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ON CHARACTERIZATIONS OF WEIGHTED COMPOSITION OPEARTORS ON NON-LOCALLY CONVEX WEIGHTED SPACES OF CONTINUOUS FUNCTIONS

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Abstract. For a system V of weights on a completely regular Hausdorff space X and a Hausdorff topological vector space E, let $CV_b(X,E)$ and $CV_0(X,E)$ respectively denote the weighted spaces of continuouse E-valued functions f on X for which $(vf)(X)$ is bounded in E and vf vanishes at infinity on X all $v \in V$. On $CV_b(X,E)(CV_0(X,E))$, consider the weighted topology, which is Hausdorff, linear and has a base of neighbourhoods of 0 consising of all sets of the form: $N(v,G) = \{f : (vf)(X) \subseteq G\}$, where $v \in V$ and G is a neighbourhood of 0 in E. In this paper, we characterize weighted composition operators on weighted spaces for the case when V consists of those weights which are bounded and vanishing at infinity on X. These results, in turn, improve and extend some of the recent works of Singh and Singh [10, 12] and Manhas [6] to a non-locally convex setting as well as that of Singh and Manhas [14] and Khan and Thaheem [4] to a larger class of operators.

Introduction

The contents of this paper are in relation with the theory of weighted composition operators on weighted spaces which are studied by Jamison and Rajagopalan [1], Singh and Summers [17], Singh and Manhas [14], Singh and one of the authors in [10, 12], Khan and Thaheem [3, 4], Manhas [6], and two of the authors in [8, 9]. In [17], Singh and Summers have made a detailed study of composition operators on locally convex weighted spaces where as multiplication operators on such spaces have been studied by Singh and Manhas [14] and their results have been generalized by Singh and Singh [10] to a larger class of operators, known as weighted composition operators. Khan and Thaheem, in a very recent work [4], have extended the work of [14] to a non-locally convex setting and their work have been further extended by Singh and Kour [8]. This paper is a continuation of earlier paper [12] in which a characterization of weighted composition operators on locally convex weighted space $CV_b(X,E)$ is presented and also it is a continuation to earlier paper [9] where we have studied weighted composition

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operators $W_{\pi,T}(W_{\theta,T})$ on non-locally convex weighted spaces $CV_b(X,E)$ and $CV_0(X,E)$ induced by $\pi : X \to E(\theta : X \to \mathbb{C})$ and $T : X \to X$.

The purpose of this paper is to characterize those weighted composition operators on non-locally convex weighted spaces which are induced by mappings $\pi : X \to B(E)$ and $T : X \to X$. These results improve and extend, in particular, some of the results contained in [4, 6, 9, 10, 12, 14].

Preliminaries

Throughout this paper we shall assume, unless stated otherwise, that X is a completely regular Hausdorff space and E is a non-trivial Hausdorff topological vector space over $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$. Then by $C(X,E)$ we denote the vector space of all continuous functions from X into E. A function $f : X \to E$ is said to vanish at infinity if for each neighbourhood N of origin in E there exists a compact subset K of X such that $f(x) \in N$ for all x in $X \backslash K$, the complement of the set K in X. A subset B of E is said to be bounded if for every neighbourhood N of 0 there exists $\varepsilon > 0$ such that $B \subseteq \varepsilon N$. Then we define

$C_0(X,E) = \{f \in C(X,E) : f \text{ vanishes at infinity on } X\}$, and
$C_b(X,E) = \{f \in C(X,E) : f(X) \text{ is bounded in } E\}$, where $f(X) = \{f(x) : x \in X\}$.

