TAMKANG JOURNAL OF MATHEMATICS
Volume 24, Number 3, Autumn 1993

CO-RS-COMPACT TOPOLOGIES

M. E. ABD EL-MONSEF, A. M. KOZAE AND A. A. ABO KHADRA

Abstract. A topology $R(\tau)$ is contructed from a given topolgy τ on a set X. $R(\tau)$ is coarser than τ, and the following are some results based on this topology:

1. Continuity and RS-continuity are equivalent if the codomain is re-topologized by $R(\tau)$.
2. The class of semi-open sets with respect to $R(\tau)$ is a topology.
3. T_2 and semi-T_2 properties are equivalent on a space whose topology is $R(\tau)$.
4. Minimal R_0-spaces are RS-compact.

1. Introduction

Throughout the present paper (X,σ) and (Y,τ) are topological spaces on which no separation axioms are assumed unless explicitly stated. A set S is said to be regular open (resp. regular closed) if S =int (cl(S)) (resp. S =cl(int(S)). A set S is said to be α-open [21] (resp. preopen [15]), if $S \subset$ int (cl int (S)), (resp. $S \subset$ int cl(S)). A set S is said to be regular semi-open [4] (resp. semi-open [13]) if there exists a regular open (resp. open) set 0 such that $0 \subset S \subset cl(0)$. It should be noticed that the complement of a regular semi-open set is also regular semi-open. The family of all regular semi-open (resp. regular open, regular closed, α-open, preopen and semi-open) sets in X is denoted by $RSO(X)$ (resp.

Received May 5, 1992.
1991 AMS Subject Classification Codes: 54A10, 54C10, 54D10, 54D20.
Key words and phrases: RS-compact spaces, RS-continuous functions, extremely disconnected, semi-T_2, α-compact spaces, R_0-spaces.

$RO(X)$, $RC(X)$, $\alpha O(X)$, $PO(X)$, $SO(X)$). A space X is said to be extremely disconnected if for every open set 0 of X, $cl(0)$ is open in X.

In 1980, Hong [9] has introduced a new class of topological spaces called RS-compact spaces which are characterized by the following property "Every regular closed cover has a finite subfamily, the interiors of whose members cover X".

Note. The definition of RS-compact space in the sense of Hong is equivalent to that of an I-compact space in the sense of Cameron [5]. In 1985 Noiri, [22] has introudced RS-compact relative to X. "A subset S of X is RS-compact relative to X if for every cover $\{V_i : i \in I\}$ of S by regular closed sets of X, there exists a finite subset I_0 of I such that $S \subset \cup\{\text{int}(V_i) : i \in I_0\}$. The relationship between RS-compacteness and ordinary compactness, with examples to illustrate that neither implies the other was studied in [22].

In 1989, Abd El-Monsef et al., [2] have introduced RS-continuous function "A function $f : X \to Y$ is called RS-continuous if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ having RS-compact complement, there exists an open set $U \subset X$ containing x such that $f(U) \subset V$". A space X is said to be almost normal space [17] if for every two disjoint regular closed subsets F_1 and F_2 of X, there exist two disjoint open sets U and V in X such that $F_1 \subset U$ and $F_2 \subset V$. A space X is semi-T_2 [14] (resp. semi-T_2' [1]) if for each $x, y \in X$, $x \neq y$, there exist U and $V \in SO(X)$ such that $x \in U$, $y \in V$ and $U \cap V = \phi$ (resp. $cl(U) \cap cl(V) = \phi$). The space (Y, τ) is R_0 [6] if for each $G \in \tau$, $x \in G$ implies $cl\{x\} \subseteq G$. We observe that in every R_0 topological space the closure of a singleton set is compact. A space X is α-compact [3] (resp. strongly compact [16]) if each cover of X by α-open (resp. preopen) sets in X has a finite subcover.

