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ON REGULAR SEMI-OPEN SETS AND S*-CLOSED SPACES

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Abstract. In this note, the regular semi-open sets, introduced in [2], are further investigated. Using covers consisting of such sets, a new class of topological spaces, called the s*-closed spaces, is defined and studied.

1. Preliminaries

By a space (X,τ) we mean a topological space on which no separation axiom is assumed. We recall the following definitions, notational conventions and characterizations. The closure (interior) of a subset A of X is denoted by ClA (resp. Int$A$). A is called regular open (regular closed) iff $A = \text{Int}ClA$ (resp. $A = Cl\text{Int}A$). The family of all regular open (regular closed) subsets of (X,τ) is denoted by $RO(X,\tau)$ (resp. $RC(X,\tau)$). A set A is said to be semi$-$ ($\alpha-$, pre$-$, $\beta-$) open subset of (X,τ) iff $A \subset Cl\text{Int}A$ [6] (resp. $A \subset \text{Int}Cl\text{Int}A$ [9], $A \subset \text{Int}ClA$ [8], $A \subset Cl\text{Int}ClA$ [1]). The complement of each semi$-$ (resp. $\alpha-$, pre$-$, $\beta-$) open set is called a semi$-$ (resp. $\alpha-$, pre$-$,$\beta-$) closed set. The family of all semi$-$open ($\alpha-$open, pre$-$open, $\beta-$open, semi$-$closed, $\alpha-$closed, pre$-$closed, $\beta-$closed) subsets of (X,τ) is denoted by $SO(X,\tau)$ (resp. $\alpha O(X,\tau)$, $PO(X,\tau)$, $\beta O(X,\tau)$, $SC(X,\tau)$, $\alpha C(X,\tau)$, $PC(X,\tau)$, $\beta C(X,\tau)$). The semi$-$closure (semi$-$interior) of a set A, denoted by $sClA$ (resp. $s\text{Int}A$), is defined in a natural way [3], $A \in SC(X,\tau)$ iff $sClA = A$ and $A \in SO(X,\tau)$ iff

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$s\mathrm{Int}A = A$. It is known that $sClA = X \setminus s\mathrm{Int}(X \setminus A)$ [3], $sClA = A \cup \mathrm{Int}ClA$ and $s\mathrm{Int}A = A \cap Cl\mathrm{Int}A$ [15]. A space (X, τ) is extremally disconnected iff $ClG \in \tau$ for every $G \in \tau$. A space (X, τ) is said to be quasi $H-$closed [12] (nearly compact [14], $s-$closed [16]) iff for every cover $\{V_\alpha : a \in \Delta\}$ of X such that $V_\alpha \in \tau$ (resp. $V_a \in \tau$, $V_a \in SO(X, \tau)$) for all $a \in \Delta$, there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{a \in \Delta_0} ClV_a$ (resp. $X = \bigcup_{\alpha \in \Delta_0} \mathrm{Int}ClV_a$, $X = \bigcup_{a \in \Delta_0} ClV_a$). A space (X, τ) is called almost regular [13] ($s-$regular [7]) iff for each $G \in RO(X, \tau)$ (resp. $G \in \tau$) containing a point $x \in X$ there exists $U \in \tau$ (resp. $U \in SO(X, \tau)$) such that $x \in U \subset ClU \subset G$ (resp. $x \in U \subset sClU \subset G$).

2 Regular Semi–Open Sets

Definition 2.1. A subset A of a space (X, τ) is said to be a regular semi–open set [2] iff $A = s\mathrm{Int}sClA$. We shall denote the class of all regular semi–open subsets of a space (X, τ) by $RSO(X, \tau)$. It is clear that $RSO(X, \tau) \subset SO(X, \tau)$.

Lemma 2.1. *For any subset A of a space (X, τ).*

$$\mathrm{Int}ClA \subset s\mathrm{Int}sClA \subset Cl\mathrm{Int}ClA$$

Proof. Straightforward.

Lemma 2.2. *Let (X, τ) be any space. Then the following statements are equivalent.*

(i) $A \in RSO(X, \tau)$.

(ii) $X \setminus A \in RSO(X, \tau)$.

(iii) $A = sCls\mathrm{Int}A$.

(iv) $A \in SO(X, \tau) \cap SC(X, \tau)$.

