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RINGS WITH A JORDAN DERIVATION WHOSE IMAGE IS CONTAINED IN THE NUCLEI OR COMMUTATIVE CENTER

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Abstract Let R be a nonassociative ring, N, L and G the left nucleus, right nucleus and nucleus respectively. It is shown that if R is a prime ring with a Jordan derivation d such that $d(R) \subseteq G$, and $(d^2(R), R) \subseteq N$ or $(d^2(R), R) \subseteq L$ then either R is associative or $2d^2 = 0$. Moreover, if $(d(R), R) = 0$ then either R is associative and commutative, or $2d = 0$. We also prove that if R is a prime ring with a derivation d and there exists a fixed positive integer n such that $d^n(R) \subseteq G$ and $(d^n(R), R) = 0$ then R is associative and $d^n = 0$, or R is associative and commutative, or $d^{2n} = (\frac{(2n)!}{n!})d^n = 0$. This partially generalize the results of [3]. We also obtain some results on prime rings with a derivation satisfying other hypotheses.

1. Introduction

Let R be a nonassociative ring. We adopt the usual notations for associators and commutators : $(x, y, z) = (xy)z - x(yz)$ and $(x, y) = xy - yx$. We shall denote the left nucleus, middle nucleus, right nucleus and nucleus by N, M, L and G respectively. Thus N, M, L and G consists of all elements n such that $(n, R, R) = 0$, $(R, n, R) = 0$, $(R, R, n) = 0$ and $(n, R, R) = (R, n, R) = (R, R, n) = 0$ respectively. An additive mapping d from R to R is called a Jordan derivation if $d(x^2) = d(x)x + xd(x)$ holds for all x in R. An additive mapping d from R to R is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all x, y in R. Obviously, every derivation is a Jordan derivation. The converse is in general not true. R is called semiprime if the only ideal of R which squares to zero is the zero ideal. R is called prime if the product of any two nonzero ideals of R is nonzero. Clearly, a prime ring is a semiprime ring. Herstein [1] proved that every Jordan derivation on a prime associative ring of characteristic not two is a derivation. Recently, Yen [3] showed the

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Theorem 1. *If R is a prime ring with a derivation d such that $d(R) \subseteq N \cap L$ or $d(R) \subseteq N \cap M$ or $d(R) \subseteq M \cap L$, then either R is associative or $d^2 = 2d = 0$.*

In Theorem 1, if the derivation is replaced by the Jordan derivation then what can we say about R and d? In fact, we prove that if R is a prime ring with a Jordan derivation d such that $d(R) \subseteq G$, and $(d^2(R), R) \subseteq N$ or $(d^2(R), R) \subseteq L$ then either R is associative or $2d^2 = 0$. Moreover, if $(d(R), R) = 0$ then either R is associative and commutative, or $2d = 0$. We also prove that if R is a prime ring with a derivation d and there exists a fixed positive integer n such that $d^n(R) \subseteq G$ and $(d^n(R), R) = 0$ then R is associative and $d^n = 0$, or R is associative and commutative, or $d^{2n} = (\frac{(2n)!}{n!})d^n = 0$. This partially generalize Theorem 1. We also obtain some results on prime rings with a derivation satisfying some hypotheses.

Assume that R has a Jordan derivation d. Thus we have

$$d(x^2) = d(x)x + xd(x) \quad \text{for all} \quad x \text{ in } R. \tag{1}$$

2. Results

Let R be a nonassociative ring. In every ring one may verify the Teichmüller identity

$$(wx, y, z) - (w, xy, z) + (w, x, yz) = w(x, y, z) + (w, x, y)z. \tag{2}$$

Suppose that $n \in N$. Then with $w = n$ in (2) we obtain

$$(nx, y, z) = n(x, y, z) \text{ for all } n \text{ in } N. \tag{3}$$

Assume that $m \in L$. Then with $z = m$ in (2) we get

$$(w, x, ym) = (w, x, y)m \quad \text{for all} \quad m \text{ in } L. \tag{4}$$

As consequences of (2), (3) and (4), we have that $N, M, L, N \cap M, M \cap L, N \cap L$ and G are associative subrings of R.

