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STABILITY OF CONVEX COMBINATIONS OF HURWITZ OR SCHUR STABLE MATRICES

ZIAD ZAHREDDINE

Abstract. Given that two systems of differential equations- or equivalently two matrices are Hurwitz (Schur) stable. We determine whether a convex combination of the specified matrices is also Hurwitz (Schur) stable. The procedure used, provides a unified approach to the testing of both types of stability. The property of aperiodicity of a convex combination of two matrices is also considered.

1. Introduction

Matrix stability plays an important role in the theory of differential equations, and has been intensively investigated over the last few decades. The concept of stability itself can be defined in many different ways depending on the context. Of great importance are stable (semistable) matrices-those whose eigenvalues have negative (nonpositive) real parts-. A matrix M is strongly stable (strongly semistable) if $M - S$ is stable (semistable) whenever S is a real symmetric positive semidefinite matrix. The concept of strong stability arises when diffusion models of biological systems are linearized at a constant equilibrium. M is D-stable if $D.M$ is stable for every choice of a positive diagonal matrix D. The concept of D-stability is an important concept in mathematical economics [1, 2, 8]. Relations between stability and D-stability were considered by the mathematical economists Arrow and NcManus [1] and Enthoven and Arrow [2], and by the mathematician Howland [4].

Most of the above stability problems have their origins in mathematical economics, where the stability of a linear dynamical system of the form $\frac{dX}{dt} = A.X$ is studied on the basis of qualitative data on the entries of the matrix. Related to the above concepts are the problems of Routh-Hurwitz and Schur-Cohn stability of a system of differential equations with real or complex coefficients, which arise mainly in problems on analysis and geometry. For some recent contributions on the latter types of stability, we mention [9] and [10]. In a very recent paper [11], a new and different approach to the stability problem is advanced.

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In sections 2 and 3 of the present work, we establish necessary and sufficient conditions for the stability of a convex combination of matrices which are stable in the Hurwitz or the Schur sense. These questions are important because they arise in the problem of stability of uncertain matrices where some or all of the entries are not known exactly. In section 4, we look at the property of aperiodicity which arises when obtaining a respose that has either no oscillations or only a finite number of oscillations. Some concluding remarks are given in section 5. Before giving the main results, we clarify our definitions.

Definition 1. A system of differential equations (or equivalently a matrix A) is Hurwitz-stable (HS), if all its eigenvalues lie in the left-half plane.

Definition 2. A is Schur-stable (SS), if all its eigenvalues lie inside the unit disc.

Definition 3. A is Hurwitz-aperiodic (HA) if all its eigenvalues are real and negative.

Definition 4. A is Schur-aperiodic (SA), if all its eigenvalues are real, positive and lie in the open interval (0,1).

2. Hurwitz Stability

The following theorem characterizes the Hurwitz stability of a convex combination of two symmetric matrices.

Theorem 2.1. *Suppose A_1 and A_2 are two $n \times n$ symmetric matrices, then the matrix $A_\lambda = \lambda A_1 + (1-\lambda)A_2$ is HS for all λ in [0,1] if and only if A_1 and A_2 are HS.*

Proof. The only if statement is obvious.

For the if statement, suppose both A_1 and A_2 are HS. It is known that a symmetric Hurwitz matrix is also negative definite and vica versa [7, theorem 13.1.4]. Therefore it would suffice to show that A_λ is negative definite for all λ in [0,1] if A_1 and A_2 are negative definite. But from

$$x^T A_\lambda x = (1-\lambda)x^T A_1 x + \lambda x^T A_2 x$$

we have

$$x^T A_\lambda x < 0 \text{ if } x^T A_1 x < 0 \text{ and } x^T A_2 x < 0$$

for all non-zero vectors x and that completes the proof.

3. Schur Stability

The following theorem corresponds to theorem 2.1 in the Schur stability case.

Theorem 3.1. *Given that A_1 and A_2 are symmetric matrices, then the matrix $A_\lambda = \lambda A_1 + (1-\lambda)A_2$ is SS for all λ in [0,1] if and only if A_1 and A_2 are SS.*

Proof. Again the only if statement is clear.

Suppose now that A_1 and A_2 are SS. It is well-known that the eigenvalues of a symmetric matrix are real [7]. If the eigenvalues of a matrix A are $\lambda_1, \lambda_2, \ldots, \lambda_n$, then the eigenvalues of $-I + A$ are given by $-1 + \lambda_1, -1 + \lambda_2, \ldots, -1 + \lambda_n$ [7, chap. 11]. Similarly, the eigenvalues of $-I + A$ are given by $-1 - \lambda_1, -1 - \lambda_2, \ldots, -1 - \lambda_n$. But, for any real λ_k it is clear that if $-1 + \lambda_k < 0$ and $-1 - \lambda_k < 0$, then $-1 < \lambda_k < 1$.