Clearly $C_0(X,E) \subset C_b(X,E)$. When $E = \mathbb{K}$ with the usual topology, these spaces are respectively denoted by $C(X)$, $C_0(X)$ and $C_b(X)$. In case $X = \mathbb{N}$, the set of all natural numbers with the discrete topology, $C_b(\mathbb{N}) = l^\infty$, the Banach algebra of all bounded sequences in $\mathbb{K}$, and $C_0(\mathbb{N}) = c_0$, the Banach space of null sequences in $\mathbb{K}$. A real-valued function f on X is called upper-semicontinuous if the set $\{x \in X : f(x) < a\}$ is open for all a in $\mathbb{R}$. By a *weight* we mean a non negative upper-semicontinuous function on X. Let V denote a family of weights on X. Then we say that $V > 0$ if for every $x \in X$ there is some $v_x \in V$ such that $v_x(x) > 0$; and that V is direct upward (or a Nachbin family) if for every pair $u, v \in V$ and every $a > 0$ there exists a $w \in V$ such that $au(x) \leq w(x)$ and $av(x) \leq w(x)$ for all x in X. Since there is no loss of generality, we hereafter assume that the sets of weights are directed upward. Now by a *system of weights* we mean a set V of weights on X which additionally satisfies that $V > 0$.

Let us now consider the following vector spaces (over $\mathbb{K}$) of continuous functions from X into E for a given system V of weights on X:

$CV_0(X,E) = \{f \in C(X,E) : vf \text{ vanishes at infinity on } X \text{ for all } v \in V\}$ and
$CV_b(X,E) = \{f \in C(X,E) : (vf)(X) \text{ is bounded in } E \text{ for all } v \in V\}$,

where $(vf)(X) = \{v(x)f(x) : x \in X\}$.

It is clear that $CV_0(X,E) \subset CV_b(X,E)$. On $CV_b(X,E)$, consider the weighted topology w_V, which is Hausdorff, linear and has a base of neighbourhoods of 0 consisting of all sets of the form:

$$N(v,G) = \{f : (vf)(X) \subseteq G\}, \text{ where } v \in V \text{ and } G \text{ is a neighbourhood of 0 in } E.$$

The space $CV_b(X, E)$, equipped with w_V is called a *weighted space*. The space $CV_0(X, E)$, being a subspace of $CV_b(X, E)$, is equipped with the topology induced by $CV_b(X, E)$.

The following are some instances of weighted spaces:

(i) If V is the set of all non-negative constant functions on X, then $CV_b(X, E) = C_b(X, E)$ and $CV_0(X, E) = C_0(X, E)$. The topology w_V in this case is the topology of uniform convergence.

(ii) If $V = \{a\chi_K : a \geq 0 \text{ and } K \subset X, K \text{ compact}\}$, where χ_K denotes the characteristic function of K, then $CV_b(X, E) = CV_0(X, E) = C(X, E)$ and w_V is the compact-open topology.

(iii) If V is the system of all non-negative weights which vanish at infinity on X, then $CV_b(X, E) = CV_0(X, E) = C_b(X, E)$ and w_V in this case is the substrict topology.

For more details on such weighted spaces, we refer to Nachbin [7], Singh and Summers [17], Khan [2] and Khan and Thaheen [3].

Let $B(E)$ denote the vector space of all continuous linear mappings from E into itself, endowed with the linear topology which has a base of neighbourhoods of 0 consisting of all sets of the form:

$U(B, G) = \{A \in B(E) : A(B) \subseteq G\}$, where B is bounded (or a finite) subset of E and G is a neighbourhood of 0 in E. By $B_u(E)$ (respectively, $B_s(E)$), we denote the space $B(E)$ when it is equipped with the uniform (respectively, strong) operator topology, that is, the topology of uniform (pointwise) convergence on bounded (finite) subsets of E.

Let $L(X, E)$ denote a vector space of functions from X into E. If $\pi : X \to B(E)$ and $T : X \to X$ are mappings such that $\pi.foT \in L(X, E)$ for every $f \in L(X, E)$, then the correspondence $f \to \pi.foT$ is a linear transformation from $L(X, E)$ into itself and we denote it by $W_{\pi,T}$ (here the multiplication of π and the composite function foT is defined as $\pi.foT(x) = \pi(x)(f(T(x))$ for all $x \in X$). In case $L(X, E)$ is a topological vector space and $W_{\pi,T}$ is continuous, it is called a weighted composition operator (in short, written as WCO) on $L(X, E)$ induced by the pair (π, T).