We assume that R has a Jordan derivation d which satisfies

$$(*) \qquad d(R) \subseteq A, \text{ where } A = N, \text{ or } M, \text{ or } L.$$

Using $(*)$ and a linearization of (1) gives

$$d(xy + yx) = d(x)y + xd(y) + d(y)x + yd(x) \in A \text{ for all } x, y, \text{ in } R. \tag{5}$$

Then with $x \in d(R)$ in (5), and using $(*)$ and noting that A is an associative subring of R, we get

$$d^2(x)y + yd^2(x) \in A \text{ for all } \; x, y \text{ in } R. \tag{6}$$

Definition. Let I be the associator ideal of R. I consists of the smallest ideal which contains all associators.

Note that I may be characterized as all finite sums of associators and right (or left) multiples of associators, as a consequence of (2). Hence we have

$$I = (R, R, R) + (R, R, R)R = (R, R, R) + R(R, R, R). \tag{7}$$

We assume that R satisfies

$$(**) \qquad (d^2(R), R) \subseteq B, \text{ where } B = N, \text{ or } L.$$

Note that if B is a Lie ideal of R i.e., $(B, R) \subseteq B$, and if $d^2(R) \subseteq B$, then we obtain $(d^2(R), R) \subseteq (B, R) \subseteq B$. Assume that $A = B = N$ or L. Using $(**)$, we get $(d^2(x), y) \in A$ for all x, y in R. Combining this with (6) yields $2d^2(x)y \in A$ and $2yd^2(x) \in A$. Thus we have

$$2d^2(R)R \subseteq A \text{ and } 2Rd^2(R) \subseteq A \text{ if } A = B = N \text{ or } L. \tag{8}$$

Applying (8), $(*)$, (3) and (4), and with $n \in 2d^2(R)$ in (3), and with $m \in 2d^2(R)$ in (4) respectively, we obtain

$$2d^2(R)(R, R, R) = 0 \text{ if } A = B = N \text{ and } 2(R, R, R)d^2(R) = 0 \text{ if } A = B = L. \tag{9}$$

Combining (9) with $(*)$ yields

$$2d^2(R)((R, R, R)R) = 0 \text{ if } A = B = N \text{ and } 2(R(R, R, R))d^2(R) = 0 \text{ if } A = B = L. \tag{10}$$

Using (7), (9) and (10) , we get

$$2d^2(R) \cdot I = 0 \text{ if } A = B = N \text{ and } I \cdot 2d^2(R) = 0 \text{ if } A = B = L. \tag{11}$$

Lemma 1. *If R is a ring with a Jordan derivation d such that $d(R) \subseteq N \cap M$ and $(d^2(R), R) \subseteq N$ (resp. $d(R) \subseteq M \cap L$ and $(d^2(R), R) \subseteq L$), then $2d^2(R)R \subseteq N \cap M$ and $2Rd^2(R) \subseteq N \cap M$ (resp. $2d^2(R)R \subseteq M \cap L$ and $2Rd^2(R) \subseteq M \cap L$).*

Proof. By symmetry, we only prove the lemma in case $d(R) \subseteq N \cap M$ and $(d^2(R), R) \subseteq N$. By (8), $2d^2(R)R \subseteq N$. Using these, (6) and (2), for all $x, y, z, w \in R$ we obtain $0 = 2(d^2(x)y, z, w) = -2(yd^2(x), z, w) = -2(y, d^2(x)z, w)$. Thus $(y, 2d^2(x)z, w) = 0$. Hence $2d^2(R)R \subseteq M$. By (6) , $2d^2(x)y + 2yd^2(x) \in N \cap M$. Since $2d^2(x)y \in M$, this implies $2yd^2(x) \in M$. By (8), $2yd^2(x) \in N$, as desired.

