TAMKANG JOURNAL OF MATHEMATICS
Volume 39, Number 1, 25-32, Spring 2008

SOME WEAKLY MAPPINGS ON INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract. In this paper, we shall introduce concepts of fuzzy semiopen set, fuzzy semiclosed set, fuzzy semiinterior, fuzzy semiclosure on intuitionistic fuzzy topological space and fuzzy open (fuzzy closed) mapping, fuzzy irresolute mapping, fuzzy irresolute open (closed) mapping, fuzzy semicontinuous mapping and fuzzy semiopen (semiclosed) mapping between two intuitionistic fuzzy topological spaces. Moreover, we shall discuss their some properties.

1. Introduction

After the introduction of the concept of fuzzy sets by Zadeh in [8] several researches were conducted on the generalization of the notion of fuzzy set. The idea of "intuitionistic fuzzy set" was first published by Atanassov in [1] and many works by the same author and his colleagues appeared in the literature [2, 3, 4]. Later, this concept was generalized to "intuitionistic L-fuzzy sets" by Atanassov and Stoeva in [5]. In [7], the author introduced the concepts of intuitionistic fuzzy topological spaces and intuitionistic fuzzy continuous mappings.

In this paper, on base [7] we shall introduce the concepts of fuzzy semiopen set, fuzzy semiclosed set and fuzzy semiinterior and fuzzy semiclosure on intuitionistic fuzzy topological spaces and fuzzy irresolute mapping, fuzzy irresolute open (closed) mapping, fuzzy semicontinuous, fuzzy semiopen (semiclosed) mapping between two intuitionistic fuzzy topological spaces. Moreover, we shall discuss their some properties.

2. Preliminaries

Definition 2.1.([5]) Let X be a nonempty set. An intuitionstic fuzzy set (**IFS** for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle : x \in X\}$$

where the mappings $\mu_A(x) : X \to I$ and $\upsilon_A : X \to I$ denote the degree of membership and the degree of nonmembership of each element $x \in X$ to the set A, respectively and $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ for each $x \in X$.

Received March 09, 2006.
2000 *Mathematics Subject Classification*. 54A40.
Key words and phrases. Intuitionistic fuzzy set, intuitionistic fuzzy irresolute mapping, intuitionistic fuzzy semicontinuous mapping.
The project is supported by Scientific Research Common Program of Beijing Municipal Commission of Education (KM200510009009).

Definition 2.2.([5]) Let X be a nonempty set, the **IFSs** A, B and $\{A_i, i \in I\}$ be in the form $A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \upsilon_B(x)\rangle : x \in X\}$ and $A_i = \{\langle x, \mu_{A_i}(x), \upsilon_{A_i}(x)\rangle : x \in X, i \in I\}$ be an arbitrary family of **IFSs** in X. Then

(1) $A \le B$ if and only if $\mu_A(x) \le \mu_B(x)$, $\upsilon_A(x) \ge \upsilon_B(x)$ for each $x \in X$;
(2) $A' = \{\langle x, \upsilon_A(x), \mu_A(x)\rangle : x \in X\}$;
(3) $\bigwedge_{i\in I} A_i = \{\langle x, \bigwedge_{i\in I} \mu_{A_i}(x), \bigvee_{i\in I} \upsilon_{A_i}(x)\rangle : x \in X\}$;
(4) $\bigvee_{i\in I} A_i = \{\langle x, \bigvee_{i\in I} \mu_{A_i}(x), \bigwedge_{i\in I} \upsilon_{A_i}(x)\rangle : x \in X\}$.

Definition 2.3.([7]) $\underline{0} = \{\langle x, 0, 1\rangle : x \in X\}$ and $\underline{1} = \{\langle x, 1, 0\rangle : x \in X\}$.

Proposition 2.4.([7]) *Let A, B, C be* **IFSs** *in X. Then*

(1) $(A \vee B)' = A' \wedge B'$, $(A \wedge B)' = A' \vee B'$;
(2) $A \le B \Rightarrow B' \le A'$;
(3) $(A')' = A$, $\underline{1}' = \underline{0}$, $\underline{0}' = \underline{1}$.

