TAMKANG JOURNAL OF MATHEMATICS
Volume 39, Number 1, 33-43, Spring 2008

ON STRONGLY PRIME Γ-NEAR RINGS

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Abstract. In this paper we prove some equivalent conditions for strongly prime $\Gamma-$ near rings N and radicals $\mathscr{P}_s(N)\,(\mathscr{P}_e(N))$ of strongly prime (equiprime) $\Gamma-$ near ring N coincides with the $\mathscr{P}_s(L)^+\left(\mathscr{P}_e(L)^+\right)$ where $\mathscr{P}_s(L)\,(\mathscr{P}_e(L))$ is strongly prime radicals(equiprime radicals) of left operator near-ring L of N.

1. Introduction

The concept of $\Gamma-$ near ring, a generalization of both the concepts near ring and $\Gamma-$ ring was introduced by Satyanarayana [11]. Later, several authors such as Satyanarayana [10], Booth and Booth, Groenewald [2, 3, 4] studied the ideal theory of $\Gamma-$ near rings. In this paper we prove some equivalent conditions for strongly prime $\Gamma-$ near rings N and radicals $\mathscr{P}_s(N)\,(\mathscr{P}_e(N))$ of strongly prime (equiprime) $\Gamma-$ near ring N coincides with the $\mathscr{P}_s(L)^+\left(\mathscr{P}_e(L)^+\right)$ where $\mathscr{P}_s(L)\,(\mathscr{P}_e(L))$ is strongly prime radicals (equiprime radicals) of left operator near-ring L of N.

2. Preliminaries

In this section we recall certain definitions needed for our purpose.

Definition 2.1. Let N be an additive group (not necessarily abelian) and Γ be a non empty set. Then N is said to be a $\Gamma-$ near ring if there exists a mapping $N\times\Gamma\times N\to N$ (The image of (a,α,b) is denoted by $a\alpha b$) satisfying the following conditions

(i) $(a+b)\alpha c = a\alpha c + b\alpha c$

(ii) $(a\alpha b)\beta c = a\alpha\left(b\beta c\right)\ \forall a,b,c\in N$ and $\alpha,\beta\in\Gamma$.

Definition 2.2. Let N be a $\Gamma-$near ring, then a normal subgroup I of $(N,+)$ is said to be left ideal (right ideal) if $a\alpha(b+i)-a\alpha b\in I\ \ \forall a,b\in N, i\in I$ and $\alpha\in\Gamma(i\alpha a\in I\ \ \forall i\in I, a\in N$ and $\alpha\in\Gamma)$. I is said to be an ideal if it is both left and right ideal of N.

Definition 2.3. A subgroup I of $(N,+)$ is said to be left (right) $\Gamma-$ subgroup of N if $N\Gamma I\subseteq I\,(I\Gamma N\subseteq I)$. I is said to be $\Gamma-$ a subgroup if it is both left and right $\Gamma-$subgroup.

Received August 7, 2006.
2000 *Mathematics Subject Classification.* 15D25, 16Y30, 16Y99.
Key words and phrases. Strongly prime, left operator, equiprime.

Definition 2.4. A $\Gamma-$near ring N is said to be 3-prime if $a,b \in N$, $a\Gamma N\Gamma b = 0$ implies $a = 0$ or $b = 0$.

Definition 2.5. Let N be a $\Gamma-$ near ring. Let $\mathscr{L}$ be the set of all mappings of N on to itself which act on the left. Then $\mathscr{L}$ is a right near ring with operations pointwise addition and composition of mappings. Let $x \in N, \alpha \in \Gamma$, define $[x,\alpha] : N \to N$ by $[x,\alpha]\, y = x\alpha y \;\; \forall y \in N$. The sub near ring L of $\mathscr{L}$ generated by the set $\{[x,\alpha]\,/\, x \in N, \alpha \in \Gamma\}$ is called the left operator near ring of N. If $I \subseteq L$, then $I^{+} = \{x \in N/\,[x,\alpha] \in I \;\forall \alpha \in I\}$. If $J \subseteq N$, $J^{+'} = \{\ell \in L/\,\ell x \in J \;\forall x \in N\}$. It is shown in [3] that I is an ideal in L implies I^{+} is an ideal in N and J is an ideal in N implies $J^{+'}$is an ideal in L.