(v) *there exists $U \in RO(X, \tau)$ such that $U \subset A \subset ClU$.*

Proof. (i) $\rightarrow$ (ii) Let $A \in RSO(X, \tau)$. Then $A = s\mathrm{Int}sClA$. By lemma 2.1, we deduce that $\mathrm{Int}ClA \subset A$, i.e. $A \in SC(X, \tau)$ and hence $X \setminus A = s\mathrm{Int}sCl(X \setminus A)$.

Consequently, $X \setminus A \in RSO(X,\tau)$.

(ii) $\rightarrow$ (iii). Obvious.

(iii) $\rightarrow$ (iv). Obvious.

(iv) $\rightarrow$ (v). Let $A \in SO(X,\tau) \cap SC(X,\tau)$. Then $\mathrm{Int}Cl A \subset A \subset Cl\mathrm{Int}A$. Taking $U = \mathrm{Int}ClA \in RO(X,\tau)$, we get

$$U \subset A \text{ and } A \subset Cl\mathrm{Int}A \subset Cl\mathrm{Int}ClA = ClU,$$

i.e. there exists $U \in RO(X,\tau)$ such that $U \subset A \subset ClU$.

(v)$\rightarrow$(i). Let $U \in RO(X,\tau)$ such that $U \subset A \subset ClA$. Then $ClU = ClA$, $U = \mathrm{Int}ClU = \mathrm{Int}ClA$ and so $\mathrm{Int}ClA \subset A$. Now, since $U \subset \mathrm{Int}A$ and $\mathrm{Int}A \subset \mathrm{Int}ClA = U$, so $U = \mathrm{Int}A$ and $ClU = Cl\mathrm{Int}A$, which implies that $A \subset Cl\mathrm{Int}A$. Accordingly,

$$\begin{aligned} s\mathrm{Int}sClA &= sClA \cap Cl\mathrm{Int}sClA \\ &= (A \cup \mathrm{Int}ClA) \cap Cl\mathrm{Int}(A \cup \mathrm{Int}ClA) \\ &= A \cap Cl\mathrm{Int}A = A, \end{aligned}$$

i.e. $A \in RSO(X,\tau)$. This completes the proof of the lemma.

Lemma 2.3. *For any space* (X,τ),

$$RO(X,\tau) \cup RC(X,\tau) \subset RSO(X,\tau).$$

Proof. If $A \in RO(X,\tau)$ or $A \in RC(X,\tau)$, then $A \in SO(X,\tau)$ and $A \in SC(X,\tau)$, which implies that $A \in RSO(X,\tau)$ by lemma 2.2.

Example 2.1. The inclusion relation in lemma 2.3 ,in general, cannot be replaced by equality. As an example let $X = \{a,b,c,d,\}$, $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$. Then $\{a,c\} \in RSO(X,\tau)$ but $\{a,c\} \notin RO(X,\tau) \cup RC(X,\tau)$.

We have the following diagram of implications, and any other, except those resulting by transitivity, can not be added, in general.

$$\begin{array}{ccccc} A \in RO(X,\tau) & \longrightarrow & A \in RSO(X,\tau) & \longleftarrow & A \in RC(X,\tau) \\ \downarrow & & \downarrow & & \\ A \in \tau \longrightarrow A \in \alpha O(X,\tau) & \longrightarrow & A \in SO(X,\tau) & & \\ \downarrow & & \downarrow & & \\ A \in PO(X,\tau) & \longrightarrow & A \in \beta O(X,\tau) & & \end{array}$$

As a sample we give the following example.

Example 2.2 Example 2.1 shows that

(i) τ and $RSO(X,\tau)$ may be not comparable, in general.

(ii) $\alpha O(X,\tau)$ and $RSO(X,\tau)$ may be not comparable, in general.

(iii) $RO(X,\tau) \neq RSO(X,\tau) \neq RC(X,\tau)$, in general.

(iv) $RSO(X,\tau)$, may be neither supratopology nor infratopology [5] on X, in general.

Lemma 2.4. *For any space (X,τ), if $A \in RSO(X,\tau)$ and $A \subset B \subset ClA$, then $B \in RSO(X,\tau)$.*

Proof. By lemma 2.2, there exists $U \in RO(X,\tau)$ such that $U \subset A \subset ClU$. Hence, $U \subset B \subset ClU$ and $B \in RSO(X,\tau)$ by using lemma 2.2 again.

