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ON S-REDUCIBLE FINSLER SPACES

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1. Introduction

To study the theory of Fields in a Finsler space F_n, it is important to introduced the Ricci tensors of F_n. The Ricci tensor

$$R_{ij} = R_{hijk}g^{hk} \tag{1.1}$$

of a Riemanian space plays the essential role in the theory of graviation. But we have very few papers ([8], [9]) concern with such tensors of $(0, 2)$ type constructed from the curvature tensors of F_n.

For the v-curvature tensor S_{hijk} of F_n, the problem is interesting because it is regarded as the Riemanian curvature tensor of the tangent Riemanian space of F_n. The v-Ricci tensor of F_n is defined from v-curvature tensor S_{hijk} as

$$S_{ij} = S_{hijk}g^{hk} \tag{1.2}$$

This tensor is symmetric and indicatory tensor. In this paper we shall study those Finsler spaces for which

$$S_{ij} = \rho h_{ij} + \mu C_i C_j \tag{1.3}$$

where ρ and μ are scalar functions, h_{ij} are components of angular metric tensor and C_i are components of contracted C-tensor i.e.

$$C_i = C^j_{ij} \tag{1.4}$$

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The v-Ricci tensor of C-reducible and semi C-reducible Finsler space is of this form. So we shall say the Finsler space whose v-Ricci tensor is of the form (1.3) as S-reducible Finsler space.

There are three kinds of torsion tensors in Cartan's theory of Finsler space F_n. Two of them are (h) hv-torsion tensor C_{ijk} and $(v)hv$-torsion tensor P_{ijk} which are symmetric in all of its indices. Since the v-curvature tensor and v-Ricci tensor are defined from the (h) hv-torsion tensor C^j_{ik} in cartan's theory of Finsler spaces as

$$\begin{aligned} S^i_{hjk} &= \partial_k C^i_{hj} + C^r_{hj}C^i_{rk} - \partial_j C^i_{hk} - C^r_{hk}C^i_{rj} \\ &= C^r_{hk}C^i_{rj} - C^r_{hj}C^i_{rk} \end{aligned} \tag{1.5}$$

Therefore, the forms of v-Ricci tensor of Finsler space will depend upon the special form of C_{ijk}. Many workers have obtained interesting forms of C_{ijk}. They are C-reducible ([3]):

$$C_{ijk} = \frac{1}{n+1}(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j) \tag{1.6}$$

Semi C-reducible ([4]):

$$C_{ijk} = \frac{p}{n+1}(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j) + \frac{q}{C^2}C_iC_jC_k \tag{1.7}$$

C_2-like ([5]):

$$C_{ijk} = \frac{1}{C^2}C_iC_jC_k \tag{1.8}$$

where p and q in (1.7) are scalars satisfying $p + q = 1$.

The S3- and S4- like Finsler spaces are characterized by relations ([2], [4])

$$L^2 S_{hijk} = S(h_{hj}h_{ik} - h_{hk}h_{ij}), \qquad n \geq 4 \tag{1.9}$$

$$L^2 S_{hijk} = h_{hj}M_{ik} + h_{ik}M_{hj} - h_{hk}M_{ij} - h_{ij}M_{hk} \tag{1.10}$$

respectively. Where S is $(o)p$-homogeneous and M_{ij} is symmetric and indicatory tensor.

The various transformations of a Finsler metric have been studied in the literature ([1]). We shall also study the conditions under which the S-reducible Finsler space is transformed to S-reducible Finsler space under these transformations of the metric.