By above some definitions, we have the following theorem:

Theorem 2.5.([7]) *Let $A, A_i (i \in I)$ be* **IFSs** *in $X, B, B_j (j \in J)$ be* **IFSs** *in Y and $f : X \to Y$ be a mapping as defined in* [7]. *Then*

(1) $A_1 \le A_2 \Rightarrow f_L^{\rightarrow}(A_1) \le f_L^{\rightarrow}(A_2)$, $B_1 \le B_2 \Rightarrow f_L^{\leftarrow}(B_1) \le f_L^{\leftarrow}(B_2)$;
(2) $A \le f_L^{\leftarrow}(f_L^{\rightarrow}(A))$, $f_L^{\rightarrow}(f_L^{\leftarrow}(B)) \le B$;
(3) $f_L^{\leftarrow}(B') = (f_L^{\leftarrow}(B))'$.

Definition 2.6.([7]) An intuitionistic fuzzy topological space (**IFTS** for short) is a pair (X, τ), where τ is a subfamily of **IFSs** in X which contains $\underline{0}$, $\underline{1}$ and is closed for any suprema and finite infima. τ is called an intuitionistic fuzzy topology on X. Each member of τ is called an intuitionistic open set (**IFCS** *for short*) and its quasi-complementation is called an intuitionistic closed set(**IFCS** for short).

Definition 2.7.([7]) Let (X, τ) be an **IFTS** and $A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle : x \in X\}$ be an **IFS** in X. Then the fuzzy interior and fuzzy closure of A are defined by

$cl(A) = \bigwedge\{C : C$ is an **IFCS** in X and $C \ge A\}$;
$int(A) = \bigvee\{D : D$ is an **IFOS** in X and $D \le A\}$.

It can be also shown that $cl(A)$ is an **IFCS**, $int(A)$ is an **IFOS** in X and A is an **IFCS** in X if and only if $cl(A) = A$; A is an **IFOS** in X if and only if $int(A) = A$.

Proposition 2.8.([7]) *Let (X, τ) be an* **IFTS** *and A, B be* **IFSs** *in X. Then the following properties hold:*

(1) $cl(A') = (int(A))'$, $int(A') = (cl(A))'$;
(2) $int(A) \le A \le cl(A)$;
(3) *if $A \le B$, then $int(A) \le int(B)$, $cl(A) \le cl(B)$.*

Definition 2.9.([7]) Let (X, τ), (Y, σ) be two **IFTSs** and let $f : X \to Y$ be a mapping. Then f is said to be fuzzy continuous if and only if the primage of each **IFS** in σ is an **IFS** in τ.

3. Intuitionistic fuzzy semiopen (semiclosed) set and some weakly mappings

Definition 3.1. Let (X,τ) be an **IFTS** and $A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle : x \in X\}$ be an **IFS** in X. Then A is called:

(1) fuzzy semiopen set (**IFSOS** for short) if and only if there exists a $B \in \tau$ such that $B \le A \le cl(B)$;
(2) fuzzy semiclosed set (**IFSCS** for short) if and only if there exists a $B' \in \tau$ such that $int(B) \le A \le B$;
(3) $sint(A) = \bigvee\{B : B$ is an **IFSOS** and $B \le A\}$;
(4) $scl(A) = \bigwedge\{C : C$ is an **IFSCS** and $A \le C\}$.

It can be also shown that $scl(A)$ is an **IFSCS** and $sint(A)$ is an **IFSOS** in X

Corollary 3.2. *Let (X,τ) be an* **IFTS** *and $A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle : x \in X\}$ be an* **IFS** *in X. Then A is an* **IFSOS** *if and only if $A \le cl(int(A))$.*

Proof. Let A be an **IFSOS**. Then there exists a $B \in \tau$ such that $B \le A \le clB$, by Definition 2.7 and Proposition 2.8 follows that $B \le A \le clB = cl(int(B)) \le cl(int(A)))$, i.e, $A \le cl(int(A))$.

Conversely, let $A \le cl(int(A))$. Then $int(A) \le A \le cl(int(A))$, let $B = int(A)$, thus there exists a $B \in \tau$ such that $B \le A \le cl(B)$. Hence A is an **IFSOS**.

Remark 3.3. From Definition 3.1, we can know that **IFOS** (**IFCS**) is **IFSOS** (**IFSCS**), but the inverses is false is shown by the following Example 3.4.

Example 3.4. Let $X = \{a,b\}$ and $A = \{\langle x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.3}, \frac{b}{0.5})\rangle : x \in X\}$.

Then the family $\tau = \{\underline{0}, \underline{1}, A\}$ of **IFSs** in X is an **IFT** on X. Let

$$C = \{\langle x, (\frac{a}{0.2}, \frac{b}{0.4}), (\frac{a}{0.6}, \frac{b}{0.6})\rangle : x \in X\}.$$

Then C is not an **IFOS**, but

$$cl(int(C)) = \{\langle x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4})\rangle : x \in X\},$$

hence $C \le cl(int(C))$, i.e, C is an **IFSOS**.