Therefore, a symmetric matrix A is SS if and only if $-I + A$ and $-I - A$ are HS. It follows that A_λ is SS for all λ in [0,1] if $-I + A_\lambda$ and $-I - A_\lambda$ are HS. But since

$$-I + A_\lambda = \lambda(-I + A_1) + (1 - \lambda)(-I + A_2)$$

and

$$-I - A_\lambda = \lambda(-I - A_1) + (1 - \lambda)(-I - A_2)$$

it follows from theorem 2.1 that $-I + A_\lambda$ is HS for all λ in [0,1] if and only if $-I + A_1$ and $-I + A_2$ are HS. And similarly, $-I + A_\lambda$ is HS for all λ in [0,1] if and only if $-I - A_1$ and $-I - A_2$ are HS.

But A_1 is SS if $-I - A_1$ and $-I + A_1$ are HS, and similarly A_2 is SS if $-I - A_2$ and $-I + A_2$ are HS. Therefore A_λ is SS for all λ in [0,1] if A_1 and A_2 are SS.

4. Aperiodicity

In this section, we look at the aperiodic property of a convex combination of symmetric matrices.

Theorem 4.1. *Suppose A_1 and A_2 are symmetric matrices, then the matrix $A_\lambda = \lambda A_1 + (1 - \lambda)A_2$ is HA for all λ in* [0,1] *if and only if A_1 and A_2 are HA.*

Proof. Since in case of symmetric matrices, Hurwitz stability is equivalent to Hurwitz aperiodicity, the proof follows from theorem 2.1.

Theorem 4.2. *If A_1 and A_2 are symmetric matrices, then $A_\lambda = \lambda A_1 + (1 - \lambda)A_2$ is SA for all λ in* [0,1] *if and only if A_1 and A_2 are SA.*

Proof. The only if statement is clear. For the if, we note that if

$$-1 + \lambda_k < 0 \text{ and } \lambda_k > 0,$$

then

$$0 < \lambda_k < 1.$$

We then follow the same steps as those in the proof of theorem 3.1.

5. Concluding Remarks

The above results offer a matrix approach to the stability problem. They also provide more insight into the interrelations between Hurwitz or Schur stability on one hand and positive or negative definite matrices on the other. Such interrelations have long been echoed by many researchers, including the legendaries Gantmacher [3], Hurwitz [5], Krein and Naimark [6] as well as Howland [4] in their works on Hermite's symmetric-matrix form of Hurwitz test, Lyapunov's second method and Markov's stability criterion. Also as mentioned above, such results are required in a variety of problems including stability studies on sets of polynomials or matrices whose coefficients are subject to perturbations.

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Department of Mathematics, Faculty of Sciences, U. A. E. University, Al-Ain, P. O. Box 17751., United Arab Emirates

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EXTREMAL PROPERTIES RELATED TO OPERATOR RADIUS

JOR-TING CHAN

Abstract. For a complex square matrix A, let $r(A)$ and $\|A\|$ denote respectively the spectral radius and the spectral norm of A. Also let $w_\rho(A)$ denote the operator radius of A. In general we have

(1) $w_\rho(A) \geq w_\rho(I_n)r(A)$, and
(2) $\rho^{-1}\|A\| \leq w_\rho(A) \leq w_\rho(I_n)\|A\|$.

In this note we study conditions on A under which there is equality in (1) or (2). The results include structural characterizations for spectral and radial matrices.

1. Introduction

Let M_n denote the vector space of all $n \times n$ complex matrices and let $\rho > 0$. A matrix $A \in M_n$ is called a ρ-contraction if there is a Hilbert space H containing $\mathbb{C}^n$, and a unitary operator U on H such that

$$A^k = \rho \cdot \operatorname{pr} U^k \Big|_{\mathbb{C}^n} \quad \text{for} \quad k = 1, 2, \ldots,$$

where pr denotes the orthogonal projection onto $\mathbb{C}^n$.

For $A \in M^n$, Holbrook [3] defined the operator radius of A by

$$w_\rho(A) = \inf\{\lambda > 0 : \frac{1}{\lambda}A \text{ is a } \rho\text{-contraction}\}.$$

This class contains the spectral norm, $\|\cdot\|$, and the numerical radius, $w(\cdot)$: they are operator radii corresponding to $\rho = 1$ and 2 respectively. The spectral radius, $r(\cdot)$, is the limit of $w_\rho(\cdot)$ as $\rho \to \infty$.