A right operator near ring R of N is defined analogously to the definition of L. Let $\mathscr{R}$ be the left near ring of all mappings of N in to itself which act on the right. If $\gamma \in \Gamma, y \in N$, we define $[\gamma, y] : N \to N$ by $x[\gamma, y] = x\gamma y$ for all $x \in N$. R is the sub near ring of $\mathscr{R}$ generated by the set $\{[\gamma, y]\,/\,\gamma \in \Gamma, y \in N\}$.

Definition 2.6. A $\Gamma-$ near ring N is said to be zero symmetric if $a\Gamma 0 = 0 \;\; \forall a \in N$.

Definition 2.7. An element x of a Γ-near ring N is called distributive if $x\alpha(a+b) = x\alpha a + x\alpha b$ for all $a,b \in N$ and $\alpha \in \Gamma$. If all the elements of a Γ-near ring N are distributive, then N is said to be distributive Γ-near ring.

Definition 2.8. An element m in a $\Gamma-$ near ring N is said to be left zero divisor if $m\alpha n = 0$ $\forall \alpha \in \Gamma$ implies that $n \neq 0$. An element n is said to be right zero divisor $m\alpha n = 0 \;\; \forall \alpha \in \Gamma$ implies that $m \neq 0$. An element in a $\Gamma-$ near ring is said to be zero divisor if it is both left and right zero divisor of N.

Definition 2.9. Let N be a Γ-near ring with left operator near ring L. If $\sum_i [d_i, \delta_i] \in L$ has the property that $\sum_i d_i \delta_i x = x \;\forall x \in N$, then $\sum_i [d_i, \delta_i]$ is called a left unity for N. A strong left unity for N is an element $[d,\delta]$ of L such that $d\delta x = x \;\forall x \in N$.

3. Strongly prime Γ− near ring

In this section we shall prove that some equivalent conditions for strongly prime $\Gamma-$ near rings.

Definition 3.1. Let N be a $\Gamma-$ near ring and $\alpha \in \Gamma$. Then the right $\alpha-$ annihilator of a subset A of N is $r_\alpha(A) = \{x \in N/\, A\alpha x = 0\}$.

Definition 3.2. A $\Gamma-$ near ring N is said to be strongly prime if for each $a \neq 0 \in N$, there exists a finite subset F of N such that $r_\alpha(a\Gamma F) = 0 \;\; \forall \alpha \in \Gamma$. F is called an insulator for a in N.

Definition 3.3. A $\Gamma-$ near ring N is said to be left weakly semiprime if $[x,\Gamma] \neq 0 \;\; \forall x \neq 0 \in N$.

Note that if N is a distributive $\Gamma-$ near-ring, then the elements of L are expressible in the form $\sum_i [x_i, \alpha_i]$.

Proposition 3.4. *If a $\Gamma-$ near ring N is distributive strongly prime then, the left operator near ring L is strongly prime.*

Proof. Let $\sum_i [x_i, \alpha_i] \neq 0 \in L$, then there exists $x \in N$ such that $\sum [x_i, \alpha_i]\, x \neq 0$, i.e., $\sum_i x_i \alpha_i x \neq 0$. Since N is strongly prime, there exists a finite subset $F = \{a_1, a_2, \ldots, a_n\}$ (say) such that for any $b \in N$,

$$\sum_i x_i \alpha_i x \Gamma F \Gamma b = 0 \text{ implies } b = 0 \tag{1}$$

Consider $G = \{[x\Gamma a_1, \Gamma], \ldots [x\Gamma a_n, \Gamma]\}$. Our claim is that G is an insulator for $\sum_i [x_i, \alpha_i]$. Let $\sum_j \left[y_j, \beta_j\right] \in L$ such that $\sum_i [x_i, \alpha_i]\, G \sum_j \left[y_j, \beta_j\right] = 0$. We shall prove that $\sum_j \left[y_j, \beta_j\right] = 0$. Now

$$\sum_i [x_i, \alpha_i]\, G \sum_j \left[y_j, \beta_j\right] = 0$$

implies

$$\sum_i [x_i, \alpha_i]\, [x\Gamma a_k, \Gamma] \sum_j \left[y_j, \beta_j\right] = 0\ \forall k = 1, 2, \ldots n.$$

Hence

$$\left(\sum_i [x_i, \alpha_i]\, [x\Gamma a_k, \Gamma] \sum_j \left[y_j, \beta_j\right]\right) z = 0\ \forall z \in N, k = 1, 2, \ldots n.$$