In case T is the identity map on X, the corresponding operator $W_{\pi,T}$ is called the multiplication operator and is denoted by M_π. On the other hand, when $\pi(x) = I$, the identity operator on E, for all $x \in X$, the corresponding WCO is called the composition operator and is denoted by C_T. For a detailed account of these operators on spaces of continuous functions, we refer to the monograph [16] of Singh and Manhas as well the recent survey article [11] of Singh and one of the authors.

A neighbourhood G of 0 in E is called *shrinkable* if $r\overline{G} \subseteq \text{int}G$ for $0 \leq r < 1$. By [5, Theorems 5 and 6], every Hausdorff topological vector space has a base of shrinkable neighbourhoods of 0 and also the Minkowski functional ρ_G of any such neighbourhood G is continuous. For details, we refer to [2, 3].

For any $t \in E$ and for $f \in C(X)$, the function f_t defined by setting $f_t(x) = f(x)t$ for all $x \in X$ clearly belongs to $C(X, E)$. In particular, the constant t-function 1_t belongs to $C(X, E)$. The conditions under which 1_t belongs to $CV_a(X, E)$ are recorded in the following proposition:

Proposition 1. *Let $\mathcal{N}$ be a base of neighbourhoods of 0 in E. Then the following statements are equivalent:*

(1.a) Every $v \in V$ is bounded (respectively, vanishes at infinity) on X.

(1.b) For every $t \in E$, $1_t \in CV_b(X,E)$ (respectively, $CV_0(X,E)$)

(1.c) Every constant selfmap on X induces a composition operator on $CV_b(X,E)$ (respectively, $CV_0(X,E)$).

Proof. We may assume that $\mathcal{N}$ consists of closed, balanced and shrinkable sets. The proof then follows from Propositions 2.1. and 2.2 of [13] by replacing the continuous seminorms p and q respectively by the Minkowski functionals ρ_G and ρ_H of shrinkable neighbourhoods G and H of 0 in E.

Characterization of WCOs

In this section, we present necessary and sufficient conditions for $W_{\pi,T}$ to be a WCO on the weighted spaces $CV_b(X,E)$ and $CV_0(X,E)$. To avoid trival cases, we assume that for every $x \in X$, there exists an $h_x \in CV_0(X)$ such that $h_x(x) \neq 0$. This holds, in particular, when X is locally compact or when each $v \in V$ vanishes at infinity on X.

In the locally convex setting, a characterization of WCO on $CV_b(X,E)$ has been presented by Singh and one of the authors in [10] under the assumption that $\pi(X)$ is equicontinuous whereas on $CV_0(X,E)$ it has been reported by Manhas in [6] but under the condition that X is a $k_{\mathbb{R}}$-space. For non-locally convex spaces, multiplication operators on weighted spaces have been studied by Khan and Thaheem [4] with the same requirement as in [6]. It can be noted that either of the condition that "$\pi(X)$ is equicontinuous" or "X is a $k_{\mathbb{R}}$-space" is needed only to make the map M_π continuous (cf. [15]). Also, the results in [4, 6, 10, 14] characterizing WCOs and multiplication operators on weighted spaces require as a hypothesis that T is continuous on whole of X and π is continuous in the uniform operator topology. But for instance when $\pi(x) = 0$ for all $x \in X$, $W_{\pi,T}$ is a continuous operator even if T is not continuous on whole of X. We now see that π is continuous in the strong operator topology as follows:

If the system V of weights satisfies condition (1.a) of Proposition 1 above, and $W_{\pi,T}$ is a WCO on $CV_a(X,E)$, where $a \in \{b,0\}$, then, for any $t \in E$, $W_{\pi,T}1_t \in CV_a(X,E)$ and $W_{\pi,T}1_t(x) = \pi(x)t$ for all $x \in X$. Further, if $\{x_\alpha\}$ is a net in X such that $x_\alpha \to x$ in X, then for every shrinkable neighbourhood G of 0 in E, we have

$$\rho_G[\pi(x_\alpha) - \pi(x)](t) = \rho_G[W_{\pi,T}1_t(x_\alpha) - W_{\pi,T}1_t(x)] \to 0$$

This implies that $\pi \in C(X, B_s(E))$. Note that π is not necessarily continuous in the uniform operator topology (see [1] for example).