2. The v-Ricci Tensor of Special Finsler Spaces

The v-curvature tensor of C-reducible Finsler space is given by ([3])

$$S_{ijkl} = \frac{1}{(n+1)^2}(h_{il}C_{jk} + h_{jk}C_{il} - h_{ik}C_{il} - h_{jl}C_{ik}) \tag{2.1}$$

where $C_{ij} = \frac{1}{2}C^rC_rh_{ij} + C_iC_j$

With help of equations (1.2) and (2.1), we have

$$S_{jk} = \rho\, h_{jk} + \mu\, C_jC_k$$

where $\rho = \frac{n-1}{(n+1)^2}C^2$ and $\mu = \frac{n-3}{(n+1)^2}$

Therefore, we have the

Theorem 2.1. *Every C-reducible Finsler space is S-reducible Finsler space.*

The v-curvature tensor for semi C-reducible Finsler space is given as ([4])

$$L^2S_{hijk} = h_{hj}M_{ik} + h_{ik}M_{hj} - h_{hk}M_{ij} - h_{ij}M_{hk} \tag{2.2}$$

where

$$M_{ij} = -L^2\frac{p^2c^2}{2(n+1)^2}h_{ij} - L^2\{\frac{p^2}{(n+1)^2} + \frac{pq}{n+1}\}C_iC_j$$

In view of equations (1.2) and (2.2), the v-Ricci tensor for semi C-reducible Finsler space can be written as

$$S_{ij} = \rho\, h_{ij} + \mu\, C_iC_j$$

where

$$\rho = \{\frac{(n-3)p^2C^2}{2(n+1)^2} - \frac{pC^2}{n+1}(\frac{p}{2} + q)\}$$

and

$$\mu = (n-3)\{\frac{p^2}{(n+1)^2} + \frac{pq}{n+1}\}$$

hence, we have the following.

Theorem 2.2. *Every semi C-reducible Finsler space is S-reducible Finsler space.*

The v-curvature tensor of $S3$-like Finsler space is written as (1.9). In view of equations (1.2) and (1.9), the v-Ricci tensor for $S3$-like Finsler space can be written as

$$S_{ij} = \rho h_{ij} + \mu C_i C_j$$

where $\rho = -\frac{S}{L^2}(n-2)$ and $\mu = 0$

Therefore, we have

Theorem 2.3. *Every S3-like Finsler space is S-reducible Finsler space.*

The v-curvature tensor for $S4$-like Finsler space is given by (1.10). In view of equations (1.2) and (1.10), the v-Ricci tensor for $S4$-like Finsler space can be written as

$$S_{ij} = \rho h_{ij} + \mu M_{ij}$$

where $\rho = -\frac{M_{hk}g^{hk}}{L^2}$ and $\mu = -(\frac{n-3}{L^2})$

Therefore, we have the followings:

Theorem 2.4. *The v-Ricci tensor of S4-like Finsler space is of the form*

$$S_{ij} = \rho h_{ij} + \mu M_{ij}$$

and

Theorem 2.5. *The S4-like Finsler space is S-reducible if and only if there exits scalars α and β with $\mu\alpha + \rho \neq 0$ such that*

$$M_{ij} = \alpha h_{ij} + \beta C_i C_j$$

In view of equations (1.8) and (1.5), we have

$$S_{hijk} = 0$$

which gives

$$S_{ij} = 0$$

Thus, we have

Theorem 2.6. *The v-Ricci tensor of C2-like Finsler space vanishes identically.*

3. The Transformation of S-Reducible Finsler Space by an h-Vector

Prasad and Srivastave ([7]) obtained the relation between v-curvature tensors with respect to $C\Gamma$ of a Finsler space (F_n, L) and (F_n, L^*), where $L^*(x, y)$ is obtained from $L(x, y)$ by the transformation

$$L^{*2}(x,y) = L^2(x,y) + (b_i y^i)^2 \tag{3.1}$$

where $b_i(x, y)$ is an h-vector in (F_n, L).