Theorem 1.1. [22]. *If $a \in RSO(X)$ and B is RS-compact relative to X, then $A \cap B$ is RS-compact relative to X.*

Theorem 1.2. [22]. *Let $A \in RO(X)$. Then A is RS-compact iff A is RS-compact relative to X.*

Theroem 1.3. [22]. *If X is RS-compact and $A \in RO(X)$, then A is RS-compact.*

Thereom 1.4. [22]. *Let X_0 be an open set of X. Then we have:*
(1) *If $A \subset X_0$, then* $\text{Int}_X(A) = \text{Int}_{X_0}(A)$.
(2) *If $V \in RSO(X)$, then $V \cap X_0 \in RSO(X_0)$.*

Theorem 1.5. [5]. *Any RS-compact is extremely disconnected.*

Theorem 1.6. [10]. *Every compact Hausdorff space is normal.*

Theorem 1.7. [12]. *A space (X, τ) is semi-T_2' iff for each $x, y \in X$, $x \neq y$, there exist $W_1, W_2 \in RC(X, \tau)$ containing x, y respectively, such that $W_1 \cap W_2 = \phi$.*

Theorem 1.8. [11]. *The following statements are equivalent for a space (X, τ):*
(a) *(X, τ) is extremely disconnected.*
(b) *For each $A \in SO(X, \tau)$, $cl(A) \in \tau$.*
(c) *For each $A, B \in SO(X, \tau)$, $cl(A \cap B) = cl(A) \cap cl(B)$.*

2. CO-RS-Compact Topologies

Let (Y, τ) be a topological space, and consider $R'(\tau) = \{U \in \tau : Y - U$ is RS-compact relative to $\tau\}$. $R'(\tau)$ is a base for a topology $R(\tau)$ on Y, called the CO-RS-compact topology on Y. We shall denote by $(Y, R(\tau))$ to be a CO-RS-compact space of (Y, τ), and $cl_{R(\tau)}(S)$ (resp. $\text{int}_{R(\tau)}(S)$) will denote the closure (resp. interior) with respect to $R(\tau)$ of a subset S of $(Y, R(\tau))$. From definition we have $R(\tau) \subset \tau$, and the following lemma is a direct consequence.

Lemma 2.1. *The function $f : (X, \sigma) \to (Y, \tau)$ is RS-continuous iff $f : (X, \sigma) \to (Y, R(\tau))$ is continuous.*

Theorem 2.1. *For any topological space (Y, τ), $(Y, R(\tau))$ is RS-compact* *space.*

Proof. Consider the $R(\tau)$ regular closed cover $\Delta = \{V_i : i \in I\}$ of Y, and V be some nonempty member of Δ, there exists $R(\tau)$ open set U such that $U \subset V \subset R - cl(U)$, and $Y - U$ is RS-compact relative to τ. By using Theorem 1.1 and Theorem 1.2. $(Y - V)$ is RS-compact subspace of Y. Assume that $A = Y - V$, thus $V_i \cap A \in RSO(A)$, for each $i \in I$ (By using Theorem 1.4), and $A = \cup\{V_i \cap A : i \in I\}$. There exists a finite subset I_0 of I such that $A = \cup\{\text{int}_A(V_i \cap A) : i \in I_0\}$. From Theorem 1.4., we have $\text{int}_A(V_i \cap A) = \text{int}_Y(V_i \cap A) \subset \text{int}_Y(V_i)$ for each $i \in I_0$. Hence $A \subset U(\text{int}_Y(V_i) : i \in I_0\}$, and $A = Y - V$ is RS-compact relative to $R(\tau)$. Thus $(Y, R(\tau))$ is RS-compact space.

Remark 2.1. From Theorem 2.1, we notice that each topological space which contains at least a proper open set with RS-compact complement has a coarser extremely disconnected topology $R(\tau)$ [not indiscrete].

Proposition 2.1. *The following statements are hold:*

(a) $RO(Y, R(\tau)) \subset RO(Y, \tau)$.

(b) $cl_\tau G = cl_R G$, *for all* $G \in RO(Y, R(\tau))$.

(c) $RSO(Y, R(\tau) \subset RSO(Y, \tau)$.

Proof. (a) Let $G \in RO(Y, R(\tau))$, then $G = \text{int}_R cl_R(G) = cl_R(G)[(Y, R(\tau))$ is extremely disconnected]. But $cl_\tau G \subset cl_R G$, implies that $G = cl_\tau G$ and $G = \text{int}_\tau cl_\tau G$.

(b) From (a), the proof is obvious.

(c) By using (b), the result follows.