This implies that

$$\sum_i [x_i, \alpha_i]\, [x\Gamma a_k, \Gamma] \sum_j \left[y_j, \beta_j\right] z = 0\ \forall z \in N, k = 1, 2, \ldots n.$$

Hence

$$\sum_i x_i \alpha_i x \Gamma F \Gamma \sum_j y_j \beta_j z = 0\ \forall z \in N.$$

By (1), $\sum_j y_j \beta_j z = 0\ \ \forall z \in N$. Therefore $\sum_j \left[y_j, \beta_j\right] = 0$. Thus L is strongly prime.

Theorem 3.5. *Let N be a left weakly semiprime and a distributive $\Gamma-$ near ring having no zero divisor, then N is strongly prime if and only if L is strongly prime.*

Proof. Suppose that L is strongly prime. To prove N is strongly prime, let $x \neq 0 \in N$. Since N is left weakly semiprime, $[x, \Gamma] \neq 0$ and since L is strongly prime, there exists a finite subset $F = \left\{\sum_{j=1}^{n} \left[y_{j_k}, \beta_{j_k}\right] / k = 1, 2, \ldots m\right\}$ (say) such that for any $\sum_\ell [z_\ell, \delta_\ell] \in L$.

$$[x, \Gamma]\, F \sum_\ell [z_\ell, \delta_\ell] = 0 \text{ implies} \sum_\ell [z_\ell, \delta_\ell] = 0 \tag{2}$$

Consider $F' = \left\{y_{j_k} \beta_{j_k} x / j = 1, 2, \ldots, n, k = 1, 2, \ldots m\right\}$. Our claim is that F' is an insulator for x. Let $y \in N$ such that $x\Gamma F' \Gamma y = 0$. We shall prove that $y = 0$. Now $x\Gamma F' \Gamma y = 0$ implies $x\Gamma y_{j_k} \beta_{j_k} x \Gamma y = 0\ \ \forall j = 1, 2, \ldots n, k = 1, 2, \ldots m$. Therefore

$$\left[x\Gamma y_{j_k} \beta_{j_k} x \Gamma y, \Gamma\right] = 0\ \forall j = 1, 2, \ldots n, k = 1, 2, \ldots m.$$

Hence

$$[x,\Gamma]\left[y_{j_k},\beta_{j_k}\right]\left[x\Gamma y,\Gamma\right]=0\ \forall k=1,2,\ldots m$$

By (2), $\left[x\Gamma y,\Gamma\right]=0$. Since N is weakly semiprime and N has no zero divisor, $y=0$ and consequently F' is an insulator for x. Therefore N is strongly prime. Converse part is follows from Proposition 3.4.

We recall that for $X\subseteq N,\langle X\rangle$ is constructed by the following recursive rules

(i) $a\in\langle X\rangle\ \ \forall a\in X$.
(ii) If $b,c\in\langle X\rangle$, then $b+c\in\langle X\rangle$
(iii) If $b\in\langle X\rangle$, $x,y\in N$, and $\gamma\in\Gamma$, then $x\gamma\left(b+y\right)-x\gamma y\in\langle X\rangle$.
(iv) If $b\in\langle X\rangle$, $x\in N$, and $\gamma\in\Gamma$, then $b\gamma x\in\langle X\rangle$
(v) If $b\in\langle X\rangle$ and $x\in N$,then $x-b\in\langle X\rangle$
(vi) Nothing else is in $\langle X\rangle$.

Definition 3.6. Suppose $X\subseteq N$ and $d\in\langle X\rangle$. We call a sequence $s_1,\ s_2,\cdots,\ s_n$ of elements of N, a generating sequence of length m for d with respect to X. If $s_1\in X$, $s_m=d$ and for each $i=2,3,\ldots m$. One of the following applies

$$\begin{aligned}
s_i &\in X\\
s_i &= s_j+s_\ell, 1\le j,\ell<i\\
s_i &= s_j\gamma x, 1\le j<i \text{ and } x\in N,\gamma\in\Gamma\\
s_i &= x\gamma\left(s_j+y\right)-x\gamma y, 1\le j<i \text{ and } x,y\in N,\gamma\in\Gamma\\
s_i &= x+s_j-x, 1\le j<i \text{ and } x\in N.
\end{aligned}$$

The complexity of d with respect to X denoted by $C_X(d)$, is the length of a generating sequence of least length for d with respect to X.