The v-curvature tensor has been given as

$$S^*_{hijk} = \sigma S_{hijk} + h_{ij}d_{hk} + h_{hk}d_{ij} - h_{ik}d_{hj} - h_{hj}d_{ik} \tag{3.2}$$

where $d_{ij} = \frac{1}{2}\alpha_1 h_{ij} + \alpha_2 m_i m_j$ and α_1 and α_2 are scalar functions

$$m_i = b_i - \frac{b_r y^r}{L} l_i$$

The contravariant component of fundamental metric tensor g^{*ij} of (F_n, L^*) has been given by

$$g^{*ij} = \sigma^{-1} g^{ij} - \frac{(1-\sigma)\beta}{L\lambda}(l^i b^j + l^j b^i) + \frac{(1-\sigma)(b^2+\sigma)}{\lambda} l^i l^j + \frac{1}{\lambda} b^i b^j \tag{3.3}$$

where $\sigma = (1 + \frac{\beta\rho}{L})$, $b^2 = b^i b_i$, $\beta = b_i y^i$ and $\rho = \frac{1}{n-1} L C^i b_i$. The (h) hv-torsion tensor has been given by

$$\begin{aligned} C^{*h}_{ij} &= C^h_{ij} + \frac{\rho}{2L\sigma}(h_{ij}m^h + h^h_j m_i + h^h_i m_j) \\ &- \frac{(1-\sigma)\beta\rho}{L^2\lambda}[\{\sigma + \frac{1}{2}(b^2 - \frac{\beta^2}{L^2})\}h^h_{ijl} + m_i m_j l^h] \\ &+ \frac{\rho}{L\lambda}[\{\sigma + \frac{1}{2}(b^2 - \frac{\beta^2}{L^2})\}h_{ij}b^h + m_i m_j b^h] \end{aligned} \tag{3.4}$$

The angular metric tensor h^*_{ij} of (F_n, L^*) has been given by

$$h^*_{ij} + l^*_i l^*_j = \sigma h_{ij} + \ell_i l_j + b_i b_j \tag{3.5}$$

where

$$L^* l^*_i = L l_i + \beta b_i$$

The v-Ricci tensor of (F_n, L^*) is obtained from equations (3.2), (3.3) and (1.3), which is given by

$$S^*_{ij} = A h_{ij} + \mu C_i C_j + B m_i m_j \tag{3.6}$$

where

$$\begin{aligned} A &= \Big[\rho + \frac{1}{\sigma}M + \frac{1}{\lambda}N(b^2 - \frac{\beta^2}{L^2}) + \frac{1}{2}\alpha_1\{\frac{1}{\sigma}(n-3) + \frac{1}{\lambda}(b^2 - \frac{\beta^2}{L^2})\}\Big], \\ M &= \frac{1}{2}(n-1)\alpha_1 + \alpha_2(b^2 - \frac{\beta}{L^2}),\ N = \frac{1}{2} + \alpha_2(b^2 - \frac{\beta^2}{L^2}) \end{aligned}$$

and

$$B = \alpha_2\{\frac{1}{\sigma}(n-3) + \frac{1}{\lambda}(b^2 - \frac{\beta^2}{L^2})\} - \frac{2}{\lambda}$$

In view of equations (1.4), (3.4), we get

$$C_i C_j = C^*_i C^*_j - M(m_i C_j + m_j C_i) - M^2 m_i m_j \tag{3.7}$$

With the help of equations (3.6), (3.5) and (3.7), we have

$$\begin{aligned} S^*_{ij} &= \rho^* h^*_{ij} + \mu^* C^*_i C^*_j + \frac{A}{\sigma}\{(\frac{L^2}{L^{*2}} - 1)l_i l_j + \frac{L\beta}{L^{*2}}(l_i b_j + l_j b_i) + (\frac{\beta^2}{L^{*2}} - 1)b_i b_j\} \\ &- \mu M(m_i C_j + m_j C_i) + (B - \mu M^2)m_i m_j \end{aligned} \tag{3.8}$$

where $\rho^* = \dfrac{A}{\sigma}$ and $\mu^* = \mu$. Thus we have the following

Theorem 3.1. *An S-reducibl Finsler space is transformed to an S-reducible Finsler space under the transformation (3.1) if and only if*

$$\frac{A}{\sigma}\{(\frac{L^2}{L^{*2}} - 1)l_i l_j + \frac{L\beta}{L^{*2}}(l_i b_j + l_j b_i) + (\frac{\beta^2}{L^{*2}} - 1)b_i b_j\} + \mu M(m_i C_j + m_j C_i) + (B - \mu M^2)m_i m_j = 0.$$

