If the former, then $IA \subseteq I \cap A \subseteq K$ and hence $A \subseteq (K : I)$. Similarly for $I \cap B \subseteq K$. Hence $(K : I)$ is a prime ideal of R and $R/(K : I) \in \mathcal{C}$. The proof concludes by recalling that $\mathfrak{B}_0(R)$ is the intersection of all prime ideals contained in R.

Recall [12] that an ideal P of R is a 3-*prime ideal* if whenever $x, y \in R$ such that $xRy \subseteq P$, then $x \in P$ or $y \in P$. Also, R is said to be a 3-*prime near-ring* if 0 is a 3-prime ideal of R.

Theorem 3. *$\mathfrak{B}_3$ is a special $\mathcal{A}$-radical.*

Proof. Let $\mathcal{C}$ be the class of all 3-prime near-rings in $\mathcal{R}_0$, and let I be an ideal of $R \in \mathcal{C} \cap \mathcal{A}$. Consider $x, y \in I$ such that $xIy = 0$. Suppose $y \neq 0$. Then for each $u \in I$, $xuRy \subseteq xIRy \subseteq xIy = 0$. Since 0 is a 3-prime ideal of R we have $xu = 0$. Thus $xI = 0$ and hence $xRI \subseteq xI = 0$. Proceed similarly to obtain $x = 0$. So 0 is a 3-prime ideal of I and hence $I \in \mathcal{C}$.

Next, take $R \in \mathcal{A}$ and $K \lhd I \lhd R$, with $K \neq I$, so that $I/K \in \mathcal{C}$. Lemma 1 gives $K \lhd R$ and hence $(K : I)$ is an ideal of R. Consider $a, b \in R$ such that $aRb \subseteq (K : I)$. So $IaRb \subseteq K$ and hence $(Ia)Ib \subseteq K$. If $Ib \not\subseteq K$, then since K is a 3-prime ideal of I, from $(Ia)I(Ib) \subseteq IaIb \subseteq K$, we obtain $Ia \subseteq K$. Hence $a \in (K : I)$. This establishes that $(K : I)$ is a 3-prime ideal of R and $R/(K : I) \in \mathcal{C}$. Conclude the proof by recalling that $\mathfrak{B}_3(R)$ is the intersection of all 3-prime ideals of R, [4], [12].

Since prime is equivalent to 3-prime in the class of rings, and since there is some confusion on the subject in the near-ring literature, it is worthwhile to point out that the concepts are not equivalent in $\mathcal{R}_0$. In fact Example 1.1 of [4] gives a d.g. near-ring which is prime but not 3-prime.

Theorem 4. *$\mathfrak{C}$ is a special $\mathcal{A}$-radical.*

Proof. Let $\mathcal{C}$ be the class of all integral near-rings in $\mathcal{R}_0$. Since for any $R \in C$, if $I \lhd R$, then I is also integral, we have $I \in \mathcal{C}$. The rest of the proof is similar enough to that of Theorem 3 to warrant leaving the details to the reader.

In [22] it is shown that $\mathfrak{N}(R)$ is the intersection of all s-prime ideals of R. Recall, [22], that a subset S of R is called an *s-system* if S contains a multiplicative semigroup S^* such that for each $s \in S$, we have $\langle s \rangle_R \cap S^* \neq \emptyset$; here $\emptyset$ is defined to be an s-system also. An ideal P of R is called an *s-prime ideal* if its complement in R, written $R\backslash P$, is an s-system. An *s-prime near-ring* is one for which 0 is an s-prime ideal. Note every s-prime ideal (near-ring) is a prime ideal (near-ring).

We say an ideal I of R is a *nilprime ideal* if I is a prime ideal and $\mathfrak{N}(R/I) = 0$. A near-ring with 0 as a nilprime ideal is called a *nilprime near-ring.* In the next result we establish that each s-prime ideal of a near-ring is a nilprime ideal and also that $\mathfrak{N}(R)$ is equal to the intersection of all the nilprime ideals of R.