There are many important properties of operator radii. For example, we have $w_\rho(\cdot)$ is invariant under unitary similarities, and if $A = A_1 \oplus A_2$, then $w_\rho(A) = \max\{w_\rho(A_1), w_\rho(A_2)\}$. For a fixed matrix A, the operator radii $w_\rho(A)$ vary with ρ in a nice way: the map $\rho \mapsto w_\rho(A)$ is convex and decreasing on $(0, \infty)$, whereas the map $\rho \mapsto \rho w_\rho(A)$ is increasing on $(1, \infty)$ and decreasing on $(0, 1)$. An account of these facts can be found in Sz.-Nagy and Foiaş [9] and Ando and Nishio [1].

Let I_n denote the $n \times n$ identity matrix. Then $w_\rho(I_n) = \rho^{-1}(2 - \rho)$ if $0 < \rho < 1$, and 1 otherwise ([2, Theorem 1]). For every A in M_n, we have the following relations ([3, Theorem 5.4, Theorem 3.1 and Theorem 4.2]):

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(1) $w_\rho(A) \geq w_\rho(I_n)r(A)$, and
(2) $\rho^{-1}\|A\| \leq w_\rho(A) \leq w_\rho(I_n)\|A\|$.

It will be of interest to study conditions under which the above inequalities become equalities. There are known results for $\rho = 1$ or 2. A matrix A for which $w(A) = r(A)$ is called a spectral matrix. If $w(A) = \|A\|$, or equivalently, $r(A) = \|A\|$, then A is a radial matrix. Their properties are given in Goldberg, Tadmor and Zwas [6] and Goldberg and Zwas [7]. One may also consult Horn and Johnson [4, Chapter 1] for a summary and extensions. On the other hand, a characterization of A for which $2w(A) = \|A\|$ can be found in Williams and Crimmins [10] and Johnson and Li[5]. We shall tackle the problems for a general ρ in the following sections.

2. The Case $w_\rho(A) = w_\rho(I_n)r(A)$

Suppose that $A \in M_n$ is spectral, i.e. $w(A) = r(A)$. Then by [4, p.61] A is unitarily similar to a matrix of the form $\lambda_1 I_{n_1} \oplus \cdots \oplus \lambda_k I_{n_k} \oplus B$, where $|\lambda_1| = \cdots = |\lambda_k| = r(A)$, $r(B) < r(A)$ and $w(B) \leq r(A)$. In general we have

Theorem 2.1. *Let $\rho > 0$ and $A \in M_n$. Then $w_\rho(A) = w_\rho(I_n)r(A)$ if and only if A is unitarily similar to a direct sum of the form $\lambda_1 I_{n_1} \oplus \cdots \oplus \lambda_k I_{n_k} \oplus B$, where $|\lambda_1| = \cdots = |\lambda_k| = r(A)$, $r(B) < r(A)$ and $w_\rho(B) \leq w_\rho(I_n)r(A)$.*

Proof. We shall only prove that if $w_\rho(A) = w_\rho(I_n)r(A)$, then A is unitarily similar to a matrix described above. The converse is obvious.

If $1 \leq \rho \leq 2$, then the condition $w_\rho(A) = w_\rho(I_n)r(A)$ becomes $w_\rho(A) = r(A)$, then as $w_\rho(A) \geq w_2(A) = w(A) \geq r(A)$, we have $w(A) = r(A)$. By the above mentioned characterization of a spectral matrix, A is unitarily similar to a matrix of the form $\lambda_1 I_{n_1} \oplus \cdots \oplus \lambda_k I_{n_k} \oplus B$, where $|\lambda_1| = \cdots = |\lambda_k| = r(A)$, $r(B) < r(A)$. Then $w_\rho(I_n)r(A) = w_\rho(A) \geq w_\rho(B)$. So A is of the desired form. Now for $0 < \rho < 2$, we have $\rho w_\rho(A) = (2-\rho)w_{2-\rho}(A)$ ([1, Theorem 3]). Hence if $w_\rho(A) = w_\rho(I_n)r(A)$ for some $0 < \rho < 1$, then $w_{2-\rho}(A) = r(A)$ for $1 < 2-\rho < 2$. By what we have proved, A is unitarily similar to a matrix of the form $\lambda_1 I_{n_1} \oplus \cdots \oplus \lambda_k I_{n_k} \oplus B$, where $|\lambda_1| = \cdots = |\lambda_k| = r(A)$, $r(B) < r(A)$ and $w_{2-\rho}(B) \leq w_{2-\rho}(I_n)r(A)$. The latter inequality can be rewritten as $\rho(2-\rho)^{-1}w_\rho(B) \leq r(A)$, i.e. $w_\rho(B) \leq w_\rho(I_n)r(A)$, as desired. We are left with the case $\rho > 2$.