By Definition 3.1, we have the following theorem.

Theorem 3.5. *Let (X,τ) be an* **IFTS** *and $A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle : x \in X\}$ be an* **IFS** *in X. Then*

(1) *A is an* **IFSOS** *if and only if $A = sint(A)$;*
(2) *A is an* **IFSCS** *if and only if $A = scl(A)$;*
(3) $\underline{0} = scl(\underline{0})$, $\underline{1} = sint(\underline{1})$;
(4) $sint(A) = (scl(A'))'$;
(5) $scl(scl(A)) = scl(A)$, $sint(sint(A)) = sint(A)$;
(6) $scl(cl(A)) = cl(A)$, $sint(int(A)) = int(A)$;

(7) *If A is an* **IFSOS** (**IFSCS**), *then* $int(sint(A)) = int(A)$ $(cl(scl(A)) = cl(A))$.

Using **IFOS**, **IFCS**, **IFSOS** and **IFSCS** we can obtain the following definitions:

Definition 3.6. Let (X,τ), (Y,σ) be two **IFTSs** and let $f : X \to Y$ be a mapping. Then f is said to be fuzzy semicontinuous if and only if the preimage of each **IFS** in σ is an **IFSOS** in X.

Definition 3.7. Let (X,τ), (Y,σ) be two **IFTSs** and let $f : X \to Y$ be a mapping. Then f is said to be fuzzy open (closed) if and only if the image of each **IFS** in $\tau(\tau^{'})$ is an **IFS** in $\sigma(\sigma^{'})$.

Definition 3.8. Let (X,τ), (Y,σ) be two **IFTSs** and let $f : X \to Y$ be a mapping. Then f is said to be fuzzy semiopen (semiclosed) if and only if the image of each **IFS** in τ is an **IFSOS**(**IFSCS**) in Y.

Definition 3.9. Let (X,τ), (Y,σ) be two **IFTSs** and let $f : X \to Y$ be a mapping. Then f is said to be fuzzy irresolute if and only if the preimage of each **IFSOS** in Y is an **IFSOS** in X.

Definition 3.10. Let (X,τ), (Y,σ) be two **IFTSs** and let $f : X \to Y$ be a mapping. Then f is said to be fuzzy irresolute open (closed) if and only if the image of each **IFSOS** (**IFSCS**) in X is an **IFSOS** (**IFSCS**) in Y.

By Definition 2.9, Definitions 3.6–3.10, we can obtain following relations:

Theorem 3.11. *Let* (X,τ), (Y,σ) *be two* **IFTSs** *and let* $f : X \to Y$ *be a mapping.*

(1) *If f is fuzzy continuous, then f is fuzzy semicontinuous;*
(2) *If f is fuzzy open (closed), then f is fuzzy semiopen (semiclosed);*
(3) *If f is fuzzy irresolute, then f is fuzzy semicontinuous.*

Remark 3.12. The inverse of Theorem 3.11 is not true. This can be seen from the following examples.

Example 3.13. Let $X = \{a,b\}$, $Y = \{c,d\}$ and

$$A = \{\langle x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.6}, \frac{b}{0.5})\rangle : x \in X\},$$

$$B = \{\langle y, (\frac{c}{0.3}, \frac{d}{0.4}), (\frac{c}{0.3}, \frac{d}{0.5})\rangle : y \in Y\}.$$

Let (X,τ) and (Y,σ) be two **IFTSs**, where $\tau = \{\underline{0}, \underline{1}, A\}$ and $\sigma = \{\underline{0}, \underline{1}, B\}$. $f : (X,\tau) \to (Y,\sigma)$ defined by $f(a) = c, f(b) = d$ is not continuous, because $f^{\leftarrow}(B) = \{\langle x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.3}, \frac{b}{0.5})\rangle : x \in X\}$ is not an **IFOS** in X, but

$$cl(int(f_L^{\leftarrow}(B))) = \{\langle x, (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4})\rangle : x \in X\}.$$

Hence $f_L^{\leftarrow}(B) \le cl(int(f_L^{\leftarrow}(B)))$, i.e, $f_L^{\leftarrow}(B)$ is an **IFSOS** in X. Therefore f is semicontinuous.