Theorem 2. *Let $\mathcal{N}$ be a base of neighbourhoods of 0 in E, $T : X \to X$ and $\pi : X \to B(E)$ such that $\pi(x) \neq 0$ for all $x \in X$. Assume that $\pi(X)$ is equicontinuous* (*or X is a $k_{\mathbb{R}}$-space*). *Then the following conditions are sufficient for the pair (π, T) to induce a WCO on $CV_b(X,E)$:*

(2.1) $\pi \in C(X, B_s(E))$;
(2.2) T is continuous;
(2.3) for every $v \in V$ and $G \in \mathcal{N}$, there eixsts $u \in V$ and $H \in \mathcal{N}$ such that

$$v(x)\rho_G(\pi(x)t) \leq uoT(x)\rho_H(t) \quad \text{for all } x \in X \text{ and } t \in E.$$

Furthermore, if the system V consists of bounded weights on X, then the above conditions (2.1)-(2.3) are also necessary for (π, T) to induce a WCO on $CV_b(X, E)$.

Proof. We may assume that $\mathcal{N}$ consists of closed, balanced and shrinkable sets.

Sufficient Part. Assume that (2.1)-(2.3) hold. It can be shown that $W_{\pi,T}$ take $CV_b(X, E)$ into itself as proved in [10] using [14, 15] with the corresponding seminorms replaced by the Minkowski functionals. To see the continuity of $W_{\pi,T}$, let $\{f_\alpha : \alpha \in \Delta\}$ be a net in $CV_b(X, E)$ such that $f_\alpha \to 0$, and let $v \in V$ and $G \in \mathcal{N}$. Using (2.3), choose $u \in V$ and $H \in \mathcal{N}$ such that $v(x)\rho_G(\pi(x)t) \leq u_(T(x)\rho_H(t)$ for all $x \in X$ and $t \in E$. Since $f_\alpha \to 0$, their exists an $\alpha_0 \in \Delta$ such that $f_\alpha \in N(u, H)$ for all $\alpha \geq \alpha_0$. Then, for any $x \in X$ and $\alpha \geq \alpha_0$, we have

$$v(x)\rho_G(\pi(x)(f_\alpha oT(x))) \leq uoT(x)\rho_H(f_\alpha oT(x)) < 1,$$

or equivalently, $v(x)W_{\pi,T}f_\alpha(x) \in G$. Thus $w_{\pi,T}$ is continuous at 0 and hence, by its linearity, on $CV_b(X, E)$.

Necessary Part. Assume that all weights in V are bounded. Then, as noted above, (2.1) holds while the proof of (2.2) is similar the one given in Theorem 2.2 of [12]. To prove (2.3), let $v \in V$ and $G \in \mathcal{N}$. Then, by continuity of $W_{\pi,T}$, there exists $u \in V$ and $H \in \mathcal{N}$ such that

$$W_{\pi,T}(N(u, H)) \subseteq N(v, G). \tag{$*$}$$

We claim that $v(x)\rho_G(\pi(x)t) \leq 3uoT(x)\rho_H(t)$ for all $x \in X$ and $t \in E$. To prove this, let $x_0 \in X$ and $s \in E$ be fixed, take $y = T(x_0)$, and set $\varepsilon = u(y)\rho_H(s)$. Then consider the following cases:

(I) $\varepsilon \neq 0$; (II) $\varepsilon = 0$; that is, (IIa) $u(y) = 0$, $\rho_h(s) \neq 0$;
(IIb) $u(y) \neq 0$, $\rho_H(s) = 0$; (IIc) $u(y) = 0, \rho_H(s) = 0$.