4. The transformation of S-reducible Finsler space by cubic transformation

Prasad, B. N. and Singh, J. N. ([6]) obtained the relation between v-curvature tensors of (F_n, L) and (F_n, L^*) by the transformation

$$L^{*3} = L^3 + \beta^3 \tag{4.1}$$

where $\beta = b_1 y^i$, $b_i(x)$ is a component of a covariant vector which is a function of position alone. They also obtained the angular metric tensor h^*_{ij} of (F_n, L^*) as

$$h^*_{ij} = p\, h_{ij} + 2\, pq\, m_1\, m_j \tag{4.2}$$

where $p = LL^{*-1}$, $q = \beta L^{*-1}$, $m_i = q l_i - p b_i$ and $l_i = \partial_i L$. The contravariant component of fundamental metric tensor g^{*ij} of (F_n, L^*) has been given by

$$g^{*ij} = p^{-1} g^{ij} - pq^3\lambda(p + qb^2)l^i l^j + q^2\lambda(l^i b^j + l^j b^i) - 2pq\lambda b^i b^j \tag{4.3}$$

where $b^i = g^{ij} b_j$, $l^i = g^{ij} l_j$, $b^2 = g^{ij} b_i b_j$
and $\lambda^{-1} = p^3 - q^3 + 2p^2 q b^2$.
The $(h)hv$-torsion tensor C^{*h}_{ij} of (F_n, L^*) has been given by

$$C^{*h}_{ij} = C^h_{ij} + \frac{q^2}{2L}(h^h_i m_j + h^h_j m_i) + pq\lambda(ql^h - 2pb^h)C_{ij} + p^2 q^3(p + qb^2)\frac{\lambda}{2L}h_{ij} l^h + \frac{p\lambda}{L} m_i m_j b^h \tag{4.4}$$

where $C_{ij} = C_{ijk}b^k$. By using the above relation they gave the relation between v-curvature tensors of (F_n, L) and (F_n, L^*) as

$$\begin{aligned} S^*_{hijk} &= PS_{hijk} + C_{ij}d_{hk} + C_{hk}d_{ij} - C_{ik}d_{hj} \\ &\quad - C_{hj}d_{ik} + h_{ij}E_{hk} + h_{hk}E_{ij} - h_{ik}E_{hj} - h_{hj}E_{ik} \end{aligned} \tag{4.5}$$

where

$$d_{hk} = \frac{p^2\lambda}{L}m_h m_k - pq^3 C_{hk} - \frac{p^2q^2\lambda}{4L}h_{hk}$$

and

$$E_{nk} = \frac{pq^2\lambda}{4L^2}[q^2(2p^3+1) - 2p^5b^2]m_h m_k - \frac{pq^4\lambda}{8L^2}(q^2 - p^2b^2)h_{hk} - \frac{p^2q^2\lambda}{4L}C_{hk}$$

In view of equation (1.2), the v-Ricci tensor S^*_{ij} of (F_n, L^*) is obtained with the help of equations (4.3), (4.5) and (1.3) as

$$\begin{aligned} S^*_{ij} &= \Big[\rho(\frac{1}{p}C_m b^m - 2pq\lambda C_{hk}b^h b^k)\frac{p^2q^2\lambda}{4L} \\ &\quad - \{\frac{1}{p}(n-3) - 2pq\lambda h_{hk}b^h b^k\}(q^2 - p^2b^2)(\frac{pq^4\lambda}{8L^2}) \\ &\quad + (\frac{1}{p}E_{hk}g^{hk} - 2pq\lambda E_{hk}b^h b^k)\Big]h_{ij} + \mu C_i C_j \\ &\quad + (\frac{1}{p}C_m b^m - 2pq\lambda C_{hk}b^h b^k)(\frac{p^2\lambda}{L}m_i m_j - pq^3 C_{ij}) \\ &\quad + \{\frac{1}{p}(n-3) - 2pq\lambda h_{hk}b^h b^k\}\Big[\frac{pq^2\lambda}{4L^2}\{q^2(2p^3+1) - 2p^5b^2\} \\ &\quad m_i m_j - \frac{p^2q^2\lambda}{4L}C_{ij}\Big] + \frac{1}{p}(C_{ij}g^{hk}d_{hk} - C^h_{im}b^m d_{hj} \\ &\quad - C^k_{jm}b^m d_{ik}) - 2pq\lambda(pS_{hijk} + C_{ij}d_{hk} - C_{ik}d_{hj} \\ &\quad - C_{hj}d_{ik} - h_{ik}E_{hj} - h_{hj}E_{ik})b^h b^k \end{aligned} \tag{4.6}$$