Propsoition 2.2. *Let X be an extremely disconnected space then:*

(a) *The union of a finite regular open sets is a regular open set.*

(b) *If X is the union of a finite number of regular open RS-compact subspaces, then X is RS-compact space.*

Lemma 2.2. *If (X, τ) is extremely disconnected, then every regular closed* *subset is regular open.*

Proof. Let $F \in RC(X,\tau)$, then $F = cl\,\text{int}\,F = \text{int}\,cl\,\text{int}\,F[(X,\tau)$ is extremely disconnected]. Thus $cl\,F = cl\,\text{int}F$, implies $\text{int}\,cl\,F = \text{int}\,cl\,\text{int}\,F = F$, and $F \in RO(X,\tau)$.

Theorem 2.2. *If $(Y, R(\tau))$ is almost normal space, then (Y,τ) is RS-compact.*

Proof. There are two cases:

(1) If there exist at least two disjoint $R(\tau)$ regular closed sets F_1, F_2. Then there exist disjoint $R(\tau)$ open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$. Hence $Y = Y - (U \cap V) = (Y - U) \cup (Y - V) = (Y - F_1) \cup (Y - F_2)$. But $(Y - F_1)$ and $(Y - F_2)$ are RS-compact subspace. [Theorem 1.3 and Theorem 2.1]. By Proposition 2.1 and Proposition 2.2, we arrive (Y,τ) is RS-compact.

(2) If there do not exist two disjoint $R(\tau)$ regular closed sets. We assume that $F_1, F_2 \in RC(Y, R(\tau))$ such that $\phi \neq F_1 \cap F_2 = F$. By Lemma 2.2, $F_1, F_2 \in RO(Y, R(\tau))$. Then $F \in RO(Y, R(\tau))$ and $F \cup (Y - F_1) \cup (Y - F_2) = F \cup (Y - (F_1 \cap F_2)) = F \cup (Y - F) = Y$. Since F, $(Y - F_1)$, $(Y - F_2)$ are RS-compact subspaces and belongs to the class of regular open sets in (Y,τ). Thus (Y,τ) is RS-compact.

A point x of X is said to be δ-adherent point of a subset S of X if int $cl(V) \cap S \neq \phi$ for every open neighborhood V of x in X. A subset S is said to be δ-closed [24] if S contains all δ-adherent points of S. In a regular space δ-closure operation is identical with the closure operation [24].

Theorem 2.3. [22]. *If X is RS-compact and A is δ-closed in X, then A is RS-compact relative to X.*

Theorem 2.4. *If (Y,τ) is regular, then (Y,τ) is RS-compact iff $\tau = R(\tau)$.*

Proof. Let $\tau = R(\tau)$, then (Y,τ) is RS-compact. Conversely, let $U \in \tau$, then $(Y - U)$ is closed in τ. But (Y,τ) is regular, then $(Y - U)$ is δ-closed. But

(Y,τ) is RS-compact, and by using Theorem 2.3, $(Y-U)$ is RS-compact relative to τ. Hence $U \in R(\tau)$, and $\tau \subset R(\tau)$. Thus $R(\tau) = \tau$.

Theorem 2.5. *Let (Y,τ) be a space such that every open set is closed, then (Y,τ) is RS-compact iff $\tau = R(\tau)$.*

Proof. Let $U \in \tau$, then $(Y-U)$ is open and closed and hence regular open set. But (Y,τ) is RS-compact, then $(Y-U)$ is RS-compact relative to τ. Thus $U \in R(\tau)$.

Theorem 2.6. *If (Y,τ) is connected, then $(Y, R(\tau))$ has the property that $cl_{R(\tau)}U = X$, for all U in $R(\tau)$.*

Proof. Let $U \in R(\tau)$, then $cl_{R(\tau)}U \in R(\tau)$, $[(Y,R(\tau))$ is extremely disconnected$]$. Thus $cl_{R(\tau)}U$ is both open and closed in $R(\tau)$, and also in τ. But (Y,τ) is connected, which implies that $cl_{R(\tau)}U = X$.

3. Separation Properties

The property of being T_1-space is expansive, that is if (Y,τ) is T_1 and $\tau \subset \tau'$ then (Y,τ') is T_1 but it is not generally contractive. The following result proves the contractivity of T_1 property from (Y,τ) to $(Y,R(\tau))$.

Lemma 3.1. *If (Y,τ) is T_1, then $(Y,R(\tau))$ is T_1.*

Proof. Let x be any point of Y, then $\{x\}$ is closed in τ and RS-compact in (Y,τ). Thus $Y-\{x\}$ is open in τ and $\{x\}$ is RS-compact. Hence $Y-\{x\}$ is open in $R(\tau)$. Thus $(Y,R(\tau))$ is T_1.