Lemma 5. *Let $\mathcal{S}$ and $\mathcal{N}$ be the sets of all s-prime ideals and nilprime ideals of R, respectively. Then $\mathcal{S} \subseteq \mathcal{N}$ and $\cap\mathcal{S} = \cap\mathcal{N} = \mathfrak{N}(R)$.*

Proof. Let P be an s-prime ideal of R. Assume $\mathfrak{N}(R/P) \neq 0$ for purposes of contradiction. Then there is an ideal Y of R such that $P \subset Y$ and $Y/P = \mathfrak{N}(R/P)$. Let $y \in Y$, $y \notin P$. Since $R\backslash P$ is an s-system, there is a multiplicative semigroup S^* contained in $R\backslash P$ such that $\emptyset \neq S^* \cap \langle y \rangle_R$. So there exists some $z \in S^* \cap \langle y \rangle_R$ and some positive integer m such that $z^m \in S^* \cap P$, a contradiction. Thus each s-prime ideal is a nilprime ideal.

Next, take K to be a nilprime ideal of R and observe that: $\mathfrak{N}(R)/K \subseteq \mathfrak{N}(R/K) = 0$. So $\mathfrak{N}(R) \subseteq K$ and hence $\mathfrak{N}(R) \subseteq \mathcal{N}$. But van der Walt has shown that $\mathfrak{N}(R) = \cap\mathcal{S}$. So $\cap\mathcal{S} = \mathfrak{N}(R) \subseteq \cap\mathcal{N} \subseteq \cap\mathcal{S}$.

Left open is the question whether every nilprime ideal is an s-prime ideal.

Theorem 6. *$\mathfrak{N}$ is a special $\mathcal{A}$-radical.*

Proof. Let $\mathcal{C}$ be the class of all nilprime near-rings in $\mathcal{R}_0$. Take $R \in \mathcal{C} \cap \mathcal{A}$ and $I \lhd R$. As in the proof of Theorem 2 we have I is a prime near-ring. Also, $\mathfrak{N}(I) \subseteq \mathfrak{N}(R) = 0$; so $I \in \mathcal{C}$.

Next, take $R \in \mathcal{A}$, $K \lhd I \lhd R$, with $I/K \in \mathcal{C}$. Proceed as in the proof of Theorem 2 to get K and $(K : I)$ are ideals of R and $(K : I)$ is a prime ideal of R. Assume $\mathfrak{N}(R/(K : I)) \neq 0$; then there is an ideal X of R such that $(K : I) \subset X$ and $X/(K : I)$ is a nil ideal of $R/(K : I)$. So $K \subseteq I \cap X$. Since $IX \nsubseteq K$ and hence $I \cap X \nsubseteq K$, we have $K \neq I \cap X$. If $x \in I \cap X$, then $x^n \in (K : I)$, for some n, and hence $Ix^n \subseteq K$. So $x^{n+1} \in K$. This yields $x + K$ is a nilpotent element in R/K. Thus $(I \cap X)/K$ is a nonzero nil ideal of I/K; but $\mathfrak{N}(I/K) = 0$. Thus $\mathfrak{N}(R/(K : I)) = 0$ and hence $R/(K : I) \in \mathcal{C}$. These remarks and Lemma 5 yield $\mathfrak{N}$ is a special $\mathcal{A}$-radical.

Note. Kaarli [15] observed that the nil radical is equal to the intersection of all the prime ideals P such that R/P is nil semisimple. He wrote that the proof is essentially that given for rings by Divinsky; see [10, p.147]. This characterization of the nil radical for near-rings can also be obtained using certain results from general radical theory. The proof we have given above has the virture of being self contained within near-ring theory and also relatively elementary.

Since $\mathfrak{J}_2$ is a special $\mathcal{R}_0$-radical one might think that for any class $\mathcal{T} \subseteq \mathcal{R}_0$ which is homomorphically closed and for which $\mathfrak{J}_0(R) = \mathfrak{J}_2(R)$ for each $R \in \mathcal{T}$, then it is immediate that $\mathfrak{J}_0$ is a special $\mathcal{T}$-radical. However, difficulties arise in pursuing this line of reasoning because there could be some $R \in \mathcal{T}$ with an ideal I such that $\mathfrak{J}_0(I) \neq \mathfrak{J}_2(I)$. Another difficulty can occur if a 0-primitive subideal of some $R \in \mathcal{T}$ is not itself an ideal of R. To circumvent these difficulties and obtain that $\mathfrak{J}_0$ or $\mathfrak{J}_1$ are special radicals on certain broad classes we introduce the following mechanism.

In the sequel we will use the following known result without further comment: if $m < n$ and G is an R-module of type n, then G is an R-module of type m. (See [20,p.50]).