Lemma 2. *If R is a ring with a Jordan derivation d such that $d(R) \subseteq G$, and $(d^2(R), R) \subseteq N$ or $(d^2(R), R) \subset L$, then the ideal C of R generated by $2d^2(R)$ is $C = 2\{d^2(R) + d^2(R)R + Rd^2(R) + R \cdot d^2(R)R\}$.*

Proof. By symmetry, we only prove the lemma in case $d(R) \subseteq G$ and $(d^2(R), R) \subseteq N$. Then by Lemma 1, we have $2d^2(R)R \subseteq N \cap M$ and $2Rd^2(R) \subseteq N \cap M$. We see that C is an additive subgroup of $(R, +)$. Thus we have

$$2(R \cdot d^2(R)R)R = 2R(d^2(R)R \cdot R) = 2R \cdot d^2(R)(R^2) \subseteq 2R \cdot d^2(R)R$$

and

$$\begin{aligned} 2R(R \cdot d^2(R)R) &= 2R(Rd^2(R) \cdot R) = 2(R \cdot Rd^2(R))R \\ &= 2(R^2)d^2(R) \cdot R \subseteq 2Rd^2(R) \cdot R = 2R \cdot d^2(R)R. \end{aligned}$$

Hence C is an ideal of R.

Theorem 2. *If R is a prime ring with a Jordan derivation d such that $d(R) \subseteq G$, and $(d^2(R), R) \subseteq N$ or $(d^2(R), R) \subseteq L$ then either R is associative or $2d^2(x) = 4d(x)d(x) = 4d(x)d(y) + 4d(y)d(x) = 0$ for all x, y in R.*

Proof. By symmetry, we only prove the theorem in case $d(R) \subseteq G$ and $(d^2(R), R) \subseteq N$. By (11), $2d^2(R) \cdot I = 0$. Using Lemma 1, we have $2d^2(R)R \subseteq N \cap M$ and $2Rd^2(R) \subseteq N \cap M$. Applying these and $d(R) \subseteq G$, we obtain that $2\{d^2(R) + d^2(R)R + Rd^2(R) + R \cdot d^2(R)R\} \cdot I = 0$. If $I = 0$, then R is associative. Assume that $I \neq 0$.

Thus by Lemma 2 and the primeness of R, $C \cdot I = 0$ implies $C = 0$. Hence $2d^2(R) = 0$. Assume that $x, y \in R$. Using this, (1) and (5), we get

$$0 = 2d^2(x^2) = 2d(d(x)x + xd(x)) = 2d^2(x)x + 4d(x)d(x) + 2xd^2(x) = 4d(x)d(x).$$

A linearization of this gives $4d(x)d(y) + 4d(y)d(x) = 0$, as desired.

Lemma 3. *If R is a ring and $(n, R) = 0$, and $n \in N \cap M$ or $n \in M \cap L$ or $n \in N \cap L$ then $n \in G$.*

Proof. We only prove the lemma in case $(n, R) = 0$, and $n \in N \cap M$. Then for all x, y in R, we have

$$(xy)n = n(xy) = (nx)y = (xn)y = x(ny) = x(yn),$$

as desired.

Theorem 3. *If R is a prime ring with a Jordan derivation d such that $(d(R), R) = 0$, and $d(R) \subseteq N \cap M$ or $d(R) \subseteq M \cap L$ or $d(R) \subseteq N \cap L$, then either R is associative and commutative, or $2d = 0$.*

Proof. By Lemma 3, $d(R) \subseteq G$. Assume that R is associative. Since $(d(R), R) = 0$, for all x, y in R we have $0 = (d(x^2), y) = 2(d(x)x, y) = 2d(x)(x, y)$. Thus $2d(x)R(x, y) = 0$. By the primeness of R, this implies $2d(x) = 0$ or $(x, y) = 0$ for all x, y in R. It is well known that the additive group $(R, +)$ can not be the union of two proper subgroups. Hence we obtain $2d(R) = 0$ or R is commutative. Assume that R is not associative.

Using Theorem 2, we have $(2d(x))^2 = 0$ for all x in R. Thus $2d(x)R$ is an ideal of R and $(2d(x)R)^2 = 0$. By the semiprimeness of R, this implies $2d(x)R = 0$. Hence $2d(R)R = 0$. Thus we see that the ideal of R generated by $2d(R)$ is $2d(R)$. By the primeness of R, $2d(R)R = 0$ implies $2d(R) = 0$, as desired.

