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INTUITIONISTIC FUZZY SEMI-GENERALIZED IRRESOLUTE MAPPINGS

R. SANTHI and K. ARUN PRAKASH

Abstract. The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy semi-generalized continuous mappings and intuitionistic fuzzy semi-generalized irresolute mappings in intuitionistic fuzzy topological space.

1. Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper [14] in 1965. Using the concept of fuzzy sets, Chang [3] introduced the concept of fuzzy topological space. In [1], Atanassov introduced the notion of intuitionistic fuzzy sets in 1986. Using the notion of intuitionistic fuzzy sets, Coker [4] defined the notion of intuitionistic fuzzy topological spaces in 1997. This approach provided a wide field for investigation in the area of fuzzy topology and its applications. One of the directions is related to the properties of intuitionistic fuzzy sets introduced by Gurcay [6] in 1997.

Continuing the work done in the paper [12], we define the notion of intuitionistic fuzzy semi-generalized continuous mappings and intuitionistic fuzzy semi-generalized irresolute mappings. We discuss characterizations of intuitionistic fuzzy semi-generalized continuous mappings and irresolute mappings. We also established their properties and relationships with other classes of early defined forms of intuitionistic continuous mappings.

2. Preliminaries

Definition 2.1 ([1]). An intuitionistic fuzzy set (IFS, for short) A in X is an object having the form

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X \}$$

where the functions $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of the membership (namely $\mu_A(x)$) and the degree of non- membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set *A*, respectively, $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$.

Corresponding author: R. Santhi.

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Definition 2.2 ([1]). Let *A* and *B* be IFS's of the forms $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle \mid x \in X\}$. Then,

- (a) $A \le B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$,
- (b) A = B if and only if $A \le B$ and $B \le A$,
- (c) $\bar{A} = \{ \langle x, \gamma_A(x), \mu_A(x) \rangle \mid x \in X \},\$
- (d) $A \cap B = \{ \langle x, \mu_A(x) \land \mu_B(x), \gamma_A(x) \lor \gamma_B(x) \mid x \in X \},\$
- (e) $A \cup B = \{ \langle x, \mu_A(x) \lor \mu_B(x), \gamma_A(x) \land \gamma_B(x) \mid x \in X \},$
- (f) $0_{\sim} = \{\langle x, 0, 1 \rangle, x \in X\}$ and $1_{\sim} = \{\langle x, 1, 0 \rangle, x \in X\}$,
- (g) $\overline{\overline{A}} = A$, $\overline{1_{\sim}} = 0_{\sim}$ and $\overline{0_{\sim}} = 1_{\sim}$.

Definition 2.3 ([1]). Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \le 1$. An intuitionistic fuzzy point (IFP), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of *X* given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta), & \text{if } x = p, \\ (0,1), & \text{otherwise.} \end{cases}$$

Definition 2.4 ([4]). An intuitionistic fuzzy topology (IFT for short) on *X* is a family τ of IFS's in *X* satisfying the following axioms:

- (i) $0_{\sim}, 1_{\sim} \in \tau$,
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
- (iii) $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement \overline{A} of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.5 ([1]). Let f be a mapping from a set X to a set Y. If

$$B = \{ \langle y, \mu_B(y), \gamma_B(y) \rangle; y \in Y \}$$

is an IFS in Y, then the *preimage* of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B)=\{\langle x,f^{-1}(\mu_B(x)),f^{-1}(\gamma_B(x))\rangle;x\in X\}$$

Definition 2.6 ([4]). Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in *X*. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of *A* are defined by

$$int(A) = \bigcup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A \},\$$

 $cl(A) = \cap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$

Note that, for any IFS *A* in (X, τ) , we have

 $cl(\bar{A}) = \overline{int(A)}$ and $int(\bar{A}) = \overline{cl(A)}$.