Theorem 7. *Let $\mathcal{W}$ be any nonempty, homomorphically closed subclass of $\mathcal{A}$. Assume that whenever $R \in \mathcal{W}$, then each R-module of type zero is also of type 2. Then $\mathfrak{J}_0$ and $\mathfrak{J}_1$ are special $\mathcal{W}$-radicals.*

Proof. Let $m = 0$ or 1. Let $\mathcal{C}$ be the class of all m-primitive near-rings in $\mathcal{R}_0$. Consider $R \in \mathcal{C} \cap \mathcal{W}$ and $I \lhd R$. So R must be 2-primitive. Recall [21, 4.49 Theorem] that each ideal of a 2-primitive near-ring is itself a 2-primitive near-ring. So $I \in \mathcal{C}$.

Next, take $R \in \mathcal{W}$, $K \lhd I \lhd R$, with $k \neq I$ and $I/K \in \mathcal{C}$. Since K is a 0-primitive ideal of I it is a prime ideal of I. Using this and $R \in \mathcal{A}$, with Lemma 1 we have that $K \lhd R$ and then $(K : I) \lhd R$. There is a type 0 I-module T such that $\mathbf{Ann}_I(T) = K$, [20, Lemma 5.3]. Since T is a monogenic I-module, there exists $t \in T$ such that $tI = T$. So $TR = tIR \subseteq tI = T$ and $T = tI \subseteq tR \subseteq T$, yielding that T is a monogenic R-module. Since T contains no nonzero proper I-ideals, it cannot contain any nonzero proper R-ideals. Thus T is a type 0 R-module and hence T is a type 2 R-module. A routine calculation shows that $(K : I) = \mathbf{Ann}_R(T)$. So $(K : I)$ is a m-primitive ideal of R and hence $R/(K : I) \in \mathcal{C}$. The proof concludes by recalling that $\mathfrak{J}_m(R)$ is the intersection of all m-primitive ideals of R, [20, pp.84-85].

At this point we introduce some terminology.

Let K, S, T and X be subsets of R, with K, T and X nonempty.
(i) We say K is (S, T)-distributive if $s(k_1 + k_2)t = sk_1t + sk_2t$, for each $k_1, k_2 \in K$, $s \in S$, and $t \in T$. (If S is empty, then delete the corresponding factors in the above equation).
(ii) We say K is (S, T)-d.g. on X if K is (S, X)-distributive and T is contained in the subgroup of R generated by X.

If the set X plays no direct role in a discussion or if it is clear from the context what X is, we often just say "K is (S, T)-d.g." for convenience.

These two concepts were first introduced in [2] and generically are called "localized distributivity conditions". (See [13] for a survey on this topic). Observe that R is a distributive near-ring if and only if R is $(\emptyset, R)$-distributive. Furthermore, R is d.g. if and only if R is $(\emptyset, R)$-d.g. on the set of distributive elements of R.

Corollary 8. *Let $\mathcal{L}$ be the class of all near-rings R such that for some $h > 0$, $m \geq 0$, then R^h is (R^m, R)-distributive. Then $\mathfrak{J}_0$ and $\mathfrak{J}_1$ are special $\mathcal{L}$-radicals.*

Proof. In [14, Proposition 4] it is shown that if R^h is (R^m, R)-distributive, then each R-module of type 0 is of type 2. In [3, Proposition 1.12] it is shown that such an R is an $\mathcal{A}$-near-ring. Thus $\mathcal{L}$ is a class as described in Theorem 7 and hence $\mathfrak{J}_0$ and $\mathfrak{J}_1$ are special $\mathcal{L}$-radicals.

It is known in the near-ring folklore that $\mathfrak{J}_1$ is not a special $\mathcal{D}$-radical. Since, to our knowledge, this fact is not yet in the literature, we make it manifest with the following example.

Example 9. Let $R = I(A_9)$, the near-ring generated by the inner automorphisms on the alternating group on nine symbols, A_9. For each $x \in R$ define α_x via: $(y)\alpha_x = x \circ y$, for each $y \in R$. (Here the operation "$\circ$" is composition of functions.) Let T be the near-ring generated by $\{\alpha_x : x \in R\}$.