Example 3.14. Let $X = \{a,b\}$, $Y = \{c,d\}$ and

$$A = \{\langle x, (\frac{a}{0.6}, \frac{b}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4})\rangle : x \in X\}, B = \{\langle y, (\frac{c}{0.2}, \frac{d}{0.4}), (\frac{c}{0.6}, \frac{d}{0.5})\rangle : y \in Y\}.$$

Let (X,τ) and (Y,σ) be two **IFTSs**, where $\tau=\{\underline{0},\underline{1},A\}$ and $\sigma=\{\underline{0},\underline{1},B\}$. $f:(X,\tau)\to(Y,\sigma)$ defined by $f(a)=c, f(b)=d$ is semicontinuous, but it is not irresolute. In fact, let $C=\{\langle y,(\frac{c}{0.5},\frac{d}{0.4}),(\frac{c}{0.5},\frac{d}{0.5})\rangle : y\in Y\}$ in Y. Then

$$cl(int(C))=\{\langle y,(\frac{c}{0.6},\frac{d}{0.5}),(\frac{c}{0.2},\frac{d}{0.4})\rangle : y\in Y\},$$

thus C is an **IFSOS**. Moreover, we know $f_L^{\leftarrow}(C)=\{\langle x,(\frac{a}{0.5},\frac{b}{0.4}),(\frac{a}{0.5},\frac{b}{0.5})\rangle : x\in X\}$, hence

$$cl(int(f_L^{\leftarrow}(C)))=\{\langle x,(\frac{a}{0.4},\frac{b}{0.4}),(\frac{a}{0.6},\frac{b}{0.5})\rangle : x\in X\},$$

i.e, $f_L^{\leftarrow}(C)\nleq cl(int(f_L^{\leftarrow}(C)))$, therefore f is not irresolute.

From definition of f, we obtain $f_L^{\rightarrow}(A)=\{\langle y,(\frac{c}{0.6},\frac{d}{0.5}),(\frac{c}{0.4},\frac{d}{0.4})\rangle : y\in Y\}$, thus $f_L^{\rightarrow}(A)\neq B$, i.e, f is not a fuzzy open mapping, but

$$cl(int(f_L^{\rightarrow}(A)))=\{\langle y,(\frac{c}{0.6},\frac{d}{0.5}),(\frac{c}{0.2},\frac{d}{0.4})\rangle : y\in Y\},$$

then $f_L^{\rightarrow}(A)\leq cl(int(f_L^{\rightarrow}(A)))$, therefore $f_L^{\rightarrow}(A)$ is a fuzzy semiopen set in Y, hence f is a semiopen mapping.

4. The properties of some weakly mappings

Theorem 4.1. *Let (X,τ), (Y,σ) be two* **IFTSs** *and let $f:X\to Y$ be a mapping. Then the following conditions are equivalent.*

(1) f is fuzzy irresolute;
(2) $f_L^{\leftarrow}(B)$ is an **IFSCS** in X for each **IFSCS** B in Y;
(3) $f_L^{\rightarrow}(scl(A))\leq scl(f_L^{\rightarrow}(A))$ for each **IFS** A in X;
(4) $scl(f_L^{\leftarrow}(B))\leq f_L^{\leftarrow}(scl(B))$ for each **IFS** B in Y;
(5) $f_L^{\leftarrow}(sint(B))\leq int(f_L^{\leftarrow}(B))$ for each **IFS** B in Y.

Proof. (1)⇒(2) is obvious.

(2)⇒(3). For any **IFS** A in X, we have $A\leq f_L^{\leftarrow}(f_L^{\rightarrow}(A))\leq f_L^{\leftarrow}(scl(f_L^{\rightarrow}(A)))$, we know that $scl(f_L^{\rightarrow}(A))$ is an **IFSCS** in Y from Definition 3.1, hence by (2), $f_L^{\leftarrow}(scl(f_L^{\rightarrow}(A)))$ is an **IFSCS** in X, thus by Definition 3.1(4) we obtain $scl(A)\leq f_L^{\leftarrow}(scl(f_L^{\rightarrow}(A)))$, therefore

$$f_L^{\rightarrow}(scl(A))\leq f_L^{\rightarrow}(f_L^{\leftarrow}(scl(f_L^{\rightarrow}(A))))\leq scl(f_L^{\rightarrow}(A)).$$