Theorem 3.1. *If (Y,τ) is Hausdorff, then $(Y,R(\tau))$ is compact.*

Proof. From Lemma (3) in [20], we have that $R(\tau) \subset c(\tau_s) \subset \tau_s \subset n(\tau) \subset \tau$. Using Lemma (4) in [19] and Theorem 2.1 in [23], the result follows.

Lemma 3.2. *If $(Y,R(\tau))$ is Hausdorff, then $(Y,R(\tau))$ is normal.*

Proof. By using Theorem 3.1, and Theorem 1.6, the proof is obvious.

Lemma 3.3. *If $(Y, R(\tau))$ is semi-T_2', then (Y, τ) is RS-compact space.*

Proof. Let $x, y \in X$ and $x \neq y$, then there exist $W_1, W_2 \in RC(Y, R(\tau))$ such that $x \in W_1$, $y \in W_2$ and $W_1 \cap W_2 = \phi$. Then $Y = Y - (W_1 \cap W_2) = (Y - W_1) \cup (Y - W_2)$. But $(Y, R(\tau))$ is RS-compact, then $(Y - W_1)$ and $(Y - W_2)$ are RS-compact relative to $R(\tau)$. By using Theorem 1.4, we can prove that they are RS-compact relative to τ. Thus (Y, τ) is RS-compact space.

Theorem 3.2. [18]. *If τ and τ' are two topologies on X, such that $\tau \subset \tau'$, then $RO(X, \tau) = RO(X, \tau')$ iff $cl_\tau G = cl_{\tau'} G$ for each $G \in \tau'$; [$cl_\tau G = cl_{\tau'} G$ is equivalent to $int_\tau F = \text{int}_{\tau'} F$, for each $F \in \tau'^c$].*

Theorem 3.3[7]. *If (X, τ) is a space, then $A \in PO(X, \tau)$ iff $A = U \cap V$, $U \in RO(X, \tau)$, V is τ-dense.*

Theorem 3.4. *If $(Y, R(\tau))$ is semi-T_2', then the following statements hold:*
(i) $RO(Y, R(\tau)) = RO(Y, \tau)$.
(ii) $PO(Y, \tau) \subset PO(Y, R(\tau))$.
(iii) $SO(Y, R(\tau)) \subset SO(Y, \tau)$.
(iv) $\alpha O(Y, R(\tau)) \subset \alpha O(Y, \tau)$.

Proof. (i) Let $U \in RO(Y, \tau)$, then $(Y - U) \in RC(Y, \tau)$. By Lemma 3.3, (Y, τ) is RS-compact, and hence extremely disconnected, thus $(Y - U) \in \tau$. By Theorem 1.3, U is RS-compact relative to τ. Then $(Y - U) \in R(\tau)$, and U is closed in $R(\tau)$, and also in τ. Thus $(Y - U) \in RO(Y, \tau)$ and also is RS-compact relative to τ. Hence $U \in R(\tau)$. Since U is open and closed in $R(\tau)$, then $U \in RO(Y, R(\tau))$. Hence $RO(Y, \tau) \subset RO(Y, R(\tau))$. But $RO(Y, R(\tau)) \subset RO(Y, \tau)$. Thus $RO(Y, R(\tau)) = RO(Y, \tau)$.

(ii) Let $A \in PO(Y, \tau)$, then by Theorem 3.3, $A = U \cap V$, where $U \in RO(Y, \tau)$ and V is τ-dense. Since $RO(Y, R(\tau)) = RO(Y, \tau)$ and each τ-dense is $R(\tau)$-dense. Thus $A \in PO(Y, R(\tau))$, and $PO(Y, \tau) \subset PO(Y, R(\tau))$.

(iii) Let $A \in SO(Y, R(\tau))$, then there exists $G \in R(\tau)$ such that $G \subset A \subset cl_{R(\tau)}G$. Since $R(\tau) \subset \tau$, then $G \in \tau$. But $RO(Y, R(\tau)) = RO(Y, \tau)$, then by Theorem 3.2 $cl_{R(\tau)}G = cl_\tau G$. Hence $A \in SO(Y, \tau)$.