Case (I): Suppose $\varepsilon \neq 0$. Then the set $D_1 = \{x \in X : u(x)\rho_H(s) < 3\varepsilon\}$ is an open neighbourhood of y and so according to Nachbin's Lemma [7, page 69], there exists an $f \in CV_b(X)$ such that $0 \leq f \leq 1$, $f(y) = 1$, and $f(X \backslash D_1) = \{0\}$. Write $h = (1/3\varepsilon)f_s$. Then $h \in CV_b(X, E)$ and for any $x \in X, \rho_H(u(x)h(x)) = (1/3\varepsilon)u(x)f(x)\rho_H(s) \leq 1$, which means that $h \in N(u, H)$. By $(*)$, $W_{\pi,T}h \in N(v, G)$ which gives $v(x)\rho_G(\pi(x)hoT(x)) \leq 1$ for every $x \in X$. This inequality when evaluated at x_0 gives $v(x_0)\rho_G(\pi(x_0)s) \leq 3\varepsilon = 3uoT(x_0)\rho_H(s)$.

Case (IIa): When $u(y) = 0$, $\rho_H(s) \neq 0$, we argue by contradiction. Suppose $v(x_0)\rho_G(\pi(x_0)s) > 0$ and write $\delta = (1/3)v(x_0)\rho_G(\pi(x_0)s)$. Then the set $D_2 = \{x \in X : u(x) < \delta[\rho_H(s)]^{-1}\}$ is an open neighbourhood of y and so again by Nachbin's lemma there exists an $f \in CV_b(X)$ such that $0 \leq f \leq 1$, $f(y) = 1$ and $f(X \backslash D_2) = \{0\}$. Write $h = (1/\delta)f_s$. Then $h \in N(u, H)$ and $W_{\pi,T}h \in N(v, G)$, which implies that $v(x)\rho_G(\pi(x)hoT(x)) \leq 1$ for every $x \in X$. This when evaluated at $x = x_0$ gives $u(x_0)\rho_G(\pi(x_0)s) \leq \delta$, which is absurd. Thus we must have $v(x_0)\rho G(\pi(x_0)s) = 0$. For cases (IIb) and (IIc), we take $D_3 = \{x \in X : u(x) < \delta + u(y)\}$ and $D_4 = \{x \in X : u(x) < \delta\}$ respectively as open neighbourhood of y and proceed in the same way as in case (IIa). Thus, for all cases, we have $v(x_0)\rho_G(\pi(x_0)s) \leq uoT(x_0)\rho_H(s)$. Since $x_0 \in X$ and $s \in E$ are arbitrarily taken, we conclude that $v(x)\rho_G(\pi(x)t) \leq 3uoT(x)\rho_H(t)$ for all $x \in X$ and $t \in E$, proving our claim as well as the proof of the theroem.

The conditions of Theorem 2 above are not sufficient for the pair (π, T) to induce a WCO on $CV_0(X, E)$ as already noted in [17, page 307] for the scalar valued space $CV_0(X)$. In the following theorem, we shall present a necessary and sufficient condition for $W_{\pi,T}$ to be a WCO on $CV_0(X, E)$, which also answers a remark given in [16, page 145] in the present setting.

Theorem 3. *Let $\mathcal{N}$ be a base of neighbourhoods of 0 in $E, T : X \to X$ and $\pi : X \to B(E)$ such that $\pi(x) \neq 0$ for all $x \in X$. Assume that $\pi(X)$ is equicontinuous* (*or X is a $k_{\mathbb{R}}$-space*)*. Then the following conditions are sufficient for the pair (π, T) to induce a WCO on $CV_0(X, E)$:*

(3.1) $\pi \in C(X, B_s(E))$;
(3.2) T is continuous;
(3.3) for every $v \in V$ and $G \in \mathcal{N}$, there exists $u \in V$ and $H \in \mathcal{N}$ such that $v(x)\rho_G(\pi(x)t) \leq uoT(x)\rho_H(t)$ for all $x \in X$ and $t \in E$.
(3.4) for every $v \in V$, $G \in \mathcal{N}$, $\varepsilon > 0$ and compact subset K of X, the set $T^{-1}(K) \cap K_t$ is compact for all $t \in E$, where $X_t = \{x \in X : v(x)\rho_G(\pi(x)t) \geq \varepsilon\}$.