Note that if R is commutative then by (5), $2d(xy) = 2\{d(x)y + xd(y)\}$ for all x, y in R. Combining this with Theorem 3 yields the

Corollary 1. *If R is a prime ring of characteristic not two with a Jordan derivation d such that $(d(R), R) = 0$, and $d(R) \subseteq N \cap M$ or $d(R) \subseteq M \cap L$ or $d(R) \subseteq N \cap L$, then d is a derivation , and either R is associative and commutative, or $d = 0$.*

In the course of the proofs of Theorems 2 and 3, we obtain the

Corollary 2. *If R is a semiprime ring with a Jordan derivation d such that $d(R) \subseteq G \cap I$ and $(d^2(R), R) \subseteq N$ or $(d^2(R), R) \subseteq L$, then $2d^2 = 0$.*

Corollary 3. *If R is a semiprime ring with a Jordan derivation d such that $(d(R), R) = 0$ and $d(R) \subseteq G \cap I$, then $2d = 0$.*

Corollary 4. *If R is a semiprime ring such that the Abelian group $(R, +)$ has no elements of order 2 and with a Jordan derivation d such that $(d(R), R) = 0$ and $d(R) \subseteq G \cap I$, then $d = 0$.*

Recall that R is a simple ring if R is the only nonzero ideal of R. In [3], we proved that if R is a simple ring with a derivation d such that $d(R) \subseteq N \cap L$ then either R is associative or $d^2 = 2d = 0$. For the prime ring case, we obtain the

Theorem 4. *If R is a prime ring with a derivation d such that $d(R) \subseteq N \cap M$ or $d(R) \subseteq M \cap L$ or $d(R) \subseteq N \cap L$, and there exists an ideal T of R such that $d(T) = 0$, then either $T = 0$ or $d = 0$.*

Proof. By symmetry, we only prove the theorem in case $d(R) \subseteq N \cap L$ and $d(T) = 0$. For all $t \in T$ and $x \in R$, using $d(T) = 0$ we have $0 = d(tx) = d(t)x + td(x) = td(x)$. Thus $Td(R) = 0$. Because of $d(R) \subseteq N \cap L$, this implies $T(Rd(R)) = (TR)d(R) \subseteq Td(R) = 0$. Hence $T(d(R) + Rd(R)) = 0$. Since d is a derivation , we get $d(R) + d(R)R = d(R) + Rd(R)$. By Lemma 1 of [3], or it is easy to see that $d(R) + Rd(R)$ is an ideal of R. Thus by the primeness of R, $T(d(R) + Rd(R)) = 0$ implies $T = 0$ or $d(R) + Rd(R) = 0$, as desired.

Corollary 5. *If R is a prime ring with a derivation d such that $d(I) = 0$, and $d(R) \subseteq N \cap M$ or $d(R) \subseteq M \cap L$ or $d(R) \subseteq N \cap L$, then either R is associative or $d = 0$.*

Theorem 5. *If R is a prime ring with a derivation d and there exists an ideal T of R such that $d(T) = 0$, and $T \subseteq N$ or $T \subseteq L$, then either $T = 0$ or $d = 0$.*

Proof. By symmetry, we only prove the theorem in case $d(T)=0$ and $T\subseteq N$. As in the proof of Theorem 4, we obtain that $Td(R)=0$, and we see that the ideal E of R generated by $d(R)$ is

$$\begin{aligned}E=&d(R)+d(R)R+d(R)R\cdot R+R\cdot d(R)R+(d(R)R\cdot R)R\\&+R(d(R)R\cdot R)+(R\cdot d(R)R)R+R(R\cdot d(R)R)+\cdots.\end{aligned}$$

Because of $T\subseteq N$ and $Td(R)=0$, we have that

$$T(d(R)R\cdot R)=(T\cdot d(R)R)R=(Td(R)\cdot R)R=0,$$

$$T\cdot(d(R)R\cdot R)R=T(d(R)R\cdot R)\cdot R=0,$$

$$T(R\cdot d(R)R)=TR\cdot d(R)R\subseteq T\cdot d(R)R=Td(R)\cdot R=0$$

and

$$T(R\cdot(R\cdot d(R)R))=TR\cdot(R\cdot d(R)R)\subseteq T(R\cdot d(R)R)=0.$$

Thus by induction we can show that $T\cdot E=0$. By the primeness of R, this implies $T=0$ or $E=0$. Hence, either $T=0$ or $d=0$, as desired.

We have a very easy consequence of Theorem 5.