Now assume that $\rho > 2$ and consider a 2×2 matrix $A_1 = \begin{pmatrix} 1 & 0 \\ a & \lambda \end{pmatrix}$. We claim that if $w_\rho(A_1) = 1$, then $a = 0$. This will be done by appealing to [3, Theorem, 2.2]:

For any $\rho > 0$, $w_\rho(A) \leq 1$ if and only if

$$\mathrm{Re}\langle (I_n - zA)x, x\rangle \geq (1 - \frac{\rho}{2})\|I_n - zA)x\|^2$$

for every $x \in \mathbb{C}^n$ and $z \in \mathbb{C}$ with $|z| \leq 1$.

Suppose $a \neq 0$. We choose an $\varepsilon > 0$ such that $(1-\frac{\rho}{2})\varepsilon > -1$. Then for $x = (u,1)^T \in \mathbb{C}^2$, where $u = \frac{1-\lambda+\varepsilon}{a}$, we have

$$\langle (I_2 - A_1)x, x\rangle = -\varepsilon < (1-\frac{\rho}{2})\varepsilon^2 = (1-\frac{\rho}{2})\|(I_2 - A_1)x\|^2.$$

Hence $w_\rho(A_1) > 1$.

In general we may assume that A is a lower triangular matrix of the form

$$\begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ a_{21} & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & \lambda_n \end{pmatrix},$$

where the eigenvalues λ_i are arranged in the order $|\lambda_1| = \cdots = |\lambda_h| > \cdots \geq |\lambda_n|$. We have to show that $a_{21} = \cdots = a_{n1} = 0, \ldots, a_{h+1,h} = \cdots = a_{nh} = 0$. Replacing A by $\frac{1}{\lambda_1}A$, we may further assume that $\lambda_1 = 1$. By the very definition, every leading principal submatrix

$$B_m = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ a_{21} & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & \lambda_m \end{pmatrix}$$

has operator radius less than or equal to one. Our previous calculation, applied to B_2, yields $a_{21} = 0$. Suppose that $a_{21} = \cdots = a_{m1} = 0 (2 \leq m < n)$ while $a_{m+1,1} \neq 0$. We consider B_{m+1}. Take an $\varepsilon > 0$ as in the 2×2 case and let $x = (u, 0, \ldots, 0, 1)^T \in \mathbb{C}^{m+1}$, where $u = \frac{1-\lambda_{m+1}+\varepsilon}{a_{m+1,1}}$. We obtain as in above

$$\langle (I_2 - B_{m+1})x, x\rangle < (1-\frac{\rho}{2})\left\|(I_2 - B_{m+1})x\right\|^2,$$

which is a contradiction. Hence $a_{21} = \cdots = a_{n1} = 0$, and $A = \lambda_1 I_1 \oplus A_1$ for an $(n-1)\times(n-1)$ matrix A_1. Provided $|\lambda_2| = |\lambda_1|$, $w_\rho(A_1) = w_\rho(I_{n-1})r(A_1)$. The proof is concluded by induction.

3. The Case $w_\rho(A) = \rho^{-1}\|A\|$ And $w_\rho(A) = w_\rho(I_n)\|A\|$

We first study the simpler case $w_\rho(A) = w_\rho(I_n)\|A\|$. Recall that if A is radial, or $w(A) = \|A\|$, then A is unitarily similar to a matrix of the form $\lambda_1 I_{n1} \oplus \cdots \oplus \lambda_k I_{n_k} \oplus B$, where $|\lambda_1| = \cdots = |\lambda_k| = r(A)$, $r(B) < r(A)$ and $\|B\| \leq r(A)$ ([4, p.45]). Now if $\rho > 1$ and $w_\rho(A) = \|A\|$, then as the map $\rho' \mapsto w_{\rho'}(A)$ is convex and monotonic decreasing, $w_{\rho'}(A) = \|A\|$ for every $\rho' \geq 1$. In particular, A is radial. If $w_\rho(A) = w_\rho(I_n)\|A\|$ for some $0 < \rho < 1$, then $w_\rho(A) = \rho^{-1}(2-\rho)\|A\|$ and $w_{2-\rho}(A) = \frac{\rho}{2-\rho}w_\rho(A) = \|A\|$ and again A is also radial. Tracing the above argument backward, we see that if A is radial, then $w_\rho(A) = w_\rho(I_n)\|A\|$ for all $\rho > 0$. We therefore have