(iv) Let $G \in \alpha O(Y, R(\tau))$, then $G \subset \mathrm{int}_R cl_R \mathrm{int}_R G \subset \mathrm{int}_\tau cl_R \mathrm{int}_\tau G = \mathrm{int}_\tau cl_\tau \mathrm{int}_\tau G$ [by Theorem 3.2]. Hence $G \in \alpha O(Y, \tau)$.

Corollary 3.1. *If $(Y, R(\tau))$ is semi-T_2' such that $\mathrm{int}_R G = \mathrm{int}_\tau G$, for each $G \in \alpha O(Y, \tau)$, then $\alpha O(Y, R(\tau)) = \alpha O(Y, \tau)$.*

Proof. If $G \in \alpha O(Y, \tau)$, then $G \subset \mathrm{int}_\tau cl_\tau \mathrm{int}_\tau G = \mathrm{int}_R cl_\tau \mathrm{int}_R G = \mathrm{int}_R cl_R \mathrm{int}_R G$. [By Theorem 3.2].

Corollary 3.2. *If $(Y, R(\tau))$ is semi-T_2', and (Y, τ) is α-compact, then $(Y, R(\tau))$ is α-compact.*

Proof. By Theorem 3.4(iv).

Corollary 3.3. *If $(Y, R(\tau))$ is semi-T_2', and $(Y, R(\tau))$ is strongly compact, then (Y, τ) is strongly compact.*

Proof. By Theorem 3.4 (ii).

Theorem 3.5. *$(Y, R(\tau))$ is semi-T_2 iff $(Y, R(\tau))$ is semi-T_2'.*

Proof. Let $(Y, R(\tau))$ be semi-T_2, and $x, y \in Y$, $x \neq y$, then there exist $U, V \in SO(Y, \tau)$ such that $x \in U$, $y \in V$ and $U \cap V = \phi$, which implies that $cl_R(U \cap V) = \phi$. Since $(Y, R(\tau))$ is extremely disconnected we have $cl_R \mathrm{int}_R U = cl_R U \in \tau$, and $cl_R \mathrm{int}_R V = cl_R V \in \tau$. But $cl_R U \cap cl_R V = cl_R(U \cap V) = \phi$. Hence $(Y, R(\tau))$ is semi-T_2'. The converse is obvious.

Theorem 3.6. *$(Y, R(\tau))$ is Hausdorff iff $(Y, R(\tau))$ is semi-T_2.*

Proof. It is similar to the proof of Theorem 3.5.

Theorem 3.7. *Let (Y, τ) be a space, then:*

a) *The class of $SO(Y, R(\tau))$ form a topology (denoted by τ') finer than $R(\tau)$.*

b) $RO(Y, R(\tau)) = RO(Y, \tau')$.

c) (Y, τ') *is* RS*-compact.*

Proof. (a) Since $(Y, R(\tau))$ is RS-compact, it is extremely disconnected and hence $SO(Y, R(\tau))$ forms a topology such that $R(\tau) \subset \tau'$.

(b) Let $G \in \tau'$, then $cl_{\tau'} G \subset cl_R G$. Conversely, let $x \in cl_R G$, and $x \in U$, $U \in \tau'$. Hence $x \in cl_R \operatorname{int}_R U \in R(\tau)$ and $cl_R \operatorname{int}_R U \cap G \neq \phi$. But $G \in \tau'$, implies $G \in SO(Y, R(\tau))$ and $G \subset cl_R \operatorname{int}_R G$, therefore $\phi \neq cl_R \operatorname{int}_R G \cap cl_R \operatorname{int}_R U \subset cl_R(\operatorname{int}_R G \cap cl_R \operatorname{int}_R U) \subset cl_R(\operatorname{int}_R G \cap int_R U) \subset cl_R(U \cap G)$, which implies that $U \cap G \neq \phi$, and so $x \in cl_{\tau'} G$. Hence $cl_R G \subset cl_{\tau'} G$. Thus $cl_R G = cl_{\tau'} G$ for each $G \in \tau'$.

(c) Using (b), the result follows.

Lemma 3.4. *If* (Y, τ) *is* R_0*, then* $(Y, R(\tau))$ *is* R_0.