Lemma 3.7. *Let N be a $\Gamma-$ near ring. If $X\neq 0$ and $X\Gamma N=0$, then $\langle X\rangle\Gamma N=0$.*

Proof. Let $X\Gamma N=0$ and suppose $x\in\langle X\rangle$ arbitrary. We use induction on $C_X(x)$. If $C_X(x)=1$, then $x\in X$ and from our assumption we have $X\Gamma N=0$. Suppose $y\Gamma N=0\ \ \forall y\in\langle X\rangle$ such that $C_X\left(y\right)<n$ and let $C_X(x)=n$. We have the following possibilities:

(i) $x=a+b$ where $a,b\in\langle X\rangle$ and $C_X(a),C_X(b)<n$. Hence

$$\begin{aligned}
x\Gamma N &= (a+b)\Gamma N\\
&= a\Gamma N+b\Gamma N\\
&= 0
\end{aligned}$$

(ii) $x=a\gamma n$ where $a\in\langle X\rangle$, $n\in N$ and $\gamma\in\Gamma$ and $C_X(a)<n$. Hence

$$\begin{aligned}
x\Gamma N &= \left(a\gamma n\right)\Gamma N\\
&\subseteq a\Gamma N\\
&= 0
\end{aligned}$$

(iii) $x = a\gamma(d+b) - a\gamma b$ where $d \in \langle X\rangle$, $a, b \in N$ and $\gamma \in \Gamma$ with $C_X(d) < n$. If m is arbitrary element of N, and $\delta \in \Gamma$, then

$$\begin{aligned} x\delta m &= \big(a\gamma(d+b) - a\gamma b\big)\delta m \\ &= a\gamma(d\delta m + b\delta m) - \big(a\gamma b\big)\delta m \\ &= a\gamma b\delta m - a\gamma b\delta m \\ &= 0. \end{aligned}$$

Hence $x\Gamma N = 0$.

(iv) If $x = a + b - a$ where $b \in \langle X\rangle$, $a \in N$ and $C_X(b) < n$. Let $m \in N$, $\gamma \in \Gamma$, then

$$\begin{aligned} x\gamma m &= (a+b-a)\gamma m \\ &= a\gamma m + b\gamma m - a\gamma m \\ &= 0. \end{aligned}$$

This completes the proof.

Corollary 3.8. *If every non zero ideal of a Γ- near ring N contains a subset F with $r_\alpha(F) = 0$, $\forall\alpha \in \Gamma$, then for each $a \in N$, $a \neq 0$, there is a $y \in N$ with $a\Gamma y \neq 0$.*

Proof. Let $a \neq 0 \in N$ and suppose F is a subset of $\langle a\rangle$ such that $r_\alpha(F) = 0 \;\; \forall\alpha \in \Gamma$. For every $n \neq 0 \in N$, we have $F\Gamma n \neq 0$ and therefore $\langle a\rangle \Gamma N \neq 0$. From Lemma 3.7, there exists $y \neq 0 \in N$ such that $a\Gamma y \neq 0$.

Theorem 3.9. *Let N be a $\Gamma-$ near ring, then N is strongly prime if and only if every non zero ideal of N contains a finite subset F with $r_\alpha(F) = 0$, $\forall\alpha \in \Gamma$.*