Theorem 3.1. *Let $\rho > 0$, $\rho \neq 1$ and $A \in M_n$. Then $w_\rho(A) = w_\rho(I_n)\|A\|$ if and only if A is a radial matrix.*

We now turn to the inequality $w_\rho(A) \geq \rho^{-1}\|A\|$. The following knowledge from Williams and Crimmins [10] is useful:

Suppose $2w(A) = \|A\| \neq 0$ and x a unit vector in $\mathbb{C}^n$ such that $\|Ax\| = \|A\|$. Then $x \perp Ax$, $A^2x = 0$ and $\operatorname{span}\{x, Ax\}$ is a reducing subspace of A.

It follows then by induction that A is unitarily similar to a matrix of the form $E \oplus \cdots \oplus E \oplus B$, where E is the 2×2 matrix $\begin{pmatrix} 0 & 0 \\ \|A\| & 0 \end{pmatrix}$, $\|B\| < \|A\|$ and $w(B) \leq w(A)$.

Theorem 3.2. *Let $\rho > 0$, $\rho \neq 1$ and $A \in M_n$. Then $w_\rho(A) = \rho^{-1}\|A\|$ if and only if A is unitarily similar to a matrix of the form $E \oplus \cdots \oplus E \oplus B$, where E is the 2×2 matrix $\begin{pmatrix} 0 & 0 \\ \|A\| & 0 \end{pmatrix}$, $\|B\| < \|A\|$ and $w_\rho(B) \leq w_\rho(A)$.*

Proof. Again we need only prove that if $w_\rho(A) = \rho^{-1}\|A\|$, then A is of the above form. There are some trivial cases. If $\rho \geq 2$ and $\rho w_\rho(A) = \|A\|$, then as $\|A\| \leq 2w(A) \leq \rho w_\rho(A)$, we have $2w(A) = \|A\|$ and the theorem follows easily. Since $\rho w_\rho(A) = (2-\rho)w_{2-\rho}(A)$ for $0 < \rho < 2$, we need only consider the situation $1 < \rho < 2$. Let C_ρ be the 2×2 matrix $\begin{pmatrix} 0 & \sqrt{\rho(2-\rho)} \\ 0 & 1-\rho \end{pmatrix}$. By [8, Theorem 3], $\rho w_\rho(A) = 2w(C_\rho \oplus A)$. Clearly $\|C_\rho \oplus A\| = \|A\|$. Take a unit vector $x \in \mathbb{C}^n$ such that $\|Ax\| = \|A\|$ and let $\tilde{x}$ be the vector $(0, \ldots, 0)^T \oplus x$ in $\mathbb{C}^{2n}$. Then $\|\tilde{x}\| = 1$ and $C_\rho \oplus A$ attains its norm at $\tilde{x}$. Since $2w(C_\rho \otimes A) = \|C_\rho \otimes A\|$, we therefore have $x \perp (C_\rho \otimes A)\tilde{x}$, $(C_\rho \otimes A)^2\tilde{x} = 0$ and $\operatorname{span}\{\tilde{x}, (C_\rho \otimes A)\tilde{x}\}$ is a reducing subspace of $C_\rho \otimes A$. By a direct verification, we have $x \perp Ax$, $A^2x = 0$ and that x and Ax span a reducing subspace of A. Hence A is unitarily similar to $E \oplus A_1$ for an $(n-2) \times (n-2)$ matrix A_1. The theorem follows by induction.

It is proved in [3, Theorm 4.5] that if $A^2 = 0$, then $w_\rho(A) = \rho^{-1}\|A\|$. Theorem 3.2 provides a weaker form of the converse: it shows that if $w_\rho(A) = \rho^{-1}\|A\|$, then A consists of a direct summand E, where $E^2 = 0$. Another observation is that unlike the situation in Theorem 3.1, there is an A for which $w_\rho(A) = \rho^{-1}\|A\|$ for a $\rho \neq 1$ but $w'_\rho(A) \neq (\rho')^{-1}\|A\|$ for some other ρ'. A simple example is given by $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \oplus (\frac{1}{3})$. Then $\|A\| = 1$, $w_\rho(A) = \rho^{-1}$ for $\rho \leq 3$ and $w_\rho(A) = \frac{1}{3}$ otherwise.

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Department of Mathematics, University of Hong Kong, Hong Kong