In view of equations (1.4) and (4.4), we get

$$\begin{aligned} C_iC_j &= C_i^*C_j^* + M(m_iC_j + m_jC_i) \\ &\quad + 2p^2q\lambda(C_iC_{j.} + C_jC_{i.}) - M^2m_im_j \\ &\quad - 2p^2q\lambda M(m_iC_{j.} + m_jC_{i.}) - 4p^4q^2\lambda C_{i.}C_{j.} \end{aligned} \tag{4.7}$$

where

$$M = -\{(p^{-1} - n + 1)\frac{q^2}{2L} - \frac{q^2}{2L} + \frac{p^2\lambda}{L}(b^2 - \beta^2/L^2)\}$$

and

$$C_{i.} = C_{ij}b^j$$

Using equations (4.2) and (4.7), the equation (4.6) can be written as

$$S_{ij}^* = \rho^*h_{ij}^* + \mu^*C_i^*C_j^* + A_{ij}^* \tag{4.8}$$

where

$$\begin{aligned} \rho^* &= \frac{1}{\rho}\Big[\rho - (\frac{1}{p}C_mb^m - 2pq\lambda C_{hk}b^hb^k)\frac{p^2q^2\lambda}{4L} \\ &\quad - \{\frac{1}{p}(n-3) - 2pq\lambda h_{hk}b^hb^k\}(q^2 - p^2b^2)(\frac{pq^4\lambda}{8L^2}) \\ &\quad + (\frac{1}{p}E_{hk}g^{hk} - 2pq\lambda E_{hk}b^hb^k)\Big], \\ \mu^* &= \mu \\ A_{ij}^* &= -2q\Big[\rho - (\frac{1}{p}C_mb^m - 2pq\lambda C_{hk}b^hb^k)\frac{p^2q^2\lambda}{4L} \\ &\quad - \{\frac{1}{p}(n-3) - 2pq\lambda h_{hk}b^hb^k\}(q^2 - p^2b^2)(\frac{pq^4\lambda}{8L^2}) \\ &\quad + (\frac{1}{p}E_{hk}g^{hk} - 2pq\lambda E_{hk}b^hb^k)\Big]m_im_j \\ &\quad + \mu\{M(m_iC_j + m_jC_i) + 2p^2q\lambda(C_iC_{j.} + C_jC_{i.}) \\ &\quad - M^2m_im_j - 2p^2q\lambda M(m_iC_{j.} + m_jC_{i.}) \\ &\quad - 4p^4q^2\lambda^2C_{i.}C_{j.}\} + (\frac{1}{p}C_mb^m - 2pq\lambda C_{hk}b^hb^k) \\ &\quad (\frac{p^2\lambda}{L}m_im_j - pq^3C_{ij}) + \{\frac{(n-3)}{p} - 2pq\lambda h_{hk}b^hb^k\} \\ &\quad \Big[\frac{pq^4\lambda}{4L^2}\{q^2(2p^3+1) - 2p^5b^2\}m_im_j - \frac{p^2q^2\lambda}{4L}C_{ij}\Big] \end{aligned}$$

$$+\frac{1}{p}(C_{ij}g^{hk}d_{hk} - C^h_{im}b^m d_{hj} - C^k_{jm}b^m d_{ik})$$
$$- 2pq\lambda(pS_{hijk} + C_{ij}d_{hk} - C_{ik}d_{hj} - C_{hj}d_{ik}$$
$$- h_{ik}E_{hj} - h_{hj}E_{ik})b^h b^k$$