Proof. Let $I \neq 0$ be an ideal in N and $a \neq 0 \in I$. Since N is strongly prime, there exists a finite subset $F \subseteq N$ such that $r_\alpha(a\Gamma F) = 0$, $\forall\alpha \in \Gamma$. Put $F_1 = a\Gamma F$. Hence F_1 is a finite subset of I with $r_\alpha(F_1) = 0$, $\forall\alpha \in \Gamma$. Conversly, let $a \neq 0 \in N$, then $\langle a\rangle \neq 0$. From our assumption, there exists a finite subset F of $\langle a\rangle$ such that $r_\alpha(F) = 0$, $\forall\alpha \in \Gamma$. It follows from the Corollary 3.8 that there exists $y \in N$ with $a\Gamma y \neq 0$. Again we use our assumption, we can find a finite subset $G_1 = \{g_1, g_2, \dots g_n\} \subseteq \langle a\Gamma y\rangle$ with $r_\alpha(G) = 0$, $\forall\alpha \in \Gamma$. For each i, let $s_{i_1}, s_{i_2}, \dots, s_{i_{m_i}}$ be the corresponding generating sequence of g_i. Each of these sequence involve a finite number of terms of the form $a\Gamma y$ or $\big(a\Gamma y\big)\Gamma t_k$, $t_k \in N$. Let $G_1 = \{a\Gamma y, \big(a\Gamma y\big)\Gamma t_k /$ these occur in the generating sequence of an element of $G\}$. Clearly G_1 is finite and $r_\alpha(G_1) \subseteq r_\alpha(G) = 0$, $\forall\alpha \in \Gamma$. Take $H = \{x / a\Gamma x \in G_1\}$. Our claim is that H is an insulator for a. Now $r_\alpha(G_1) = 0$ implies that for any $n \in N$, $G_1\alpha n = 0$, $\forall\alpha \in \Gamma$ implies $n = 0$. Since $a\Gamma H \subseteq G_1$, we have H is an insulator for a and consequently N is strongly prime.

Proposition 3.10. *Let N be zero symmetric $\Gamma-$ near ring then the following are equivalent.*

(1) *N is strongly prime $\Gamma-$ near ring.*

(2) *Every non zero right $\Gamma-$ subgroup of N contains a finite subset F such that $r_\alpha(F) = 0$, $\forall\alpha \in \Gamma$.*

(3) *Every non zero right ideal of N contains a finite subset F such that $r_\alpha(F) = 0, \ \forall \alpha \in \Gamma$.*
(4) *Every non zero ideal of N contains a finite subset F such that $r_\alpha(F) = 0, \ \forall \alpha \in \Gamma$.*

Proof. (1) $\Longrightarrow$ (2) : Let $I \neq 0$ be a right $\Gamma-$ subgroup of N and let $a \neq 0 \in I$. Since N is strongly prime, a has an insulator F such that $r_\alpha(a\Gamma F) = 0, \ \forall \alpha \in \Gamma$. Let $G = a\Gamma F$. Then $G \subseteq I$ and $r_\alpha(G) = 0, \ \forall \alpha \in \Gamma$.

(2) $\Longrightarrow$ (3) $\Longrightarrow$ (4) is obvious.

(4) $\Longrightarrow$ (1) : It follows from Theorem 3.9.

Proposition 3.11. *Let N be a zero symmetric $\Gamma-$ near ring with DCC on right annihilators, then N is 3-prime if and only if N is strongly prime.*

Proof. Suppose N is strongly prime. To prove N is 3-prime, let $a, b \in N$ such that $a \neq 0$ and $b \neq 0$. Since N is strongly prime, there exists a finite subset F of N such that $a\Gamma F\Gamma b \neq 0$. Hence $a\Gamma N\Gamma b \neq 0$. Conversly, let $I \neq 0$ be an ideal in N and for each $\alpha \in \Gamma$, consider the collection of right $\alpha-$ annililators $\{r_\alpha(F)\}$, where F runs over all finite subset of I. From our hypothesis, there exists a minimal element $M = r_\alpha(F_0)$. If $M \neq 0$, let $m \neq 0 \in M$ and $a \neq 0 \in I$. Since N is 3-prime, there exists $n \neq 0 \in N$ such that $m\Gamma n\Gamma a \neq 0$. Hence $n\Gamma a \neq 0$. Let $S = r_\alpha(F_0 \cup \{n\Gamma a\})$ $\forall \alpha \in \Gamma$. Now $m \in M$ but $m \notin S$ implies that S is smaller than M, a contraction. This forces that $M = (0)$. Hence for every non zero ideal I of N, there exists a finite subset F such that $r_\alpha(F) = 0 \ \ \forall \alpha \in \Gamma$ and consequently N is strongly prime.

4. Radicals of strongly prime $\Gamma-$ near rings

In this section we shall prove that strongly prime radical $\mathscr{P}_s(N)$ of N coincides with $\mathscr{P}_s(L)^+$ where $\mathscr{P}_s(L)$ is the strongly prime radical of the left operator near ring L of N.