(3)⇒(4). For any **IFS** B in Y, let $f_L^{\leftarrow}(B)=A$, by (3), we have

$$f_L^{\rightarrow}(scl(f_L^{\leftarrow}(B))\leq scl(f_L^{\rightarrow}(f_L^{\leftarrow}(B)))\leq scl(B),$$

this implies

$$scl(f_L^{\leftarrow}(B)\leq f_L^{\leftarrow}(f_L^{\rightarrow}(scl(f_L^{\leftarrow}(B))))\leq f_L^{\leftarrow}(scl(B)).$$

(4)⇒(5). For any **IFS** B in Y, by $sint(B)=(scl(B'))'$ and (4), we have

$$\begin{aligned} f_L^{\leftarrow}(sint(B)) &= f_L^{\leftarrow}((scl(B'))') \\ &= (f_L^{\leftarrow}(scl(B')))' \\ &\le (scl(f_L^{\leftarrow}(B')))' \\ &= ((sint(f_L^{\leftarrow}(B)))')' \\ &= sint(f_L^{\leftarrow}(B)) \end{aligned}$$

(5)⇒(1). Let B be an **IFSOS** in Y, then $B = sint(B)$ from Theorem 3.5, by (5) we obtain $f_L^{\leftarrow}(B) \le sint(f_L^{\leftarrow}(B))$. On the other hand $f_L^{\leftarrow}(B) \ge sint(f_L^{\leftarrow}(B))$ by Definition 3.1, thus $f_L^{\leftarrow}(B) = sint(f_L^{\leftarrow}(B))$, therefore $f_L^{\leftarrow}(B)$ is an **IFSOS** in X from Theorem 3.5.

Theorem 4.2. *Let (X,τ), (Y,σ) be two* **IFTSs** *and let $f: X \to Y$ be a mapping.Then the following conditions are equivalent.*
(1) *f is fuzzy semicontinuous;*
(2) *$f_L^{\leftarrow}(B)$ is an* **IFSCS** *in X for each* **IFCS** *B in Y;*
(3) *$scl(f_L^{\leftarrow}(B)) \le f_L^{\leftarrow}(cl(B))$ for each* **IFS** *B in Y;*
(4) *$f_L^{\leftarrow}(int(B)) \le sint(f_L^{\leftarrow}(B))$ for each* **IFS** *B in Y;*
(5) *$f_L^{\leftarrow}(B) \le cl(sint(f_L^{\leftarrow}(B)))$ for each* **IFOS** *B in Y.*

Proof. (1)⇒(2) is obvious.

(2)⇒(3). Let B be **IFS** in Y. Then $cl(B)$ is an **IFCS**, so by (2), $f_L^{\leftarrow}(cl(B))$ is an **IFSCS** in X. Noting that $B \le cl(B)$, we obtain $f_L^{\leftarrow}(B) \le f_L^{\leftarrow}(cl(B))$, hence $scl(f_L^{\leftarrow}(B)) \le f_L^{\leftarrow}(cl(B))$ from Definition 3.1.

(3)⇒(4). This proof is easily and therefore omitted.

(4)⇒(5). For any **IFOS** B in Y. Then $B = int(B)$, thus $f_L^{\leftarrow}(B) = f_L^{\leftarrow}(int(B)) \le sint(f_L^{\leftarrow}(B))$, hence $f_L^{\leftarrow}(B) = sint(f_L^{\leftarrow}(B))$, therefore $f_L^{\leftarrow}(B)$ is an **IFSOS** from Theorem 3.5. Thus $f_L^{\leftarrow}(B) \le cl(int(f_L^{\leftarrow}(B)))$ from Corollary 3.2.

(5)⇒(1). It follows immediately from Corollary 3.2 and therefore omitted.

Theorem 4.3. *Let (X,τ), (Y,σ) be two* **IFTSs** *and let $f: X \to Y$ be a mapping. Then the following conditions are equivalent.*
(1) *f is fuzzy irresolute open;*
(2) *$f_L^{\rightarrow}(sint(A)) \le sint(f_L^{\rightarrow}(A))$ for each* **IFS** *A in X;*
(3) *$sint(f_L^{\leftarrow}(B)) \le f_L^{\leftarrow}(sint(B))$ for each* **IFS** *B in Y;*
(4) *For any* **IFS** *A in X,* **IFS** *B in Y and let A be the* **IFSCS** *such that $f_L^{\leftarrow}(B) \le A$. Then there exists an* **IFSCS** *C in Y and $B \le C$ such that $f_L^{\leftarrow}(C) \le A$.*

Proof. (1)⇒(2). By Definition 3.1(3), we have $sint(A) \le A$, hence $f_L^{\rightarrow}(sint(A)) \le f_L^{\rightarrow}(A)$ and by Definition 3.1, we know $sint(A)$ is an **IFSOS** in X, thus $f_L^{\rightarrow}(sint(A)) \le sint(f_L^{\rightarrow}(A))$.