Therefore, we have the following:

Theorem 4.1. *An S-reducible Finsler space is transformed to an S-reducible Finsler space under the transformation (4.1) if and only if*

$$A^*_{ij} = 0.$$

5. The transformation of S-reducible Finsler space by one Form

Let (F_n, L^*) be a Finsler space obtained from a Finsler space (F_n, L) by the transformation

$$L^{*2}(x,y) = L^2(x,y) + \beta^2 \tag{5.1}$$

where $\beta = b_i(x)y^i$ is of form in (F_n, L). Such a transformation was fiest introduced by M. Matsumoto ([1]). The contravariant component of metric tensor g^{*ij} of (F_n, L^*) has been given as

$$g^{*ij} = g^{ij} - \frac{1}{1+b^2}b^i b^j \tag{5.2}$$

where $b^i = g^{ij}b_j$ and $b^2 = g^{ij}b_ib_j$

We also gave the (h) hv-torsion tensor C^{*i}_{jk} of (F_n, L^*) as

$$C^{*i}_{jk} = C^i_{jk} - \frac{1}{1+b^2}C_{.jk}b^{-1} \tag{5.3}$$

where $C_{.jk} = C_{ijk}b^i$

By using the above relation, he obtained a relation between v-curvature tensors of (F_n, L) and (F_n, L^*) as

$$S^*_{ijkl} = S_{ijkl} + \frac{1}{1+b^2}(C_{.ik}C_{.jl} - C_{.il}C_{.jk}) \tag{5.4}$$

With the help of equations (5.3) and (1.4), we get

$$C_iC_j = C_i^*C_j^* - \frac{C_{..i}C_{..j}}{(1+b^2)^2} + (C_iC_{..j} + C_jC_{..i})/(1+b^2) \tag{5.5}$$

From (5.1), we get the angular metric tensor $h_{ij}^* = L^*\dfrac{\partial^2 L^*}{\partial y^i \partial y^j}$ of (F_n, L^*) as

$$h_{ij}^* = h_{ij} + \frac{\beta^2}{L^{*2}}l_il_j + \frac{L^2}{L^{*2}}b_ib_j + \frac{L\beta}{L^{*2}}(l_ib_j + l_jb_i) \tag{5.6}$$

In view of equations (1.2), (1.3), (5.2) and (5.4), the v-Ricci tensor is obtained as given below:

$$\begin{aligned} S_{ij}^* &= \rho h_{ij} + \mu C_iC_j + \frac{1}{1+b^2}(C_{.j}^kC_{.jk} - C_.C_{.ij}) \\ &\quad - \frac{1}{1+b^2}\left[S_{.ij.} + \frac{1}{1+b^2}(C_{..j}C_{..i} - C_{...}C_{.ij})\right] \end{aligned} \tag{5.7}$$

Using equations (5.5) and (5.6), equation (5.7) can be written as

$$S_{ij}^* \;=\; \rho^* h_{ij}^* + \mu^* C_i^*C_j^* + B_{ij}^* \tag{5.8}$$

where $\rho^* = \rho$ and $\mu^* = \mu$ and

$$\begin{aligned} S_{ij}^* &= \rho\Big\{\frac{\beta^2}{L^{*2}}l_il_j + \frac{L^2}{L^{*2}}b_ib_j + \frac{L\beta}{L^{*2}}(l_ib_j + l_jb_i) \\ &\quad + \mu\frac{C_iC_{..j} + C_jC_{..i}}{1+b^2} - \frac{C_{..i}C_{..j}}{(1+b^2)^2} + \frac{1}{1+b^2}(C_{.j}^kC_{.ik} - C_.C_{.ij} - S_{.ij.}) \\ &\quad - \frac{1}{(1+b^2)^2}(C_{..j}C_{..i} - C_{...}C_{.ij} \end{aligned}$$

Therefore, we have the following:

Theorem 5.1. *An S-reducible Finsler space is transformed to an S-reducible Finsler space under the tranaformation* (5.1) *if and only if*

$$B_{ij}^* \;=\; 0$$

6. The transformation of S-reducible Finsler space by β-change

Let (F_n, L^*) be a Finsler space obtained from a Finsler space (F_n, L) by the transformation ([1])

$$L^*(x,y) \;=\; L(x,y) + \beta(x,y) \tag{6.1}$$

where $\beta = b_i(x)y^i$.