Notation 4.1. *For a Γ-near ring N, the prime radical and the set of all nilpotent elements are denoted by $\mathscr{P}_o(N)$ and $\mathscr{N}(N)$ respectively.*

Definition 4.2. An ideal I of a Γ-near ring N is said to be 2-primal if $\mathscr{P}_o\left(\frac{N}{I}\right) = \mathscr{N}\left(\frac{N}{I}\right)$.

A Γ-near ring N is called strongly 2-primal if every ideal I of N is 2-primal . If the zero ideal of N is 2-primal, then N is called 2-primal. This equivalent to $\mathscr{P}_o(N) = \mathscr{N}(N)$.

The following theorem charactersize 2-primalness for ideals in Γ-near rings. The proof is minor modification of proof of the corresponding theorem in Near-ring theory [1], and we omit it.

Theorem 4.3. *Let I be an ideal of a Γ-near ring N. Then*
(i) *I is a completely semiprime ideal if and only if I is both a semiprime and 2-primal ideal.*
(ii) *If $N\Gamma I \subseteq I$, then the following are equivalent:*
  (a) *I is completely prime ideal;*
  (b) *I is both a prime and a completely semiprime;*

(c) *I is both a prime and a 2-primal ideal.*

Lemma 4.4. *If* Γ*-near ring* N *is a strongly 2-primal, then every prime ideal of* N *is completely prime.*

Proof. It follows from Theorem 4.3.

Definition 4.5. An ideal I of a $\Gamma-$ near ring N is said to be strongly prime if for each $a \notin I$, there exists a finite subset F such that for any $b \in N$, $a\Gamma F\Gamma b \subseteq I$ implies that $b \in I$. F is called an insulator for a.

Proposition 4.6. *Let* N *be a distributive* $\Gamma-$ *near ring. If* P *is a strongly prime ideal of* N*, then* $P^{+\prime} = \{\ell \in L / \ell x \in P\ \forall x \in N\}$ *is a strongly prime ideal of* L*.*

Proof. Suppose that P is a strongly prime ideal of N. We shall prove that $P^{+\prime}$ is a strongly prime ideal of L. Let $\sum_i [x_i, \alpha_i] \notin P^{+\prime}$, then there exists $x \in N$ such that $\sum_i [x_i, \alpha_i]\, x \notin P$, that is $\sum_i x_i \alpha_i x \notin P$. Since P is strongly prime in N, there exists a finite subset $F = \{f_1, f_2, \cdots, f_n\}$ of N such that for any $b \in N$,

$$\sum_i x_i \alpha_i x \Gamma F \Gamma b \subseteq P \text{ implies } b \in P. \tag{3}$$

Consider the collection $F' = \{[x\Gamma f_1, \Gamma], \cdots, [x\Gamma f_n, \Gamma]\}$. Our claim is that F' is an insulator for $\sum_i [x_i, \alpha_i]$. Let $\sum_j [y_j, \beta_j] \in L$ such that $\sum_i [x_i, \alpha_i]\, F' \sum_j [y_j, \beta_j] \subseteq P^{+\prime}$. To prove $\sum_j [y_i, \beta_i] \in P^{+\prime}$. Now

$$\sum_i [x_i, \alpha_i]\, F' \sum_j [y_j, \beta_j] \subseteq P^{+\prime}$$

implies

$$\sum_i [x_i, \alpha_i]\, [x\Gamma f_i, \Gamma] \sum_j [y_j, \beta_j] \subseteq P^{+\prime}\ \ \forall\ i = 1, 2, \cdots, n,$$

$$\text{i.e.,}\quad \left(\sum_i [x_i, \alpha_i]\, [x\Gamma f_i, \Gamma] \sum_j [y_j, \beta_j]\right) z \subseteq P\ \ \forall\ z \in N, i = 1, 2, \cdots, n.$$

Hence

$$\sum_i x_i \alpha_i x \Gamma F \Gamma \sum_j y_j \beta_j z \subseteq P\ \ \forall\ z \in N.$$

By (3), $\sum_j y_j \beta_j z \in P\ \ \forall\ z \in N$. i.e., $\sum_j [y_j, \beta_j]\, z \in P\ \ \forall z \in N$. Hence $\sum_j [y_j, \beta_j] \in P^{+\prime}$ and therefore F' is an insulator for $\sum_i [x_i, \alpha_i] \notin P^{+\prime}$ and consequently $P^{+\prime}$ is a strongly prime ideal of L.