Furthermore, the above conditions (3.1)-(3.4) are necessary for (π, T) to induce a WCO on $CV_0(X, E)$ if all the weights in V vanish at infinity on X.

Proof. We may assume that $\mathcal{N}$ consists of closed, balanced and shrinkable sets.

Sufficient Part. Assume that (3.1)-(3.4) hold. Then, by Theorem 2 above, $W_{\pi,T}$ is a WCO on $CV_b(X, E)$. We show that $CV_0(X, E)$ is invariant under $W_{\pi,T}$. For this, let $f \in CV_0(X, E)$, $v \in V$ and $G \in \mathcal{N}$. By (3.3) choose $u \in V$ and $H \in \mathcal{N}$ such that

$$v(x)\rho_G(\pi(x)t) \leq uoT(x)\rho_H(t) \text{ for all } x \in X \text{ and } t \in E. \qquad (**)$$

Since $f \in CV_0(X, E)$, given any $\varepsilon > 0$ there exists a compact subset K of X such that $u(x)\rho_H(f(x)) < \varepsilon$ for all $x \in X \backslash K$. Also, for any $t \in E$, set $K_t = T^{-1}(K) \cap X_t$ is compact, in view of (3.4). Now if $x \notin K_t$, then either $x \notin T^{-1}(K)$ or $x \notin X_t$. First suppose that $x \notin T^{-1}(K)$. Then $T(x) \in X \backslash K$, and so we have $uoT(x)\rho_H(foT(x)) < \varepsilon$. By $(**)$, this implies that $v(x)\rho_G(\pi(x)(foT(x))) < \varepsilon$. If $x \notin X_t$, then $v(x)\rho_G(\pi(x)t) < \varepsilon$

and in particular this implies that $v(x)\rho_G(\pi(x)(foT(x))) < \varepsilon$. Thus $vW_{\pi,T}f : X \to E$ vanishes at infinity. Since $v \in V$ was arbitrary, we conclude that $W_{\pi,T}f \in CV_0(X,E)$.

Necessary Part. Assume that all the weights in V vanish at infinity and let $W_{\pi,T}$ be a WCO on $CV_0(X,E)$. Then the conditions (3.1)-(3.3) can be proved in a similar way as in Theorem 2 above. To prove (3.4), take $v \in V$, $G \in \mathcal{N}$, $\varepsilon > 0$ and let K be a compact subset of X. Then by Nachbin's Lemma [7, page 69] there exists an $f \in CV_0(X)$ such that $0 \leq f \leq 1$ and $f(K) = \{1\}$. For any $t \in E$, the function $f_t \in CV_0(X,E)$ since weights are vanishing. So $W_{\pi,T}f_t \in CV_0(X,E)$, which means that the set $A = \{x \in X : v(x)\rho G(\pi(x)(foT(x))) \geq \varepsilon\}$ is compact. We note that $K_t = T^{-1}(K) \cap X_t$ is a closed subset of A. Hence K_t is compact.

Remarks. (i) Theorems 2 and 3 above improve Theorem 3.1 of [4], Theorem 2.1 of [6], Theorem 3.2 of [10], Theorem 2.1 of [14] and Theorems 3.3 and 3.4 of [9] as well as extend Theorem 2.2 of [12] to a non-locally convex setting.

(ii) On non-locally convex weighted spaces $FV_b(X,E)$ and $FV_0(X,E)$ of E-valued functions (not necessarily continuous) on a topological space X, weighted composition operators have been studied by two of the authors in [8].

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