For basic facts about this near-ring see [19]. [1, Example 2], [21, p.429, no.65], [4, Example 1.1]. It is known that T is a d.g. near-ring with unity, 0-primitive, with $\mathfrak{J}_1(T) \neq 0$. So T contains a miminal ideal I such that $I^2 \neq 0$ and $I \subseteq \mathfrak{J}_1(T)$. From [18, Corollary 6] and [15, Theorem 5], we have that I is a 1-primitive near-ring. The assumption that $\mathfrak{J}_1$ is a special $\mathcal{D}$-radical leads to a contradiction, for if this were so, then $\mathfrak{J}_1(I) = I \cap \mathfrak{J}_1(T)$. However, $0 = \mathfrak{J}_1(I)$ and $I \cap \mathfrak{J}_1(T) = I$.

Although this example slams shut the door to getting that $\mathfrak{J}_1$ is a special $\mathcal{D}$-radical, the next result shows some progress in that direction can be made.

Corollary 10. Let $\mathcal{U}$ be the class of all near-rings R such that for some $m \geq 0$, then R is (R^m, R)-d.g. and for each ideal I of R there exists $n \geq 0$ such that I is (I^n, I)-d.g.. Then $\mathcal{J}_1$ is a special $\mathcal{U}$-radical.

Proof. In [14, Proposition 9] it is shown that if R is (R^m, R)-d.g., then each R-module of type 1 is of type 2. In [7, Corollary 3.8] it is shown that such an R is an $\mathcal{A}$-near-ring. Using these facts, the remainder of the proof is similar to the proof of Theorem 7.

We next list some examples, or construction schemes to produce examples, of near-rings which satisfy the hypothesis of Corollary 10, but not necessarily that of Corollary 8.

Example Schemes 11. (i) Let $R = A \oplus B$, where A is a simple d.g. near-ring and B is a distributive near-ring. In particular, take $A = E(G)$, where G is a finite simple nonabelian group, and B is one of the examples given in [9, Section 26].

(ii) Let R be a d.g. near-ring which is not distributive, yet all of its proper ideals are d.g. themselves. In particular, if every proper ideal is either square zero or a ring, then this holds. Two concrete examples are number 36 on the group S_3 and number 139 on the group D_8 in the appendix of [21, pp.411 and 418]. The latter is of interest because it is a d.g. near-ring with unity; it has three proper, nonzero ideals: two of them are rings and the other one is square zero.

Corollary 12.

(i) *If* $\mathfrak{p}$ *is any one of* $\mathfrak{B}_0, \mathfrak{B}_3, \mathfrak{C}$, *or* $\mathfrak{N}$ *then* $\mathfrak{p}(I) = I \cap \mathfrak{p}(R)$, *for each* $R \in \mathcal{A}$ *and each* $I \lhd R$.

(ii) *If* $R \in \mathcal{L}$ *and* $I \lhd R$, *then* $\mathfrak{J}_0(I) = I \cap \mathfrak{J}_0(R)$.

(iii) *If* $R \in \mathcal{L} \cap \mathcal{U}$ *and* $I \lhd R$, *then* $\mathfrak{J}_1(I) = I \cap \mathfrak{J}_1(R)$.

Proof. For each part we make use of the following result due to Kaarli [17]: if $\mathfrak{p}$ is a special $\mathcal{T}$-radical and $R \in \mathcal{T}$, then $\mathfrak{p}(I) = I \cap \mathfrak{p}(R)$, for each $I \lhd R$. Then (i) follows from Theorems 2, 3, 4, and 6; (ii) follows form Corollary 8; and (iii) follows from Corollaries 8 and 9.

Immediately form Corollary 12 together with Lemma 5 we have the following characterization of near-rings which are ideals of nil semisimple near-rings.

Corollary 13. *If* $\mathfrak{N}(R) = 0$ *then* R *is a subdirect product of nilprime near-rings. If* $R \in \mathcal{A}$, *then each ideal of* R *is a subdirect product of nilprime near-rings.*

In this sense nilprime near-rings can be considered to be basic building blocks for structure in $\mathcal{R}_0$.

If $\mathfrak{p}$ is a Hoehnke radical on $\mathcal{R}_0$, but is not a special $\mathcal{R}_0$-radical, an interesting problem is to find the largest class $\mathcal{T}$ for which $\mathfrak{p}$ is a special $\mathcal{T}$-radical. One would like $\mathcal{T}$ to contain $\mathcal{D}$. We leave unresolved the question of whether or not $\mathfrak{J}_0$ is a special $\mathcal{A}$-radical.

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