Lemma 2.5. *For any space (X,τ), $A \in RSO(X,\tau)$ iff there exist an open set G and a closed set F such that*

$$\text{Int}F \subset G \subset A \subset F \subset ClG$$

Proof. Straightforward.

Lemma 2.6. *For any space (X,τ),*

$$PO(X,\tau) \cap RSO(X,\tau) = RO(X,\tau)$$

Proof. Suppose that $A \in PO(X,\tau)$ and $A \in RSO(X,\tau)$. Then $A \subset \mathrm{Int}ClA$ and $A = s\mathrm{Int}sClA$. Therefore,

$$A = s\mathrm{Int}(A \cup \mathrm{Int}ClA) \;=\; s\mathrm{Int}\mathrm{Int}ClA = \mathrm{Int}ClA,$$

i.e. $A \in RO(X,\tau)$. The reverse implication follows directly from the above diagram.

Corollary 2.1. *For any space* (X,τ),

$$PC(X,\tau) \cap RSO(X,\tau) = RC(X,\tau)$$

Proof. Obvious.

Remarks. Lemma 2.6 and corollary 2.1 may be not true in general even if $A \in \beta O(X,\tau)$ (resp. $A \in \beta C(X,\tau)$) instead of $A \in PO(X,\tau)$ (resp. $A \in PC(X,\tau)$). In example 2.1, $\{a,c\} \in \beta O(X,\tau) \cap RSO(X,\tau)$, but $\{a,c\} \notin RO(X,\tau)$.

Lemma 2.7. *For each subset A of a space (X,τ), the sets* $s\mathrm{Int}sClA$, $\mathrm{Int}sClA$, $s\mathrm{Int}ClA$ *and* $\mathrm{Int}ClA$ *are rgular semi–open sets.*

Proof. We shall prove that $s\mathrm{Int}sClA \in RSO(X,\tau)$ and the proof of the other parts is then obvious. We have by lemma 2.1

$$\mathrm{Int}ClA \subset s\mathrm{Int}sClA \subset Cl\mathrm{Int}ClA,$$

where $\mathrm{Int}ClA \in RO(X,\tau)$. The result follows directly by using lemma 2.2.

Corollary 2.2. *The closure and the semi–closure of a semi–open subset A of a space (X,τ) are regular semi–open.*

Proof. Since $A \in SO(X,\tau)$, so $ClA = Cl\mathrm{Int}A$ and $sClA \subset Cl\mathrm{Int}sClA$. Therefore, ClA, $sClA \in SO(X,\tau)$. But ClA, $sClA \in SC(X,\tau)$, hence the result follows by lemma 2.2.

Corollary 2.3. *For each subset A of a space (X,τ), the sets* $Cl\mathrm{Int}A$, $Cls\mathrm{Int}A$, *s*$Cl\mathrm{Int}A$ *and* s$Cls\mathrm{Int}A$ *are regular semi–open.*

Proof. This follows directly by corollary 2.2 and by observing that $\mathrm{Int}A$, $s\mathrm{Int}A \in SO(X,\tau)$.

Lemma 2.8. *For any space (X,τ), if $Y \in \alpha O(X,\tau)$ and $A \in RSO(X,\tau)$ then $A \cap Y \in RSO(Y,\tau_Y)$.*

Proof. Since $A \in RSO(X,\tau)$, so $A \in SO(X,\tau)$ and $X \setminus A \in SO(X,\tau)$. Therefore, $A \cap Y \in SO(Y,\tau_Y)$ and $(X \setminus A) \cap Y \in SO(Y,\tau_Y)$ [9]. But $(X \setminus A) \cap Y = Y \setminus (A \cap Y)$ and hence $Y \setminus (A \cap Y) \in SO(Y,\tau_Y)$. Consequently, $A \cap Y \in RSO(Y,\tau_Y)$.

Lemma 2.8 may be not true, in general, even if $Y \in RSO(X,\tau)$, as the following example shows.

Example 2.3. Taking $Y = \{a,c,d\}$ and $A = \{b,c,d\}$ in example 2.1, then $A \cap Y \notin RSO(Y,\tau_y)$.

Lemma 2.9. *For any space (X,τ), if $Y \in RO(X,\tau)$ and $A \in RSO(Y,\tau_y)$, then $A \in RSO(X,\tau)$.*

Proof. By lemma 2.2, we have $A \in SO(Y,\tau_y)$ and $Y \setminus A \in SO(Y,\tau_y)$. This implies that $A \in SO(X,\tau)$ and $Y \setminus A \in SO(X,\tau)$ [10]. Since Y is regular open in X, so it is regular semi–open in X and hence $X \setminus Y \in SO(X,\tau)$. Therefore, $(Y \setminus A) \cup (X \setminus Y) = X \setminus A \in SO(X,\tau)$. Accordingly, $A \in SC(X,\tau)$. By lemma 2.2, $A \in RSO(X,\tau)$.

3. S^*–Closed Spaces

Definition 3.1. A filterbase $\mathcal{F}$ in a space (X,τ) s^*–converges to a point $x_0 \in X$ iff for each $A \in RSO(X,\tau)$ such that $x_0 \in A$, there exists an $F \in \mathcal{F}$ such that $F \subset A$.

Definition 3.2. A filterbase $\mathcal{F}$ in a space (X,τ) s^*-accumulates to $x_0 \in X$ iff for each $A \in RSO(X,\tau)$ such that $x_0 \in A$ and each $F \in \mathcal{F}$, $F \cap A \neq \phi$.

The following lemma is an easy consequence of the above definitions.

Lemma 3.1. *If $\mathcal{F}$ is a maximal filterbase in a space (X,τ), then $\mathcal{F}$ s^*-accu-nmulates to $x_0 \in X$ iff $\mathcal{F}$ s^*-converges to x_0.*

Theorem 3.1. *A filterbase $\mathcal{F}$ in a space (X,τ) s^*-converges to $x_0 \in X$ iff for each $A \in SO(X,\tau)$ such that $x_0 \in A$, there exists an $F \in \mathcal{F}$ such that $F \subset sClA$.*

Proof. (Necessity). Suppose that $\mathcal{F}$ s^*-converges to $x_0 \in X$ and $A \in SO(X,\tau)$ such that $x_0 \in A$. By corollary 2.2, $sClA \in RSO(X,\tau)$ and $x_0 \in sClA$. Therefore, there exists an $F \in \mathcal{F}$ such that $F \subset sClA$.

(Sufficiency). Let the condition be satisfied and let $A \in RSO(X,\tau)$ such that $x_0 \in A$. Then $A \in SO(X,\tau)$ and therefore, there exists an $F \in \mathcal{F}$ such that $F \subset sClA$. By lemma 2.2, $A \in SC(X,\tau)$. Consequently, $sClA = A$ and the proof is complete.

The following theorem can be proved similarly.

Theorem 3.2. *A filterbase $\mathcal{F}$ in a space (X,τ) s^*-accumulates to $x_0 \in X$ iff for each $A \in SO(X,\tau)$ such taht $x_0 \in A$ and each $F \in \mathcal{F}$, $F \cap sClA \neq \phi$.*

Definition 3.3. A space (X,τ) is said to be s^*-closed space iff each regular semi-open cover of X has a finite subcover.

Theorem 3.3. *Each s^*-closed space is nearly compact and $s-$closed.*

Proof. Obvious.

It is known that the one point compactification of a finite discrete space is not $s-$closed [16], and by theorem 3.3, is therefore not s^*-closed. But it is nearly compact.

Theorem 3.4. *A space (X,τ) is s^*-closed iff for each semi-open cover*

$\{A_a : a \in \Delta\}$ of X, there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{a\in\Delta_0} sClA_a$.

Proof. **(Necessity).** Let (X,τ) be s^*−closed and $\{A_a : a \in \Delta\}$ be a semi−open cover of X. By corollary 2.2, we have $sClA_a \in RSO(X,\tau)$ for every $a \in \Delta$ and $X = \bigcup_{a\in\Delta} sClA_a$. Therefore, there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{a\in\Delta_0} sClA_a$.

(Sufficiency). Let $\{A_a : a \in \Delta\}$ be any regular semi−open cover of X. Then, by lemma 2.2, $A_a \in SO(X,\tau)$ and $A_a \in SC(X,\tau)$, for each $a \in \Delta$. Therefore, by the hypothesis, there exists a finite subset Δ_0 of Δ such that

$$X = \bigcup_{a\in\Delta_0} sClA_a = \bigcup_{a\in\delta_0} A_a,$$

which shows that (X,τ) is s^*−closed.

