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REMARKS ON THE PAPER "A NOTE ON EVERITT TYPE INTEGRAL INEQUALITY" OF B. G. PACHPATTE

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Abstract. In the present note we give an interpolating inequality for Pachpatte's inequality from [1]. A discrete analogouos is also given.

B. G. Pachpate [1] proved the following theorem:

Theorem A. *Let p, q be real-valued continuous functions defined on $I = [a,b]; p'$ exists and is continuous on I. Let f, g be real-valued continuous functions defined on I which are twice continuously differentiable on I and $f(a) = f(b) = g(a) = g(b) = 0$. Then*

$$(1)\qquad (\int_a^b [p(t)f'(t)g'(t) + q(t)f(t)g(t)]dt)^2$$
$$\leq \frac{1}{2}\{(\int_a^b M[f(t)]^2 dt)(\int_a^b g^2(t)dt) + (\int_a^b M[g(t)]^2 dt)(\int_a^b f^2(t)dt)\},$$

where

$$M[f(t)] = -(p(t)f'(t))' + q(t)f(t).$$

Equality holds in (1) if either f is null or g is null on I.

Here we shall note that the following interpolation inequality for (1) is valid: If the conditions of Theorem A are valid then

$$(2)\qquad (\int_a^b [p(t)f'(t)g'(t) + q(t)f(t)g(t)]dt)^2$$
$$\leq \{(\int_a^b M[f(t)]^2 dt)(\int_a^b g^2(t)dt)(\int_a^b M[g(t)]^2 dt)(\int_a^b f^2(t)dt)\}^{1/2}$$
$$\leq \frac{1}{2}\{(\int_a^b M[f(t)]^2 dt)(\int_a^b g^2(t)dt) + (\int_a^b M[g(t)]^2 dt)(\int_a^b f^2(t)dt)\}.$$

As in [1] we have

$$(3)\qquad \int_a^b p(t)f'(t)g'(t)dt = -\int_a^b (p(t)f'(t))'g(t)dt,$$

$$(4)\qquad \int_a^b p(t)f'(t)g'(t)dt = -\int_a^b (p(t)g'(t))'f(t)dt.$$

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From (3) and (4) we observe that

$$\left(\int_a^b [p(t)f'(t)g'(t)+q(t)f(t)g(t)]dt\right)^2$$
$$=\left(\int_a^b M[f(t)]g(t)dt\right)\left(\int_a^b M[g(t)]f(t)dt\right).$$

Using Cauchy's inequality to each integral on the right side we obtain the first inequality in (2). The second inequality in (2) is a simple consequence of the arithmetic mean-the geometric mean inequality.

Similarly, we can prove discrete analogous of the previous results. For example the following result is valid:

Let a, b, p, q be real n-tuples, and let Δ and M_n be operators defined by

$$\Delta a_i = a_{i+1} - a_i, \quad M_i(a) = -\Delta(p_{i-1}\Delta a_{i-1}) + q_i a_i .$$

If $a_1 = a_n = b_1 = b_n = 0$, then we have

$$\left(\sum_{i=1}^{n-1}(p_i\Delta a_i\Delta b_i + q_i a_i b_i)\right)^2 \le \left\{\sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2 \sum_{i=2}^{n-1} M_i(a)^2 \sum_{i=2}^{n-1} M_i(b)^2\right\}^{1/2}.$$

Reference

[1] B. G. Pachpatte, "A Note on Everitt Type Integral Inequality." *Tamkang Journal of Mathematics* Vol. 18, No. 2, 1987, pp. 7-10.

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