Corollary 6. *If R is a prime associative ring and there exists an ideal T of R such that $d(T)=0$, then either $T=0$ or $d=0$.*

Theorem 6. *If R is a prime ring with a derivation d and there exists an ideal T of R such that $T\subseteq G$ and $d(T)\subseteq G$, then R is associative, or $T=0$, or $d=0$.*

Proof. By the hypotheses, for all $t\in T, x\in R$, we have $d(tx)=d(t)x+td(x)\in G$ and so $d(t)x\in G$. Thus $d(T)R\subseteq G$. Hence by (3), we get $d(T)(R,R,R)=0$ and so $d(T)((R,R,R)R)=0$. Thus by (7), $d(T)\cdot I=0$. Using $d(T)\subseteq G$ and $d(T)R\subseteq G$, we see that the ideal W of R generated by $d(T)$ is

$$W=d(T)+d(T)R+Rd(T)+R\cdot d(T)R.$$

Applying $d(T)\cdot I=0$, $d(T)\subseteq G$ and $d(T)R\subseteq G$, we obtain $W\cdot I=0$. By the primeness of R, this implies $W=0$ or $I=0$. If $I=0$, then R is associative. Assume that $W=0$. Then $d(T)=0$. By Theorem 5, we have that $T=0$ or $d=0$. This completes the proof of Theorem 6.

Proposition. *If R is a ring with a derivation d, then the associative subrings N, M and L of R are invariant under d, i.e., $d(A)\subseteq A$, where $A=N$ or M or L.*

Proof. Assume that $m\in M$, $x,y\in R$. Then by the definition of d, we obtain

$$\begin{aligned}(x,d(m),y)=&(xd(m))y-x(d(m)y)=(d(xm)-d(x)m)y-x(d(my)-md(y))\\=&d(xm)y-d(x)(my)-xd(my)+(xm)d(y)=d((xm)y)-d(x(my))\\=&d((x,m,y))=d(0)=0.\end{aligned}$$

Thus $d(m) \in M$. Hence $d(M) \subseteq M$. The proofs of other cases are similar.

Theorem 7. *If R is a prime ring with a derivation d and there exists a fixed positive integer n such that $d^n(R) \subseteq G$ and $(d^n(R), R) = 0$, then R is associative and $d^n = 0$, or R is associative and commutative, or $d^{2n} = (\frac{(2n)!}{n!})d^n = 0$.*

Proof. Note that $d^{2n}(R) \subseteq d^{2n-1}(R) \subseteq \cdots \subseteq d^{n+2}(R) \subseteq d^{n+1}(R) \subseteq d^n(R) \subseteq G$, or by the Proposition we have $d^i(R) \subseteq G$ for all integers $i \geq n$. Since $(d^n(R), R) = 0$, we obtain $(d^i(R), R) = 0$, $i \geq n$. Assume that $x, y, z \in R$. Using $d^n(R) \subseteq G$, we get $d^n(d^n(x)d^{n-1}(y)) \in G$. Expanding this by Leibniz's formula , and applying $d^i(R) \subseteq G$, $i \geq n$ and noting that G is an associative subring of R, we have $d^{2n}(x)d^{n-1}(y) \in G$. Thus $d^{2n}(R)d^{n-1}(R) \subseteq G$. Using this and $(d^i(R), R) = 0$, $i \geq n$, and argue as above, we get $d^{2n}(x)d^{n-1}(d^n(y)d^{n-2}(z)) \in G$ and so $d^{2n}(x)(d^{2n-1}(y)d^{n-2}(z)) \in G$. Hence $(d^{2n}(R)d^{2n-1}(R))d^{n-2}(R) \subseteq G$. Continuing in this manner, and applying $d^i(R) \subseteq G$ and $(d^i(R), R) = 0$, $i \geq n$, we finally obtain

$$(d^{2n}(R)d^{2n-1}(R) \cdots d^{n+2}(R))d(R) \subseteq G.$$

Argue as above, we have $(d^{2n}(R)d^{2n-1}(R) \cdots d^{n+2}(R))d(d^n(x)y) \subseteq G$ and so $(d^{2n}(R)d^{2n-1}(R) \cdots d^{n+2}(R))d^{n+1}(x)y \subseteq G$. Therefore we get

$$(d^{2n}(R)d^{2n-1}(R) \cdots d^{n+1}(R))R \subseteq G. \tag{12}$$

Using $d^i(R) \subseteq G$, $i \geq n$, we have

$$(d^{2n}(R)d^{2n-1}(R) \cdots d^{n+1}(R)) \subseteq G. \tag{13}$$

By (3), combining (12) with (13) yields

$$(d^{2n}(R)d^{2n-1}(R) \cdots d^{n+1}(R))(R, R, R) = 0. \tag{14}$$

Using (13) and (14), we get

$$(d^{2n}(R)d^{2n-1}(R) \cdots d^{n+1}(R)) \cdot (R, R, R)R = 0. \tag{15}$$

By (7), combining (14) with (15) yields

$$(d^{2n}(R)d^{2n-1}(R) \cdots d^{n+1}(R)) \cdot I = 0. \tag{16}$$

Applying (13), $(d^i(R), R) = 0$ and $d^i(R) \subseteq G, i \geq n$, we see that the ideal U of R generated by $(d^{2n}(R)d^{2n-1}(R) \cdots d^{n+1}(R))$ is

$$U = (d^{2n}(R)d^{2n-1}(R) \cdots d^{n+1}(R)) + (d^{2n}(R)d^{2n-1}(R) \cdots d^{n+1}(R))R. \tag{17}$$

Using (13), (16) and (17), we obtain $U \cdot I = 0$. By the primeness of R, $U = 0$ or $I = 0$. If $I = 0$, then R is associative. By Theorem 1 of [2], either $d^n = 0$ or R is commutative.

Assume that $U = 0$. Thus we have

$$d^{2n}(R)d^{2n-1}(R)\cdots d^{n+2}(R)d^{n+1}(R) = 0. \tag{18}$$

As above, we see that the ideal V of R generated by $(d^{2n}(R)d^{2n-1}(R)\cdots d^{n+2}(R))$ is

$$V = (d^{2n}(R)d^{2n-1}(R)\cdots d^{n+2}(R)) + (d^{2n}(R)d^{2n-1}(R)\cdots d^{n+2}(R))R. \tag{19}$$

Applying $d^i(R) \subseteq G$ and $(d^i(R), R) = 0, i \geq n$, (18) and (19), we get $V^2 = 0$. By the semiprimeness of R, this implies $V = 0$. Hence we obtain

$$d^{2n}(R)d^{2n-1}(R)\cdots d^{n+2}(R) = 0. \tag{20}$$

Continuing in this way, we can finally show that $d^{2n}(R) = 0$. Thus we get $0 = d^{2n}(d^{2n-2}(x)y) = (2n)d^{2n-1}(x)d^{2n-1}(y)$. Using this, $(d^i(R), R) = 0$ and $d^i(R) \subseteq G$, $i \geq n$, we have that $(2n)d^{2n-1}(x)R$ is an ideal of R and $((2n)d^{2n-1}(x)R)^2 = 0$. By the semiprimeness of R, this implies $(2n)d^{2n-1}(x)R = 0$. Hence $(2n)d^{2n-1}(R)R = 0$. Thus the ideal of R generated by $(2n)d^{2n-1}(R)$ is $(2n)d^{2n-1}(R)$. Therefore $(2n)d^{2n-1}(R) = 0$. Continuing in this manner, and applying $d^i(R) \subseteq G$ and $(d^i(R), R) = 0, i \geq n$, we finally obtain $(2n)(2n-1)\cdots(2n-(n-1))d^n(R) = 0$. Thus $(\frac{(2n)!}{n!})d^n = 0$. This completes the proof of Theorem 7.

Corollary 7. *If R is a prime ring with a derivation d and there exists a fixed positive integer n such that $d^n(R) \subseteq G$ and $(d^n(R), R) = 0$, and char R does not divide $(\frac{(2n)!}{n!})$ then either R is associative and commutative, or $d^n = 0$.*

Corollary 8. *If R is a semiprime ring with a derivation d and there exists a fixed positive integer n such that $d^n(R) \subseteq G \cap I$ and $(d^n(R), R) = 0$, then $d^{2n} = (\frac{(2n)!}{n!})d^n = 0$.*

Theorem 7 partially generalizes Theorem 1.

In [4], we extended Theorem 7 to s-derivation d with $sd = ds$.

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