Theorem 3.5. *For any space (X,τ), the following statements are equivalent.*

(i) *(X,τ) is s^*−closed.*

(ii) *For each semi−open cover $\{A_a : a \in \Delta\}$ of X, there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{a\in\Delta_0} sClA_a$.*

(iii) *For each family $\{F_a : a \in \Delta\}$ of semi−closed subsets of X such that $\bigcap_{a\in\Delta} F_a = \phi$, there exists a finite subset Δ_0 of Δ such that $\bigcap_{a\in\Delta_0} F_a = \phi$.*

(iv) *for each family $\{F_a : a \in \Delta\}$ of semi−closed subsets of X, if $\bigcap_{a\in\Delta_0} s\text{Int}F_a \neq \phi$ for every finite subset Δ_0 of Δ, then $\bigcap_{a\in\Delta} F_a \neq \phi$.*

(v) *for each family $\{A_a : a \in \Delta\}$ of regular semi−open subsets of X such that $\bigcap_{a\in\Delta} A_a = \phi$, there exists a finite subset Δ_0 of Δ such that $\bigcap_{a\in\Delta_0} A_a = \phi$.*

(vi) *for each family $\{A_a : a \in \Delta\}$ of regular semi−open subsets of X, if $\bigcap_{a\in\Delta_0} A_a \neq \phi$ for every finite subset Δ_0 of Δ, then $\bigcap_{a\in\Delta} A_a \neq \phi$.*

(vii) *Every filterbase $\mathcal{F}$ in X s^*−accumulates to some point $x_0 \in X$.*

(viii) *Every maximal filterbase $\mathcal{F}$ in X s^*−converges to some point $x_0 \in X$.*

Proof. By theorem 3.4 we have (i)$\leftrightarrow$(ii) and the equivalencies (iii)$\leftrightarrow$(iv) and (v)$\leftrightarrow$(vi) are obvious.

(ii)$\to$(iii). Let $\{F_a : a \in \Delta\}$ be a family of semi-closed subsets of X such that $\bigcap_{a\in\Delta} F_a = \phi$. Therefore, $X = \bigcup_{a\in\Delta} (X\setminus F_a)$, where $X \setminus F_a \in SO(X,\tau)$ for all $a \in \Delta$. By (ii), there exists a finite subset Δ_0 of Δ such that

$$X = \bigcup_{a\in\Delta_0} sCl(X \setminus F_a) = \bigcup_{a\in\Delta_0} (X\setminus s\mathrm{Int}F_a) = X\setminus(\bigcap_{a\in\Delta_0} s\mathrm{Int}F_a),$$

which implies that $\bigcap_{a\in\Delta_0} s\mathrm{Int}F_a = \phi$.

(iii)$\to$(v). Let $\{A_a : a \in \Delta\}$ be a family of regular semi-open subsets of X such that $\bigcap_{a\in\Delta} A_a = \phi$. By lemma 2.2, $A_a \in SO(X,\tau) \cap SC(X,\tau)$ for all $a \in \Delta$. Using (iii), there exists a finite subset Δ_0 of Δ such that $\bigcap_{a\in\Delta_0} s\mathrm{Int}A_a = \phi = \bigcap_{a\in\Delta_0} A_a$.

(v)$\to$(i). Let $\{A_a : a \in \Delta\}$ be any regular semi-open cover of X. Therefore, $\bigcap_{a\in\Delta} (X \setminus A_a) = \phi$ and by lemma 2.2, $X \setminus A_a \in RSO(X,\tau)$ for all $a \in \Delta$. Using (v), there exists a finite subset Δ_0 of Δ such that $\bigcap_{a\in\Delta_0} (X \setminus A_a) = \phi$. This implies that $\bigcup_{a\in\Delta_0} A_a = X$ and X is s^*-closed.

(i)$\to$(viii). Suppose that $\mathcal{F} = \{F_a : a \in \Delta\}$ is a maximal filterbase in X which does not s^*-converge to any point in X. Therefore, by lemma 3.1, $\mathcal{F}$ does not s^*-accumulate to any point in X. This implies that for every $x \in X$, there exist $A(x) \in RSO(X,\tau)$ containing x and $F_{a(x)} \in \mathcal{F}$ such that $F_{a(x)} \cap A(x) = \phi$. Hence, the family $\{A(x) : x \in X\}$ is a regular semi-open cover of X and by the hypothesis, there is a finite subfamily $\{A(x_i) : i = 1,2,\ldots,n\}$ such that $X = \bigcup_{i=1}^{n} A(x_i)$. Since $\mathcal{F}$ is a filterbase in X, there exists an $F_0 \in \mathcal{F}$ such that $F_0 \subset \bigcap_{i=1}^{n} F_{a(x_i)}$. Accordingly, $F_0 \cap A(x_i) = \phi$ for all $i \in \{1,2,\ldots,n\}$. This implies that

$$\phi = F_0 \cap (\bigcup_{i=1}^{n} A(x_i)) = F_0 \cap X,$$

i.e. $F_0 = \phi$. This is a contradiction and consequently, $\mathcal{F}$ must s^*−converges to some point $x_0 \in X$.

(viii)→(vii). Follows directly from lemma 3.1 and the fact that every filterbase is contained in a maximal filterbase.

(vii)→(v). Let $\{A_a : a \in \Delta\}$ be a family of regular semi−open subsets of X such that $\bigcap_{a\in\Delta} A_a = \phi$. Suppose that for every finite subfamily $\{A_{a_i} : i = 1, 2, \ldots, n\}$, $\bigcap_{i=1}^{n} A_{a_i} \neq \phi$. Therefore,

$$\mathcal{F} = \{\bigcap_{i=1}^{n} A_{a_i} : n \in N, \quad a_i \in \Delta\}$$

forms a filter base in X. Using (vii), $\mathcal{F}$ s^*−accumulates to some point $x_0 \in X$. This implies that, for every $A(x_0) \in RSO(X,\tau)$ containing x_0, $F \cap A(x_0) \neq \phi$ for every $F \in \mathcal{F}$. Since $\bigcap_{F\in\mathcal{F}} F = \phi$, there exists $F_0 \in \mathcal{F}$ such that $x_0 \notin F_0$, which implies that there exists $a_0 \in \Delta$ such that $x_0 \notin A_{a_0}$. Accordingly, $x_0 \in X \setminus A_{a_0}$ and $X \setminus A_{a_0} \in RSO(X,\tau)$ by using lemma 2.2. Therefore, there exists $a_0 \in \Delta$ such that $F_0 \cap (X \setminus A_{a_0}) = \phi$ contradicting the fact that $\mathcal{F}$ s^*−accumulates to x_0. This completes the proof.

Theorem 3.6. *Each s−regular and s^*−closed space is compact.*

Proof. Let $\{G_a : a \in \Delta\}$ be any open cover of an s−regular and s^*−closed space (X,τ). Then for each $x \in X$, there exists an $a(x) \in \Delta$ such that $x \in G_{a(x)}$. Since (X,τ) is s−regular, there exists $A(x) \in SO(X,\tau)$ such that $x \in A(x) \subset sClA(x) \subset G_{a(x)}$. Therefore, the family $\{sClA(x) : x \in X\}$ is a regular semi−open cover of (X,τ) (by using corollary 2.2). Since (X,τ) is s^*−closed, there exists a finite subfamily $\{A(x_i) : i = 1, 2, \ldots, n\}$ such that $X = \bigcup_{i=1}^{n} sClA(x_i) \subset \bigcup_{i=1}^{n} G_{a(x_i)}$, which completes the proof.

The following example shows that the condition of s−regularity in theorem 3.6 can not be dropped.

Example 3.1. Let $X = (0,1)$ with the topology τ consisting of X, ϕ and all subsets of X of the form $(0, 1 - 1/n)$, where $n = 2, 3, \ldots$. Then (X, τ) is neither s−regular nor compact, but it is s^*−closed because the only non−empty regular semi−open set in (X, τ) is X itself.

Using theorem 3.4 and lemma 4.1 of [11], we get the following result.

Corollary 3.1. *Each extremally disconnected s−closed space is s^*−closed.*

Using corollary 3.1 and theorem 3.4 of [4], we get the following reslut.

Corollary 3.2. *If a spase (X, τ) is nearly compact (or quasi H−closed) and extremally disconnected, then (X, τ) is s^*−closed.*

Using corollary 3.1 and theorem 3.5 of [4], we get the following result.

Corollary 3.3. *Each almost regular and s−closed space is s^*−closed.*

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