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CENTRAL SEPARABLE ALGEBRAS OVER LOCAL-GLOBAL RINGS II

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All rings in this paper are commutative with 1, all modules are unitary right modules.

A ring R is a local-global ring (so called a ring with many units) if a polynomial f in $R[x_1, \ldots, x_n]$ represents a unit over R_P for every maximal ideal P in R, then f represents a unit over R [3,5]. Such rings include semilocal rings and von Neumann regular rings.

And a central separable algebra A over a ring R is said to have the cancellation property if, for any central separable R-algebras X and Y, the fact that $A \otimes_R X$ and $A \otimes_R Y$ are isomorphic implies that X and Y are isomorphic.

In [7], Roy and Sridharan showed that every central separable algebra over a semilocal ring has the cancellation property. In this paper, we show that every central separable algebra over a local-global ring has the cancellation property. Using this property, we classify central separable algebras over a connected local-global ring R by their classes in the Brauer group $Br(R)$ and the ranks. And, by this classification, we can classify free symmetric inner product spaces of rank 2 or 3 over a connected local-global ring with 2 a unit by Arf and Witt invariant, c. f. [2].

Now we consider the cancellation property over a local-global ring, using Ojanguren and Sridharan's equivalent conditions for the property [6] and Estes and Guralnick's n-th root property over a local-global ring [3].

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Theorem. *Let R be a local-global ring, then every central separable algebras over R has the cancellation property.*

Proof. Let A be a central separable algebra over R, and P a projective A-module with $P^n \cong A^n$ for some positive integer n. Then A is module finite by Theorem 2.1 of [1], and P is a finitely generated A-module. Thus, by Theorem 2.11 of [3], P and A are isomorphic as A-modules, and thus P is an A-bimodule routinely. Hence, by Theorem of [6], A has the canellation property.

This Theorem is a generalization of a result by Roy and Sridharan [7]. And, using this Theorem, we can classify central separable algebras over a connected local-global ring.

Corollary 1. *Let A and B be central separable algebras over a local-global ring R. If A and B are in the same class of $Br(R)$, and they have the same rank, then A and B are isomorphic.*

Proof. Since A and B are in the same class of $Br(R)$, there are R-progenerators P and Q such that $A \otimes_R \mathrm{End}_R(P, P) \cong B \otimes_R \mathrm{End}_R(Q, Q)$. But, since P and Q are finitely generated projective R-modules, they are finitely generated free R-modules by Theorem of [5]. And A and B also are finitely generated free R-modules by Theorem of [5] again. Thus, since A and B have the same rank, P and Q have the same rank by the invariant dimension property over a commutative ring. Therefore $\mathrm{End}_R(P, P)$ and $\mathrm{End}_R(Q, Q)$ are isomorphic as R-algebras. But they are central separable R-algebras. Hence A and B are isomorphic by the Theorem above.

This classification has an implication in the theory of quadratic forms.

Corollary 2. *Let R be a connected local-global ring with 2 a unit. If two free symmetric inner product spaces of rank 2 or 3 over R have the same rank and the same Arf and Witt invariant, then they are isomorphic.*

Proof. Proposition II. 5.1 and Proposition II. 5.2 of [2], Theorem 4.3 of [4], and Corollary 1.

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