Proof. Let $x \in Y$, and $x \in G \in R(\tau)$, then $G \in \tau$, and $cl_\tau\{x\} \subset G$. Since $cl_\tau\{x\}$ is compact in (Y, τ), implies that it is compact in $(Y, R(\tau))$, and nearly compact in $(Y, R(\tau))$. But $(Y, R(\tau))$ is extremely disconnected, then $cl_\tau\{x\}$ is RS-compact relative to $R(\tau)$, which implies that it is RS-compact relative to τ. Thus $cl_\tau\{x\}$ is closed in $(Y, R(\tau))$, implies that $cl_R\{x\} \subset cl_\tau\{x\}$. Hence $cl_R\{x\} = cl_\tau\{x\}$, and $(Y, R(\tau))$ is R_0.

Theorem 3.8. *Minimal* R_0*-spaces are* RS*-compact spaces.*

Proof. Using Lemma 3.4, the result follows.

Acknowledgement

We would like to thank the referee for valuable comments and suggestions.

References

[1] M. E. Abd El-Monsef, "Studies on some pretopological concepts", *Ph. D. Thesis*, Tanta University (1980).

[2] M. E. Abd El-Monsef, R. A. Mahmoud and A. A. Nasef, "Functions based on compactness", (to appear).

[3] R. H. Atia, S. N. El-deeb and A. S. Mashhour, "α-compactness and α-homeomorphism", (preprint).

[4] D. E. Cameron, "Properties of S-closed spaces", *Proc. Amer. Math. Soc.*, 72 (3) (1978), 581-585.

[5] ________, "Some maximal topologies which are QHC" *Proc. Amer. Math. Soc.*, 75 (1) (1979), 149-156.

[6] A. S. Davis, "Indexed systems of neighborhoods for general topological spaces", *Amer. Math. Month.*, 68 (1961), 886-893.

[7] M. Ganster, "Preopen sets and resolvable spaces", *to appear in Kyungpook Math. J.*

[8] D. B. Gauld, M. Mrsevic, I. L. Reilly, and M. K. Vamanamurthy, "Colindelof topologies and L-continuous functions", *Clasnik, Math.*, 19 (39) (1984), 297-308.

[9] W. C. Hong, "RS-compact spaces", *J. Korean. Math. Soc.*, 17 (1980), 39-43.

[10] S. T. Hu, "Elements of general topology", Holden Day, IM, (1972).

[11] D. S. Jankovic, "On locally irreducible spaces", *Ann. de la Soc. Sci de Bruxelles*, T, 97, II, (1983), 59-72.

[12] A. M. Kozae, "Studies on some maximal and minimal topological concepts", *Ph. D. Thesis*, Tanta University (1988).

[13] N. Levine, "Semi-open sets and semi continuity in topological spaces", *Amer. Math. Monthly* 70 (1963), 36-41.

[14] S. N. Maheshwari and R. Prasad, "Some new separation axioms", *Ann. Soc. Sci. Bruxelles*, T. 3 (89) (1975), 395-407, MR 52 # 6660.

[15] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, "On pretopological spaces", *Bull. Math. de la Soc. R. S. de Roumanie* 28 (76) (1984), 39-45.

[16] A. S. Mashhour, M. E. Abd El-Monsef, I. A. Hasanein and T. Noiri, "Strongly compact spaces", *Delta J. Sci.* 8 (1) (1984), 30-46.

[17] A. S. Mashhour, F. S. Mahmoud, I. A. Hasanein and M. A. Fath Alla, "On some generalization of compactness", (to appear).

[18] J. Mioduszewski and L. Rudolf, "H-closed and extremely disconnected spaces", *Dissertations Math.*, 66 (1969).

[19] M. Mrsevic, I. L. Reilly and M. K. Vamanamurthy, "On semi-regularization topologies", *J. Austral. Math. Soc.*, (Series A) 38 (1985), 40-54.

[20] M. Mrsevic and I. L. Reilly, "On N-continuity and Co-N-closed topologies", *Ricerche di Math.*, 36 (1) (1987), 33-43.

[21] O. Njasted, "On some classes of nearly open sets", *Pac. J. of Math.* 15 (3) (1965), 961-970.

[22] T. Noiri, "On RS-compact spaces", *J. Korean Math. Soc.*, 22 (1985), No. 1, pp. 19-34.

[23] M. K. Singal and A. Mathur, "On nearly compact spaces II", *Boll, Della Un. Math. Italiana*, (4) 9 (1974), 670-678.

[24] N. V. Velicko, "H-closed topological spaces", *Amer. Math. Soc. Transl.*, (2) 78 (1968), 103-118.

Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt.