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CORRIGENDUM TO f-CONSERVATIVE MATRIX SEQUENCES

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The purpose of this short note is to add the relevant terminology on some spaces of double sequences and is therefore to rectify [1] of Başar.

Quite recently, the class $(\lambda : \mu)$ has been characterized by Başar in [1], for a matrix sequence $\mathcal{A} = (A_p)$ by setting $\lambda, \mu = f$ or fs; where f and fs denote the spaces of almost convergent single sequences and series, respectively. Since the $\mathcal{A}$-transform of every $x \in f$ or fs is the double sequence $\mathcal{A}x = ((Ax)_n^p)$, we require to define the space μ, appearing in each class $(\lambda : \mu)$ of [1], such that $\mu \subset W$; where W denotes the linear space of all real double sequences. So, we shall define the subspaces F and Fs of W, via uniform f-column limits of an infinite matrix $X = (x_{ij})$, $(i, j = 0, 1, \ldots)$, as follows:

$$F = \{(x_{ij}) \in W : \lim_q (\sum_{k=0}^{q-1} x_{i+k,j})/q = b \text{ uniformly in } i, j \text{ for some } b\}$$

and

$$Fs = \{(x_{ij} \in W : \lim_q (\sum_{k=0}^{q-1} y_{i+k,j})/q = b \text{ uniformly in } i, j \text{ for some } b\},$$

where $y_{ij} = \Sigma_{k=0}^{i} x_{kj}$. Let us also denote the spaces derived in the case $b = 0$ form the spaces F and Fs, by F_0 and F_0s respectively. We should remark to the reader here that the concept of almost convergence regarded in above spaces is more different than the almost convergence of double sequences introduced by Móricz-Rhoades in [2]. It is trivial in the case $x_{ij} = x_i$ for all j that the

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above spaces of double sequences reduce to the corresponding spaces of single. sequences.

At this stage, Lemmas B and D of [1] attain the following statements:

Lemma B. *Given* $\mathcal{A} = (A_p^n)$. *THen* $\mathcal{A} \subset (f : c)$ *iff* $\mathcal{A} \subset (c : C)$, *and (2.1) holds; where* C *denotes the linear space of double sequences such that* $\lim_1 x_{ij}$ *exists uniformly in* j.

Lemma D. *Let* $(u_{ij}) \in W$ *with* $s_{ij} = \Sigma_{k=0}^{i} u_{kj}$ *(equivalently* $u_{0j} = s_{0j}$ *and* $u_{ij} = s_{ij} - s_{i-1,j}$ *for* $i \geq 1$) *for all* i, j. *Then the transformation* $g : Fs \to F$ *defined by* $g(u_{ij}) = (s_{ij})$, *is a linear isomorphism.*

Above lemma renders that $(s_{ij}) \in F$ whenever $(u_{ij}) \in Fs$, and conversely.

Finally, we express our thanks to the referee for his valuable comments which has been useful for us to prepare the present note, at the completion to the report on "Certain matrix sequences on $c(p)$ and $cs(p)$" submitted to Mathematical Japonica (2828).

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f-CONSERVATIVE MATRIX SEQUENCES

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Abstract. The main purpose of this paper is to determine the necessary and sufficint conditions on the matrix sequence $\mathcal{A} = (A_p)$ in order that $\mathcal{A}$ contained in one of the classes $(f : f), (f : fs), (fs : f)$ and $(fs : fs)$, where f and fs respectively denote the spaces of all almost convergent real sequences and series. Our results are more general than those of Duran [3] and Solak [7]. Additionally, theorems of Steinhaus type concerning some subclasses of above matrix classes, are also given.

I. Introduction

In this paper m, c, c_0 and bs have their usual meanings. The shift operator S is defined on m by $(Sx)_n = x_{n+1}$. A Banach limit L is defined on m, as a non-negative linear functional, such that $L(Sx) = L(x)$ and $L(e) = 1$, ([1],p.32), where $e = (1, 1, \cdots)$. A sequence $x \in m$ is said to be almost convergent to the generalized limit x_0 if all Banach limits of x is x_0 [5], and denoted by f-lim $x = x_0$. It is proved by Lorentz [5] that f-lim $x = x_0$ if and only if $\lim_p(x_n + \ldots + x_{n+p-1})/p = x_0$ uniformly in n. It is well-know that a convergent sequece is almost convergent such that its limit and its generalized limit are equal. Given an infinite series Σa_n, it is said to be almost convergent if its sequence of partial sums is almost convergent. By f and fs, we denote the spaces of all almost convergent real sequences and series, respectively.