Proposition 4.7. *Let* N *be a distributive strongly 2-primal* Γ*- near ring with strong left unity.If* Q *is a strongly prime ideal of* L*, then* $Q^+ = \{x \in N / [x, \alpha] \in Q\ \ \forall\ \alpha \in \Gamma\}$ *is a strongly prime ideal of* N*.*

Proof. Suppose Q is a strongly prime ideal of L. We shall prove that Q^+ is a strongly prime ideal of N. Let $x \notin Q^+$, then there exists $\alpha \in \Gamma$ such that $[x,\alpha] \notin Q$.Since Q is a strongly prime ideal of L, there exists a finite subset $F = \left\{ \sum_{j=1}^{n} \left[y_{j_k}, \beta_{j_k} \right] \mid k = 1,2,\ldots,m \right\}$ (say) such that for any $\sum_{\ell} [z_\ell, \delta_\ell] \in L$,

$$[x,\alpha]\, F \sum_{\ell} [z_\ell, \delta_\ell] \subseteq Q \text{ implies that } \sum_{\ell} [z_\ell, \delta_\ell] \in Q. \tag{4}$$

Consider $F' = \left\{ y_{j_k} \beta_{j_k} x / j = 1,2,\cdots,n, k = 1,2,\cdots,m \right\}$. Our claim is that F' is an insulator for x. Let $a \in N$ such that $x\Gamma F' \Gamma a \subseteq Q^+$. To prove $a \in Q^+$. Now $x\Gamma F' \Gamma a \subseteq Q^+$ implies

$$\left[x\Gamma F' \Gamma a, \Gamma \right] \subseteq Q,$$
$$\text{i.e., } \left[x\Gamma y_{j_k} \beta_{j_k} x \Gamma a, \Gamma \right] \subseteq Q, \ \forall j = 1,2,\cdots,n, k = 1,2,\cdots,m.$$

This implies that

$$[x,\Gamma]\, F\, [x\Gamma a, \Gamma] \subseteq Q. \tag{5}$$

In particular $[x,\alpha]\, F\, [x\Gamma a, \Gamma] \subseteq Q$. By (4) $[x\Gamma a, \Gamma] \subseteq Q$. Now since Q is strongly prime in L, Q is prime in L. By Proposition 3.3 [3], Q^+ is prime ideal of N. Since N is strongly 2-primal, Q^+ is completely prime in N. Hence $x\Gamma a \in Q^+$ and $x \notin Q^+$ implies $a \in Q^+$. Thus Q^+ is strongly prime in N.

Proposition 4.8. *Let N be a distributive strongly 2-primal $\Gamma-$ near ring with strong left unity and L, a left operator near ring of N. Then $\mathscr{P}_s(N) = \mathscr{P}_s(L)^+$.*

Proof. Let P be a strongly prime ideal of L. Then by Proposition 4.7, P^+ is a strongly prime ideal of N. Moreover $\left(P^+\right)^{+'} = P$ [2, Proposition 5]. Suppose Q is strongly prime in N, then by Proposition 4.6, $Q^{+'}$ is strongly prime in L and $\left(Q^+\right)^{+'} = Q$ [2, Proposition 5]. Thus the mapping $P \to P^+$ defines a 1-1 correspondence between the set of strongly prime ideals of L and N.

Hence $\mathscr{P}_s(L)^+ = (\cap P)^+ = \cap P^+ = \mathscr{P}_s(N)$.

5. Radicals of equiprime

In this section we shall prove that equiprime radical $\mathscr{P}_e(N)$ of N coincides with $\mathscr{P}_e(L)^+$ where $\mathscr{P}_e(L)$ is the equiprime radical of left operator near ring L of N.

Definition 5.1. Let N be a Γ-near ring, and P be an ideal in N. Then P is said to be equiprime if $a, x, y \in N, a \notin P, a\alpha n\beta x - a\gamma n\delta y \in P \ \forall n \in N, \alpha, \beta, \gamma, \delta \in \Gamma$ implies $x - y \in P$.