(2)⇒(3). Let $A = f_L^{\leftarrow}(B)$. Form (2) we have

$$f_L^{\rightarrow}(sint(f_L^{\leftarrow}(B))) \le sint(f_L^{\rightarrow}(f_L^{\leftarrow}(B))) \le sint(B),$$

this implies

$$sint(f_L^{\leftarrow}(B)) \le f_L^{\leftarrow}(f_L^{\rightarrow}(sint(f_L^{\leftarrow}(B)))) \le f_L^{\leftarrow}(sint(B)),$$

i.e,

$$sint(f_L^{\leftarrow}(B)) \leq f_L^{\leftarrow}(sint(B)).$$

(3)⇒(4). Let A be an **IFSCS** in X and B be an **IFS** in Y such that $f_L^{\leftarrow}(B) \leq A$, hence $A' \leq f_L^{\leftarrow}(B')$, we know that A' is an **IFSOS**, thus $sint(A') = A' \leq sint(f_L^{\leftarrow}(B'))$ form Proposition 2.8, therefore $A' \leq sint(f_L^{\leftarrow}(B')) \leq f_L^{\leftarrow}(sint(B'))$, this implies $A \geq (f_L^{\leftarrow}(sint(B')))' = f_L^{\leftarrow}(scl(B))$, let $C = scl(B)$, then C satisfies condition of (4).

(4)⇒(1). Let D be an **IFSOS** in X, $B = (f_L^{\rightarrow}(D))'$, $A = D'$. Then A is an **IFSCS**, hence $f_L^{\leftarrow}(B) = f_L^{\leftarrow}((f_L^{\rightarrow}(D))') = (f_L^{\leftarrow}(f_L^{\rightarrow}(D)))' \leq D' = A$, by (4), there exists an **IFSCS** C and $B \leq C$ such that $f_L^{\leftarrow}(C) \leq A = D'$, thus $D \leq (f_L^{\leftarrow}(C))'$, thus $f_L^{\rightarrow}(D) \leq f_L^{\rightarrow}(f_L^{\leftarrow}(C')) \leq C'$. On the other hand by $B \leq C$, $f_L^{\rightarrow}(D) = B' \geq C'$, hence $f_L^{\rightarrow}(D) = C'$. Since C' is an **IFSOS**, we have $f_L^{\rightarrow}(D)$ is an **IFSOS**.

Analogously, we can prove following theorems:

Theorem 4.4. *Let (X,τ), (Y,σ) be two* **IFTSs** *and let $f: X \to Y$ be a mapping. Then the following conditions are equivalent.*

(1) *f is fuzzy irresolute closed;*
(2) *$f_L^{\rightarrow}(scl(A) \geq scl(f_L^{\rightarrow}(A))$ for each* **IFS** *A in X;*
(3) *$scl(f_L^{\leftarrow}(B)) \geq f_L^{\leftarrow}(scl(A))$ for each* **IFS** *B in Y;*
(4) *For any* **IFS** *A in X and* **IFS** *B in Y, let A be the* **IFSOS** *such that $f_L^{\leftarrow}(B) \leq A$. Then there exists an* **IFSOS** *C in Y and $B \leq C$ such that $f_L^{\leftarrow}(C) \leq A$.*

Theorem 4.5. *Let (X,τ), (Y,σ) be two* **IFTSs** *and let $f: X \to Y$ be a mapping. Then the following conditions are equivalent.*

(1) *f is fuzzy semiopen;*
(2) *$f_L^{\rightarrow}(int(A) \leq sint(f_L^{\rightarrow}(A))$ for each* **IFS** *A in X;*
(3) *$int(f_L^{\leftarrow}(B)) \leq f_L^{\leftarrow}(sint(B))$ for each* **IFS** *B in Y.*

Theorem 4.6. *Let (X,τ) and (Y,σ) be two* **IFTSs** *and let $f: X \to Y$ be a mapping. Then the following conditions are equivalent.*

(1) *f is fuzzy semiclosed;*
(2) *$int(cl(f_L^{\rightarrow}(A))) \geq f_L^{\rightarrow}(cl(A))$ for each* **IFS** *A in X;*
(3) *$scl(f_L^{\rightarrow}(A)) \leq f_L^{\rightarrow}(cl(A))$ for each* **IFS** *A in X.*

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