Let $A = (a_{nk})$ be an infinite matrix of real numbers a_{nk} $(n, k = 0, 1, \cdots)$ and λ, μ two non-empty subsets of the space s of all real sequences. We say that the matrix A defines a transformation from λ into μ, if for every sequence $x = (x_k) \in \lambda$ the sequence $Ax = ((Ax)_n)$ exists and is in μ, where $(Ax)_n = \sum_k a_{nk}x_k$. For simplicity in notation, here and after we write $\sum_k$ instead of $\sum_{k=0}^{\infty}$. By $(\lambda : \mu)$, we denote the class of all such matrices. If there is some notion of limit or sum in λ and μ, then we write $(\lambda : \mu; P)$ to denote the subclass of $(\lambda : \mu)$ which preserve the limit or sum. We say that $A \in (\lambda : \mu)$ is f-multiplicative r if f-lim $Ax = r(f-\text{lim } x)$ for all $x \in \lambda$, where $\lambda, \mu = f$. In conformance with the nature of the space fs and the class $(\lambda : \mu)$, it will be, of course, convenient to restate this definition in the cases of $\lambda, \mu(\lambda \text{ or } \mu) = fs$. We also denote the class of all such matrices by $(\lambda : \mu)_r$. It is evident in the case $r = 1$ that the class $(\lambda : \mu)_r$ coincides

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with the class $(\lambda:\mu;P)$ and hence the set inclusions $(\lambda:\mu;P)\subset(\lambda:\mu)_r\subset(\lambda:\mu)$ are immediate.

Let $\mathcal{A}$ denote the sequence of real matrices $A_p=(a_{nk}(p))$. We write for a sequence $x=(x_k)$, $(\mathcal{A}x)_n^p=\sum_k a_{nk}(p)x_k$ if it exists for each n,p and $\mathcal{A}x=((\mathcal{A}x)_n^p)_{n,p=0}^{\infty}$. A sequence x is said to be $\mathcal{A}$-summable to x_0 if $\lim_n(\mathcal{A}x)_n^p=x_0$ uniformly in p. To denote the matrix sequence $\mathcal{A}$ contained in the class $(\lambda:\mu)$, we write $\mathcal{A}\subset(\lambda:\mu)$. If $a_{nk}(p)=a_{nk}$ for all p, then $\mathcal{A}$ is reduced to the usual summability method A and $a_{nk}(p)=1$ $(n=k)$ for all p, $=0$ $(n\neq k)$ for all p, then $\mathcal{A}$ corresponds to the identity matrix I which is equivalent to the ordinary convergence. Similarly, the method f which is equivalent to the almost convergence introduced by Lorentz [5], the almost sumability method introduced by King [4], etc,. can be defined by this new method $\mathcal{A}$. So, the method $\mathcal{A}$ is more general and more comprehensive than the usual summability method A.

By the f-conservativity of any summability method, we mean the method belonging to one of the classes $(f:f),(f:fs),(fs:f)$ or $(fs:fs)$. The object of this study is to characerize the matrix sequences contained in the f-conservative matrix classes and in this way to fill up some gaps in the existing literature.

II. Matrix sequences from f into f and fs

In this section, we give necessary and sufficient conditions on the matrix sequence $\mathcal{A}=(A_p)$ in order that $\mathcal{A}\subset(f:f),(f:fs)$.

We start with following two lemmas which require in the proof of Theorem 2.1. The first one is due to Stieglitz [8], and the other one is obtained from Folgerung 8 of Stieglitz [8] with $\mathcal{A}=(A_p^n)$.

Lemma A. *Given $\mathcal{B}=(B_p)$. Then the following three statements are equivalent:*
(a) *$\mathcal{B}x$ exists for all $x\in m$,*
(b) *$\mathcal{B}x$ exists for all $x\in c_0$,*
(c) $\sum_k |b_{nk}(p)|<\infty,(n,p)$.

Lemma B. *Given $\mathcal{A}=(A_p^n)$. Then $\mathcal{A}\subset(f:c)$, and only if $\mathcal{A}\subset(c:c)$, and*

$$\lim_q\sum_k|\Delta[a_{qk}^n(p)-a_k]|=0 \tag{2.1}$$

uniformly in n,p; where $a_k=\lim_q a_{qk}^n(p)$ uniformly in n,p for each k and $\Delta[a_{qk}^n(p)-a_k]=a_{qk}^n(p)-a_k-[a_{q,k+1}^n(p)-a_{k+1}]$.

Theorem 2.1. *$\mathcal{A}\subset(f:f)$ if and only if, all limits being uniform in p,*

$$\sup_{n,p}\sum_k|a_{nk}(p)|<\infty, \tag{2.2}$$

$$f-\lim a_{nk}(p)=a_k \text{ for each } k, \tag{2.3}$$

$$f-\lim \sum_k a_{nk}(p) = a, \tag{2.4}$$

$$\lim_q \sum_k \frac{1}{q+1} \mid \sum_{i=0}^{q} \Delta[a_{n+i,k}(p) - a_k] \mid = 0 \text{ uniformly in } n. \tag{2.5}$$

Proof. Necessity. Let $\mathcal{A} \subset (f : f)$ and $x \in f$. Since $f \subset m$, the necessity of (2.2) is immediate by Lemma A. The necessities of (2.3) and (2.4) are obvious, since $e^k, e \in f$, where e^k is the sequence whose only non-zero term is 1 in the k^{th} place.

To prove the necessity of (2.5), we define the double sequence $\mathcal{B} = (B_p^n)$ of infinite matrices such that

$$b_{qk}^n(p) = \frac{1}{q+1} \sum_{i=0}^{q} a_{n+i,k}(p). \tag{2.6}$$

Let $S_n = \frac{1}{n+1} \sum_{i=0}^{n} S^i$, where S^i denotes the composition of the shift operator with itself i times. Then $\lim_q (Bx)_{nq}^p$ exists uniformly in n, p for all $x \in f$, since $(Bx)_{nq}^p = S_q(Ax)_n^p$. Hence, $\mathcal{B} \subset (f : c)$ and thus we get by (2.1) that

$$\lim_q \sum_k \mid \Delta[b_{qk}^n(p) - b_k] \mid = 0 \text{ uniformly in } n, p$$

which is equivalent to (2.5).

Sufficiency. Suppose the conditions (2.2)-(2.5) hold and $x \in f$. Then, one can easily observe that $B_p^n = (b_{qk}^n(p))$ in (2.6), satisfies the conditions of Lemma B and thus we have $\mathcal{B} \subset (f : c)$. This implies the existence of $lim_q\ S_q(Ax)_n^p$ uniformly in n, p which completes the proof.

As an immediate consequence of Theorem 2.1, we have

Corollary 2.2 **(a)** $\mathcal{A} \subset (f : f)_r$ *if and only if* (2.2) *holds,* (2.3) *and* (2.5) *hold with* $a_k = 0$ *for each* k *and* (2.4) *also holds with* $a = r$.

(b) $\mathcal{A} \subset (f : f_0)$ *if and only if* (2.2) *holds,* (2.3) *and* (2.5) *hold with* $a_k = 0$ *for each* k *and* (2.4) *also holds with* $a = 0$, *where* f_0 *denotes the space of all sequences which are almost convergent to zero.*

Now, we can give a theorem of Steinhaus type. For this, we need the following lemma due to Başar-Solak [2]:

Lemma C. $\mathcal{A} \subset (m : f)$ *if and only if* (2.2), (2.3) *hold, and*

$$\lim_q \sum_k \frac{1}{q+1} \mid \sum_{i=0}^{q} a_{n+i,k}(p) - a_k \mid = 0 \text{ uniformly in } n, p. \tag{2.7}$$

Theorem 2.3. *The classes* $(m : f)$ *and* $(f : f)_r$ *are disjoint.*

Proof. Suppose that the converse of this is true and $\mathcal{A} \subset (m : f) \cap (f : f)_r$. Then, by combining (2.7) and (2.3) of Corollary 2.2(a), we have

$$\lim_q \sum_k \frac{1}{q+1} \mid \sum_{i=0}^{q} a_{n+i,k}(p) \mid = 0 \text{ uniformly in } n, p$$

which contradicts (2.4) of Corollary 2.2(a).

In the special case $\mathcal{A} = A$, Theorem 2.1 is reduced to Theorem 2 of Duran [3]. We now state without proof the following easy lemma.

Lemma D. *Given an infinite series* Σu_n *with* $s_k = \sum_{i=0}^{k} u_i$ (or $u_o = s_0$ and $u_k = s_k - s_{k-1}$ for $k \geq 1$). *Then the transformation* $g : fs \to f$, *defined by* $g(u) = s$, *is a linear isomorphism.*

Above lemma renders that $s \in f$ whenever $u \in fs$, and conversely. This terminology is used throughout.

Theorem 2.4. $\mathcal{A} \subset (f : fs)$ *if and only if, all limits being uniform in* p,

$$\sup_{n,p} \sum_k \mid \sum_{i=0}^{n} a_{ik}(p) \mid < \infty, \tag{2.8}$$

$$f - \lim \sum_{i=0}^{n} a_{ik}(p) = a_k \text{ for each } k, \tag{2.9}$$

$$f - \lim \sum_k \sum_{i=0}^{n} a_{ik}(p) = a, \tag{2.10}$$

$$\lim_q \sum_k \frac{1}{q+1} \mid \sum_{i=0}^{q} \sum_{j=0}^{n+i} \Delta[a_{jk}(p) - a_k] \mid = 0 \text{ uniformly in } n. \tag{2.11}$$

Proof. Let $\mathcal{A} \subset (f : fs)$ and $x \in f$. N0w, consider the following equality obtained from the n, m^{th} partial sums of $(Ax)_i^p$:

$$\sum_{i=0}^{n} \sum_{k=0}^{m} a_{ik}(p) x_k = \sum_{k=0}^{m} \sum_{i=0}^{n} a_{ik}(p) x_k; \ n, m, p = 0, 1, \cdots$$

which yields by letting $m \to \infty$ that

$$\sum_{i=0}^{n} \sum_k a_{ik}(p) x_k = \sum_k \sum_{i=0}^{n} a_{ik}(p) x_k; \ n, p = 0, 1, \cdots.$$

Then $\mathcal{B}x \in f$ and hence $\mathcal{B} \subset (f : f)$, since $g(\mathcal{A}x) = \mathcal{B}x$, where $B_p = (b_{nk}(p))$ with $b_{nk}(p) = \sum_{i=0}^{n} a_{ik}(p)$ for all n, k and p. Thus, we obtain the proof by the equivalence of the methods $\mathcal{A}$ and $\mathcal{B}$.

By Theorem 2.4, we have

Corollary 2.5 (a) $\mathcal{A} \subset (f : fs)_r$ if and only if (2.8) holds, (2.9) and (2.11) hold with $a_k = 0$ for each k and (2.10) also holds with $a = r$.

(b) $\mathcal{A} \subset (f : f_0 s)$ if and only if (2.8) holds, (2.9) and (2.11) hold with $a_k = 0$ for each k and (2.10) also holds with $a = 0$, where $f_0 s$ denotes the space of all series which are almost convergent to zero.

We now give the following lemma due to Başar-Solak [2], and nextly give a theorem of Steinhaus type whose proof is similar to that of Theorem 2.3.

Lemma E. *$\mathcal{A} \subset (m : fs)$ if and only if (2.8), (2.9) hold, and*

$$\lim_q \sum_k \frac{1}{q+1} \mid \sum_{i=0}^{q} \sum_{j=0}^{n+i} a_{jk}(p) - a_k \mid = 0 \text{ uniformly in } n, p. \tag{2.12}$$

Theorem 2.6. *The classes $(m : fs)$ and $(f : fs)_r$ are disjoint.*

In the special case $\mathcal{A} = A$, Theorem 2.4 is reduced to Theorem 2.4 of Solak [7].

III. Matrix Sequences from fs into f and fs

In this section, we establish necessary and sufficient conditions on the matrix sequence $\mathcal{A} = (A_p)$ in order that $\mathcal{A} \subset (fs : f), (fs : fs)$.

Theorem 3.1. *$\mathcal{A} \subset (fs : f)$ if and only if*

$$\sup_{n,p} \sum_k \mid \Delta a_{nk}(p) \mid < \infty, \tag{3.1}$$

$$\lim_k a_{nk}(p) = 0 \textit{ for each } n, p, \tag{3.2}$$

$$f - \lim a_{nk}(p) = a_k \textit{ uniformly in } p \textit{ for each } k, \tag{3.3}$$

$$\lim_q \sum_k \frac{1}{q+1} \mid \sum_{i=0}^{q} \Delta^2[a_{n+i,k}(p) - a_k] \mid = 0 \textit{ uniformly in } n, p; \tag{3.4}$$

where $\Delta^2[a_{n+i,k}(p) - a_k] = \Delta\{\Delta[a_{n+i,k}(p) - a_k]\}$.

Proof. Necessity. Let $\mathcal{A} \subset (fs : f)$ and $u \in fs$. Now, to show the necessity of (3.2), we assume that (3.2) is not satisfied for some n, p and obtain a contradiction as in Theorem 2.1 of Öztürk [6]. Indeed, under this assumption we can find some $u \in fs$ such that $\mathcal{A}u$ does not belong to f. For example, if we choose $u = ((-1)^n) \in fs$ then $(\mathcal{A}u)_n^p = \sum_k a_{nk}(p)(-1)^k$ which does not coverge for each n, p. That is to say that $\mathcal{A}$-transform of the series $\Sigma(-1)^n$, which belongs to fs, does not even exist. But this contradicts the fact that $\mathcal{A}$ is f-conservative. Hence, (3.2) is necessary. The proof of the necessity of (3.3) also follows as in (2.3).

Let us consider the equality

$$\sum_{k=0}^{m} a_{nk}(p)u_k = \sum_{k=0}^{m-1} \Delta a_{nk}(p)s_k + a_{nm}(p)s_m;\ m,\ n,\ p = 0, 1, \cdots \tag{3.5}$$

obtained by applying the Abel's partial summation to the m^{th} partial sums of $\mathcal{A}u$. By (3.2), it is obtained on taking the limit as $m \to \infty$ in (3.5) that

$$\sum_k a_{nk}(p)u_k = \sum_k \Delta a_{nk}(p)s_k;\ n, p = 0, 1, \cdots. \tag{3.6}$$

It follows by passing to f-limit in (3.6) that $\mathcal{B} = (B_p) \subset (f : f)$, where $B_p = (b_{nk}(p))$ with $b_{nk}(p) = \Delta a_{nk}(p)$ for all n, k and p. Therefore $\mathcal{B} = (B_p)$ satisfies (2.2),(2.5) and these are equivalent to (3.1),(3.4), respectively.

Sufficiency. Suppose the conditions (3.1)-(3.4) hold and $u \in fs$. Again consider $\mathcal{B} = (b_{nk}(p))$ in (3.6). Therefore, it is immediate that "$\mathcal{B} = (b_{nk}(p))$ satisfies (2.2),(2.3) and (2.5) if and only if $\mathcal{A} = (a_{nk}(p))$ satisfies (3.1),(3.3) and (3.4), respectively." Additionally, we have by (3.2) and (3.3) that

$$f - \lim \sum_k b_{nk}(p) = f - \lim a_{n,0}(p) = a_0 \text{ uniformly in } p.$$

Hence, $\mathcal{B} \subset (f : f)$ and this yields by passing to f-limit in (3.6) that $\mathcal{A}u \in f$. This means that every element of fs is almost $\mathcal{A}$-summable and thus the proof is completed.

By Theorem 3.1, we have

Corollary 3.2 (a) *$\mathcal{A} \subset (fs : f)_r$ if and only if (3.1), (3.2) hold, and (3.3), (3.4) also respectively hold with $a_k = r, \Delta^2 a_k = 0$ for each k.*

(b) *$\mathcal{A} \subset (fs : f_0)$ if and only if (3.1), (3.2) hold, and (3.3), (3.4) also hold with $a_k = 0$ for each k.*

We now give the following lemma due to Başar-Solak [2] and later give a theorem of Steinhaus type.