The contravariant component of metric tensor has been given as

$$g^{*ij} \;=\; LL^{*-1}g^{ij} - LL^{*-2}(y^ib^j + y^jb^i) + (Lb^2 + \beta)L^{*-3}y^iy^j \tag{6.2}$$

where $b^i = g^{ij}b_j$ and $b^2 = g^{ij}b_ib_j$

The (h) hv-torsion tensor C^{*i}_{jk} of (F_n, L^*) has been given as

$$\begin{aligned} C^{*j}_{ik} &= C^j_{ik} - \frac{1}{2}L^{*-1}(h^j_i m_k + h^j_k m_i + h_{ik}m^j) \\ &\quad - \{L^{*-1}C_{.ik} - \frac{1}{2}L^{*-2}(mh_{ik} - 2m_im_k)\}y^j \end{aligned} \tag{6.3}$$

where $h_{ij} = g_{ij} - L^{-2}y_iy_j$, $m_i = \beta L^{-2}y_i - b_i$

The v-curvature tensor of (F_n, L^*) has been given as

$$\begin{aligned} S^*_{ijkl} &= L^*L^{-1}S_{ijkl} + \frac{1}{4}m^2(L^*L^{-1})(h_{il}h_{jk} - h_{ik}h_{jl}) \\ &\quad + \frac{1}{2}L^{-1}(h_{il}C_{.jk} - h_{ik}C_{.jl} + h_{jk}C_{.il} - h_{jl}C_{.ik}) \\ &\quad + \frac{1}{4}(L^*L)^{-1}(h_{il}m_jm_k - h_{ik}m_jm_l + h_{jk}m_im_l - h_{jl}m_im_k) \end{aligned} \tag{6.4}$$

where $m^2 = g_{ij}m^im^j$

With the help of equations (1.4) and (6.3), we get

$$C_iC_j = C^*_iC^*_j + (\frac{n+1}{2})L^{*-1}(C_im_j + C_jm_i) - (\frac{n+1}{2})^2L^{*-2}m_im_j \tag{6.5}$$

From (6.1), we have the following relation between angular metric tensor of (F_n, L) and (F_n, L^*)

$$h_{ij} \;=\; (\frac{L}{L+\beta})h^*_{ij} \tag{6.6}$$

In view of equation (1.2) and using also the relations (6.2), (6.4), (1.3), we have

$$S^*_{ij} = (\rho + A)h_{ij} + \mu C_i C_j + BC_{ij} + Db_i b_j \tag{6.7}$$

where

$$A = \frac{1}{4}\{m^2 L^{*-2}(n-2) + \frac{1}{2}L^{-1}C_. + \frac{1}{4}L^{-2}m^2\}$$
$$B = \frac{1}{2}L^{*-1}(n-3) \quad \text{and} \quad D = \frac{1}{4}L^{-2}(n-3)$$

Using equation (6.5), (6.6) and (6.7), the v-Ricci tensor of (F_n, L^*) is given as

$$S^*_{ij} = \rho^* h^*_{ij} + \mu^* C^*_i C^*_j + D^*_{ij} \tag{6.8}$$

where

$$\rho^* = (\frac{\rho + A}{L + \beta})L, \qquad \mu^* = \mu$$

and

$$D^*_{ij} = \frac{\mu}{2}(n+1)L^{*-1}(C_i m_j + C_j m_i) + \{D - \mu(\frac{n+1}{2})^2 L^{*-2}\}m_i m_j + BC_{ij}$$

Therefore, we have the

Theorem 6.1. *An S-reducible Finsler space is transformed to an S-reducible Finsler space under the transformation* (6.1) *if and only if*

$$D^*_{ij} = 0.$$

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