Proposition 5.2. *Let N be a Γ-near ring. If P is an equiprime ideal of N, then $P^{+'} = \{\ell \in L / \ell x \in P \ \forall x \in N\}$ is an equiprime ideal of L.*

Proof. Let $\ell \notin P^{+\prime}$ and $\ell', \ell'' \in N$ such that $\ell' - \ell'' \notin P^{+\prime}$. From definition of $P^{+\prime}$, there exist $a, b \in N$ such that $\ell a \notin P$ and $(\ell' - \ell'') b \notin P$, that is $\ell a \notin P$ and $\ell' b - \ell'' b \notin P$. From the hypothesis, there exists $c \in N$ such that

$$\begin{aligned} &(\ell a)\alpha c\beta(\ell' b) - (\ell a)\gamma c\delta(\ell'' b) \notin P, \ \forall \alpha,\beta,\gamma,\delta \in \Gamma \\ \text{i.e., } &[\ell a,\alpha][c,\beta]\ell' b - [\ell a,\gamma][c,\delta]\ell'' b \notin P, \ \forall \alpha,\beta,\gamma,\delta \in \Gamma \\ \text{i.e., } &\ell[a,\alpha][c,\beta]\ell' b - \ell[a,\gamma][c,\delta]\ell'' b \notin P, \ \forall \alpha,\beta,\gamma,\delta \in \Gamma. \end{aligned}$$

Hence

$$\left(\ell[a\alpha c,\beta]\ell' - \ell[a\gamma c,\delta]\ell''\right) b \notin P, \ \forall \alpha,\beta,\gamma,\delta \in \Gamma.$$

This proves that

$$\ell[a\alpha c,\beta]\ell' - \ell[a\gamma c,\delta]\ell'' \notin P^{+\prime}, \ \forall \alpha,\beta,\gamma,\delta \in \Gamma$$

and consequently $P^{+\prime}$ is an equiprime ideal of L.

Proposition 5.3. *Let N be a distributive $\Gamma-$ near ring. If Q is an equiprime ideal of L, then $Q^+ = \{x \in N / [x,\alpha] \in Q \ \ \forall \alpha \in \Gamma\}$ is an equiprime ideal of N.*

Proof. Let $x \notin Q^+$ and $a, b \in N$ such that $a - b \notin Q^+$. We claim that $x\Gamma N\Gamma a - x\Gamma N\Gamma b \not\subset Q^+$. Since $x \notin Q^+$ and $a - b \notin Q^+$, then there exist $\alpha, \beta \in \Gamma$ such that $[x,\alpha] \notin Q$ and $[a-b,\beta] \notin Q$ implies that $[x,\alpha] \notin Q$ and $[a,\beta] - [b,\beta] \notin Q$. Since Q is a equiprime ideal in L, there exists $\ell = \sum_i [y_i,\beta_i] \in L$ such that $[x,\alpha]\ell[a,\beta] - [x,\alpha]\ell[b,\beta] \notin Q$. Hence $[x\alpha\ell a - x\alpha\ell b,\beta] \notin Q$. This implies that $x\alpha\ell a - x\alpha\ell b \notin Q^+$.

$$\begin{aligned} \text{i.e., } \ & x\alpha\sum_i [y_i,\beta_i]\, a - x\alpha\sum_i [y_i,\beta_i]\, b \notin Q^+ \\ \text{i.e., } \ & x\alpha\sum_i y_i\beta_i a - x\alpha\sum_i y_i\beta_i b \notin Q^+. \end{aligned}$$

But clearly

$$x\alpha\sum_i y_i\beta_i a - x\alpha\sum_i y_i\beta_i b \in x\Gamma N\Gamma a - x\Gamma N\Gamma b.$$

Thus $x\Gamma N\Gamma a - x\Gamma N\Gamma b \not\subset Q^+$ and consequently Q^+ is an equiprime ideal of N.

Theorem 5.4. *Let N be a distributive $\Gamma-$ near ring with left operator near ring L, then $\mathscr{P}_e(L)^+ = \mathscr{P}_e(N)$.*

Proof. Let P be an equiprime ideal of L. Then by Proposition 5.3, P^+ is an equiprime ideal of N. Moreover $(P^+)^{+\prime} = P$ by [2, Proposition 5]. Suppose Q is an equiprime ideal in N, then by Proposition 5.2, $Q^{+\prime}$ is an equiprime ideal in L and $\left(Q^{+\prime}\right)^+ = Q$ by [2, Proposition 5]. Thus the mapping $P \to P^+$ defines a 1-1 correspondence between the set of equiprime ideals of L and N. Hence $\mathscr{P}_e(L)^+ = (\cap P)^+ = \cap P^+ = \mathscr{P}_e(N)$.

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