Lemma F. *$\mathcal{A} \subset (bs : f)$ if and only if (3.1), (3.2), (3.3) hold, and*

$$\lim_q \sum_k \frac{1}{q+1} \left| \sum_{i=0}^{q} \Delta[a_{n+i,k}(p) - a_k] \right| = 0 \text{ uniformly in } n, p. \tag{3.7}$$

Theorem 3.3 *The classes $(bs : f)$ and $(fs : f)_r$ are disjoint.*

Proof. Suppose that the converse of this is true and let $\mathcal{A} \subset (bs : f) \cap (fs : f)_r$. Then, the both series $\sum_k \Delta a_{nk}(p)$ and $\sum_k \frac{1}{q+1} \sum_{i=0}^{q} \Delta a_{n+i,k}(p)$ are uniformly convergent in n, p. Therefore, we have by (3.3) of Corollary 3.2 (a) that

$$\lim_q \sum_k \frac{1}{q+1} \sum_{i=0}^{q} \Delta a_{n+i,k}(p) = f - \lim a_{n,0}(p) = r \text{ uniformly in } p. \tag{3.8}$$

On the other hand it follows by combining (3.3) of Corollary 3.2 (a) and (3.7) that

$$\lim_q \sum_k \frac{1}{q+1} \mid \sum_{i=0}^{q} \Delta a_{n+i,k}(p) \mid = 0 \text{ uniformly in } n, p. \tag{3.9}$$

Then, (3.9) trivially implies that $\lim_q \mid \sum_k \frac{1}{q+1} \sum_{i=0}^{q} \Delta a_{n+i,k}(p) \mid = 0$ uniformly in n, p, which contradicts (3.8) and this completes the proof.

In the special case $\mathcal{A} = A$, Theorem 3.1 is reduced to Theorem 2.2 of Solak [7].

Now, we can give

Theorem 3.4. *$\mathcal{A} \subset (fs : fs)$ if and only if*

$$\sup_{n,p} \sum_k \mid \sum_{i=0}^{n} \Delta a_{ik}(p) \mid < \infty, \tag{3.10}$$

$$\lim_k a_{nk}(p) = 0 \textit{ for each } n, p, \tag{3.11}$$

$$f - \lim \sum_{i=0}^{n} a_{ik}(p) = a_k \textit{ uniformly in } p \textit{ for each } k, \tag{3.12}$$

$$\lim_q \sum_k \frac{1}{q+1} \mid \sum_{i=0}^{q} \sum_{j=0}^{n+i} \Delta^2 [a_{jk}(p) - a_k] \mid = 0 \textit{ uniformly in } n, p. \tag{3.13}$$

Proof. This is easily obtained by the similar kind of argument that of Theorem 2.4. We are now ready to give the following corollary:

Corollary 3.5 (a) *$\mathcal{A} \subset (fs : fs)_r$ if and only if* (3.10), (3.11) *hold, and* (3.12), (3.13) *also hold with $a_k = r$, $\Delta^2 a_k = 0$ for each k, respectively.*

(b) *$\mathcal{A} \subset (fs : f_0 s)$ if and only if* (3.10), (3.11) *hold, and* (3.12), (3.13) *also hold with $a_k = 0$ for each k.*

We shall give a lemma due to Başar-Solak [2] and later state a theorem of Steinhaus type whose proof is similar to that of Theorem 3.3.

Lemma G. *$\mathcal{A} \subset (bs : fs)$ if and only if* (3.10), (3.11), (3.12) *hold, and*

$$\lim_q \sum_k \frac{1}{q+1} \mid \sum_{i=0}^{q} \sum_{j=0}^{n+i} \Delta [a_{jk}(p) - a_k] \mid = 0 \text{ uniformly in } n, p. \tag{3.14}$$

Theorem 3.6. *The classes $(bs : fs)$ and $(fs : fs)_r$ are disjoint.*

Finally, we should state in the case $\mathcal{A} = A$ that Theorem 3.4 is reduced to Theorem 2.3 of Solak [7].

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