Definition 2.7. An IFS $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X\}$ in an IFTS (X, τ) is called

- (i) intuitionistic fuzzy semi open set (IFSOS) if $A \subseteq cl(int(A))$. [6]
- (ii) intuitionistic fuzzy α -open set (IF α OS) if $A \subseteq int(cl(int(A)))$.[6]
- (iii) intuitionistic fuzzy preopen set (IFPOS) if $A \subseteq int(cl(A))$.[6]
- (iv) intuitionistic fuzzy regular open set (IFROS) if int(cl(A)) = A.[6]
- (v) intuitionistic fuzzy semi-pre open set (IFSPOS) if there exists $B \in IFPO(X)$ such that $B \subseteq A \subseteq cl(B)$.[13]

An IFS *A* is called intuitionistic fuzzy semi closed set, intuitionistic fuzzy α -closed set, intuitionistic fuzzy preclosed set, intuitionistic fuzzy regular closed set and intuitionistic fuzzy semi-preclosed set, respectively (IFSCS, IF α CS, IFPCS, IFRCS and IFSPCS resp), if the complement \overline{A} is an IFSOS, IF α OS, IFPOS, IFROS and IFSPOS respectively.

The family of all intuitionistic fuzzy semi open (resp. intuitionistic fuzzy α -open, intuitionistic fuzzy preopen, intuitionistic fuzzy regular open and intuitionistic fuzzy semi-preopen) sets of an IFTS (*X*, τ) is denoted by IFSO(*X*) (resp IF α (*X*), IFPO(*X*), IFRO(*X*) and IFSPO(*X*)).

Definition 2.8 ([12]). An intuitionistic fuzzy set *A* of an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy semi-generalized closed set (IFSGCS) if scl(*A*) \subseteq *U*, whenever $A \subseteq U$ and *U* is intuitionistic fuzzy semi-open set.

The complement \overline{A} of intuitionistic fuzzy semi-generalized closed set A is called intuitionistic fuzzy semi-generalized open set (IFSGOS).

Definition 2.9 ([12]). An intuitionistic fuzzy topological space (X, τ) is said to be intuitionistic fuzzy semi- $T_{1/2}$ space, if every intuitionistic fuzzy *sg*-closed set in *X* is intuitionistic fuzzy semi- closed in *X*.

Definition 2.10 ([8]). Let $p(\alpha, \beta)$ be an IFP of an IFTS(X, τ). An IFS A of X is called an intuitionistic fuzzy neighbourhood (IFN) of $p(\alpha, \beta)$, if there exists an IFOS B in X such that $p(\alpha, \beta) \in B \subseteq A$.

Definition 2.11. Let $f : X \to Y$ be a mapping from an IFTS *X* into an IFTS *Y*. The mapping *f* is called

(i) *intuitionistic fuzzy continuous*, if $f^{-1}(B)$ is an IFOS in X, for each IFOS B in Y.[6]

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- (ii) *intuitionistic fuzzy semi-continuous*, if $f^{-1}(B)$ is an IFSOS in X, for each IFOS B in Y.[6]
- (iii) *intuitionistic fuzzy pre-continuous*, if $f^{-1}(B)$ is an IFPOS in X, for each IFOS B in Y.[6]
- (iv) *intuitionistic fuzzy* α *-continuous*, if $f^{-1}(B)$ is an IF α OS in X, for each IFOS B in Y.[6]
- (v) *intuitionistic fuzzy semi-pre continuous*, if $f^{-1}(B)$ is an IFSPOS in *X*, for each IFOS *B* in *Y*.[13]
- (vi) *intuitionistic fuzzy completely continuous*, if $f^{-1}(B)$ is an IFROS in *X*, for each IFOS *B* in *Y*.[15]

Lemma 2.12 ([15]). Let $g: X \to X \times Y$ be the graph of a function $f: X \to Y$. If A is an IFS of X and B is an IFS of Y, then $g^{-1}(A \times B)(x) = (A \cap f^{-1}(B))(x)$.

3. Intuitionistic fuzzy semi-generalized continuous mappings

In this section we introduce intuitionistic fuzzy semi-generalized continuous mappings and studied some of the properties regarding it.

Definition 3.1. Let *A* be an IFS in an IFTS (X, τ) . Then the intuitionistic fuzzy semi-generalized interior and intuitionistic fuzzy semi-generalized closure of A are defined as follows.

sgint(A) = \cup {G | G is an IFSGOS in X and G \subseteq A}, sgcl(A) = \cap {K | K is an IFSGCS in X and A \subseteq K}.

Example 3.2. Let $X = \{a, b\}$.

Let
$$A = \left\langle x, \left(\begin{array}{c} \underline{a} & , \ \underline{b} \\ 0.2 & 0.3 \end{array} \right), \left(\begin{array}{c} \underline{a} & , \ \underline{b} \\ 0.7 & 0.7 \end{array} \right) \right\rangle$$

$$B = \left\langle x, \left(\begin{array}{c} \underline{a} & , \ \underline{b} \\ 0.4 & 0.7 \end{array} \right), \left(\begin{array}{c} \underline{a} & , \ \underline{b} \\ 0.6 & 0.1 \end{array} \right) \right\rangle$$

Then $\tau = \{0_{\sim}, 1_{\sim}, A, B\}$ is an IFTS on *X*.

Then sgint(*B*) = $A \cup 0_{\sim} = A$ and sgcl(*B*) = 1_{\sim} .

Proposition 3.3. If A be an IFS in X, then $A \leq sgcl(A) \leq scl(A) \leq cl(A)$.

Proof. The result follows from Definition.

Theorem 3.4. If A is an IFSGCS in X, then sgcl(A) = A.

Proof. Since *A* is an IFSGCS, sgcl(*A*) is the smallest IFSGCS which contains *A*, which is nothing but *A*. Hence sgcl(*A*) = *A*. \Box

Theorem 3.5. If A is an IFSGOS in X, then sgint(A) = A.

Proof. Similar to the above theorem.

Definition 3.6. Let (X, τ) and (Y, κ) be IFT's. A mapping $f : X \to Y$ is called intuitionistic fuzzy semi-generalized continuous (intuitionistic fuzzy *sg*-continuous), if $f^{-1}(B)$ is an IFSGCS in *X* for every IFCS *B* in *Y*.

Theorem 3.7. *Every intuitionistic fuzzy continuous mapping is an intuitionistic fuzzy sg-continuous mapping.*

Proof. Let *B* be an IFCS in *Y*. Then by our assumption, $f^{-1}(B)$ is an IFCS in *X*. In [12], it has been proved that every intuitionistic fuzzy closed set is an intuitionistic fuzzy *sg*-closed set in *X*. Thus $f^{-1}(B)$ IFSGCS in *X*. Hence *f* is an intuitionistic fuzzy *sg*-continuous mapping.

The following example shows that the converse of above theorem is not true in general.

Example 3.8. Let $X = \{a, b\}, Y = \{c, d\}.$

Let
$$A = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}\right) \right\rangle$$

$$B = \left\langle x, \left(\frac{c}{0.7}, \frac{d}{0.8}\right), \left(\frac{c}{0.3}, \frac{d}{0.2}\right) \right\rangle.$$

Then $\tau = \{0_{\sim}, 1_{\sim}, A\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, B\}$ are IFTS on *X* and *Y* respectively. Define a mapping $f: (X, \tau) \to (Y, \kappa)$ by f(a) = c, f(b) = d. Clearly *f* is intuitionistic fuzzy *sg*-continuous map.

Now we have $f^{-1}(B) = \left\langle x, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.7 & 0.8 \end{pmatrix}, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.3 & 0.2 \end{pmatrix} \right\rangle$. $f^{-1}(B) \notin \tau$, which shows that f is not an intuitionistic fuzzy continuous map.

Theorem 3.9. Every intuitionistic fuzzy α -continuous mapping is an intuitionistic fuzzy sgcontinuous mapping.

Proof. Let *B* be an IFCS in *Y*. Since *f* is an intuitionistic fuzzy α -continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy α -closed set in *X*. In [12], it has been proved that every IF α CS is an intuitionistic fuzzy *sg*-closed set in *X*. Thus $f^{-1}(B)$ IFSGCS in *X*. Hence *f* is an intuitionistic fuzzy *sg*-continuous mapping.

The following example shows that the converse of the above theorem is not true in general.

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Example 3.10. Let $X = \{a, b\}, Y = \{u, v\}.$

Let
$$A = \left\langle x, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.7 & 0.5 \end{pmatrix}, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.3 & 0.5 \end{pmatrix} \right\rangle$$

$$B = \left\langle y, \begin{pmatrix} \underline{u} & , \underline{v} \\ 0.25 & 0.3 \end{pmatrix}, \begin{pmatrix} \underline{u} & , \underline{v} \\ 0.2 & 0.2 \end{pmatrix} \right\rangle.$$

Then $\tau = \{0_{\sim}, 1_{\sim}, A\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, B\}$ are IFTS on *X* and *Y* respectively. Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by f(a) = u, f(b) = v. Clearly *f* is intuitionistic fuzzy *sg*-continuous map.

Now we have
$$f^{-1}(B) = \left\langle x, \left(\begin{array}{c} \underline{a} & , \underline{b} \\ 0.25 & 0.3 \end{array} \right), \left(\begin{array}{c} \underline{a} & , \underline{b} \\ 0.2 & 0.2 \end{array} \right) \right\rangle$$
.
 $\operatorname{cl}(f^{-1}(B)) = 1_{\sim}, \operatorname{int}(\operatorname{cl}(f^{-1}(B))) = \operatorname{int}(1_{\sim}) = 1_{\sim}$

cl(int(cl($f^{-1}(B)$))) = cl(1_{\sim}) = 1_{\sim} . Thus cl(int(cl($f^{-1}(B)$))) $\not\subseteq f^{-1}(B)$, which shows that f is not an intuitionistic fuzzy α -continuous map.

Thus the class of intuitionistic fuzzy α -continuous maps is properly contained in the class of intuitionistic fuzzy *sg*-continuous maps.

Forthcoming theorem and example shows that the class of intuitionistic fuzzy semi-continuous maps is properly contained in the class of intuitionistic fuzzy *sg*-continuous maps.

Theorem 3.11. Every intuitionistic fuzzy semi-continuous mapping is intuitionistic fuzzy sgcontinuous mapping.

Proof. Let $f : X \to Y$ be any function from IFTS *X* in to *Y* such that *f* is intuitionistic fuzzy semi-continuous. By definition of intuitionistic fuzzy semi-continuous, $f^{-1}(A)$ is IFSCS in *X* for every IFCS *A* in *Y*. In [12], it has been proved that every intuitionistic fuzzy semi-closed set is an intuitionistic fuzzy *sg*-closed set in *X*. Thus $f^{-1}(B)$ IFSGCS in *X*. Hence *f* is an intuitionistic fuzzy *sg*-continuous mapping.

The converse of the above theorem is not true as seen from the following example. \Box

Example 3.12. Let $X = \{a, b\}, Y = \{u, v\}.$

Let
$$A = \left\langle x, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.2 & 0.4 \end{pmatrix}, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.6 & 0.25 \end{pmatrix} \right\rangle$$

$$B = \left\langle y, \begin{pmatrix} \underline{u} & , \underline{v} \\ 0.3 & 0.5 \end{pmatrix}, \begin{pmatrix} \underline{u} & , \underline{v} \\ 0.4 & 0.5 \end{pmatrix} \right\rangle.$$

Then $\tau = \{0_{\sim}, 1_{\sim}, A\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, B\}$ are IFTS on *X* and *Y* respectively. Define a mapping $f: X \to Y$ by f(a) = u, f(b) = v. Clearly *f* is intuitionistic fuzzy *sg*-continuous map.

Now we have $f^{-1}(B) = \left\langle x, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.3 & 0.5 \end{pmatrix}, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.4 & 0.5 \end{pmatrix} \right\rangle$.

 $cl(f^{-1}(B)) = 1_{\sim}$. int $[cl(f^{-1}(B))] = int(1_{\sim}) = 1_{\sim}$. Thus $int[cl(f^{-1}(B))] \not\subseteq f^{-1}(B)$, which shows that '*f*' is not intuitionistic fuzzy semi-continuous mapping.

Theorem 3.13. *Every intuitionistic fuzzy sg-continuous mapping is intuitionistic fuzzy semipre continuous mapping.*

Proof. Let *B* be an IFCS in *Y*. Since *f* is intuitionistic fuzzy *sg*-continuous map, $f^{-1}(B)$ is an intuitionistic fuzzy *sg*-closed set in *X*. In paper [12], it has been proved that, every intuitionistic fuzzy *sg*-closed set is an intuitionistic fuzzy semi-pre closed set. Therefore $f^{-1}(B)$ is an IFSPCS in *X*. Hence *f* is an intuitionistic fuzzy semi-pre continuous mapping.

The converse of the above theorem is not true as seen from the following example.

Example 3.14. Let $X = \{a, b\}, Y = \{u, v\}.$

Let
$$A = \left\langle x, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.4 & 0.5 \end{pmatrix}, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.1 & 0.3 \end{pmatrix} \right\rangle$$

$$B = \left\langle y, \begin{pmatrix} \underline{u} & , \underline{v} \\ 0.15 & 0.3 \end{pmatrix}, \begin{pmatrix} \underline{u} & , \underline{v} \\ 0.5 & 0.7 \end{pmatrix} \right\rangle.$$

Then $\tau = \{0_{\sim}, 1_{\sim}, A\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, B\}$ are IFTS on *X* and *Y* respectively. Define a mapping $g : (X, \tau) \rightarrow (Y, \kappa)$ by g(a) = u, g(b) = v. Clearly *g* is intuitionistic fuzzy semi-pre continuous map. Infact we have

$$g^{-1}(B) = \left\langle x, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.15 & 0.3 \end{pmatrix}, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.5 & 0.7 \end{pmatrix} \right\rangle$$

 $scl(g^{-1}(B)) = 1_{\sim} \not\subseteq A$. Hence *g* is not intuitionistic fuzzy semi-generalized continuous mapping.

Remark 3.15. Intuitionistic fuzzy pre-continuity is independent from intuitionistic fuzzy *sg*-continuity.

The proof follows from the following examples.

Example 3.16. Let $X = \{a, b\}, Y = \{u, v\}.$

Let
$$A = \left\langle x, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.3 & 0.4 \end{pmatrix}, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.7 & 0.6 \end{pmatrix} \right\rangle$$

$$B = \left\langle y, \begin{pmatrix} \underline{u} & , \underline{v} \\ 0.6 & 0.5 \end{pmatrix}, \begin{pmatrix} \underline{u} & , \underline{v} \\ 0.4 & 0.5 \end{pmatrix} \right\rangle.$$

Then $\tau = \{0_{\sim}, 1_{\sim}, A\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, B\}$ are IFTS on *X* and *Y* respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \kappa)$ by f(a) = u, f(b) = v. Clearly *f* is intuitionistic fuzzy *sg*-continuous map. Infact we have

$$f^{-1}(B)\left\langle x, \left(\begin{array}{cc}\underline{a} & , \underline{b} \\ 0.15 & 0.3\end{array}\right), \left(\begin{array}{cc}\underline{a} & , \underline{b} \\ 0.5 & 0.7\end{array}\right)\right\rangle$$

 $(cl(f^{-1}(B)) = 1_{\sim} \cap \overline{A} = \overline{A}$. $int[cl(f^{-1}(B))] = int(\overline{A}) = 0_{\sim} \cup A = A$. Hence $f^{-1}(B) \not\subseteq A = int[cl(f^{-1}(B))]$ which shows that f is not an intuitionistic fuzzy pre-continuous map.

Example 3.17. Let $X = \{a, b\}, Y = \{u, v\}.$

Let
$$A = \left\langle x, \left(\begin{array}{c} \underline{a} & , \ \underline{b} \\ 0.4 & 0.5 \end{array} \right), \left(\begin{array}{c} \underline{a} & , \ \underline{b} \\ 0.5 & 0.6 \end{array} \right) \right\rangle$$

$$B = \left\langle y, \left(\begin{array}{c} \underline{u} & , \ \underline{v} \\ 0.2 & 0.3 \end{array} \right), \left(\begin{array}{c} \underline{u} & , \ \underline{v} \\ 0.4 & 0.7 \end{array} \right) \right\rangle.$$

Then $\tau = \{0_{\sim}, 1_{\sim}, A\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, B\}$ are IFTS on *X* and *Y* respectively. Define a mapping $h: (X, \tau) \to (Y, \kappa)$ by h(a) = u, h(b) = v. Clearly *h* is intuitionistic fuzzy pre-continuous map. Infact we have

$$h^{-1}(B) = \left\langle x, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.2 & 0.3 \end{pmatrix}, \begin{pmatrix} \underline{a} & , \underline{b} \\ 0.4 & 0.7 \end{pmatrix} \right\rangle$$

 $scl(h^{-1}(B)) = 1_{\sim}$. $h^{-1}(B) \subset A$, but $scl(h^{-1}(B)) \not\subset A$, which shows that f is not an intuitionistic fuzzy sg-continuous map.



The above diagram shows the relationships between intuitionistic fuzzy *sg*-continuous mappings and some other mappings. The reverse implications are not true in the above diagram.

Theorem 3.18. Let $f : X \to Y$ be a mapping from a IFTS X into an IFTS Y. Then the following statements are equivalent.

- (i) *f* is intuitionistic fuzzy sg-continuous mapping.
- (ii) $f^{-1}(B)$ is an IFSGOS in X, for every IFOS B in X.

Proof. (i) \Rightarrow (ii) Let *B* be an IFOS in *Y*, then \overline{B} is an IFCS in *Y*. Since *f* is intuitionistic fuzzy *sg*-continuous mapping $f^{-1}(\overline{B})$ is an IFSGCS in *X*. Then $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$, implies $f^{-1}(B)$ is an IFSGOS in *Y*.

(ii) \Rightarrow (i) Let *B* be an IFCS in *Y*. By our assumption $f^{-1}(\overline{B})$ is an IFSGOS in *X* for every IFOS \overline{B} in *Y*. But $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$, which in turn implies $f^{-1}(B)$ is an IFSGCS in *X*. Hence *f* is intuitionistic fuzzy *sg*-continuous mapping.

Theorem 3.19. Let $f : X \to Y$ be an intuitionistic fuzzy sg-continuous mapping. Then the following statements hold.

- (i) $f(\operatorname{sgc1}(A)) \leq c1(f(A))$, for every intuitionistic fuzzy set A in X.
- (ii) $sgc1(f^{-1}(B)) \le f^{-1}(c1(B))$ for every intuitionistic fuzzy set B in Y.

Proof. (i) Let $A \le X$. Then cl(f(A)) is an intuitionistic fuzzy closed set in *Y*. Since *f* is intuitionistic fuzzy *sg*-continuous, $f^{-1}[cl[f(A)]]$ is intuitionistic fuzzy *sg*-closed in *X*. Since $A \le f^{-1}(f(A)) \le f^{-1}[cl[f(A)]]$ and $f^{-1}[cl[f(A)]]$ is intuitionistic fuzzy *sg*-closed, implies $sgcl(A) \le f^{-1}[cl(f(A))]$. Hence $f[sgcl(A)] \le cl[f(A)]$.

(ii) Replacing *A* by $f^{-1}(B)$ in (i), we get

$$f[\operatorname{sgcl}(f^{-1}(B))] \le \operatorname{cl}[f(f^{-1}(B))] = \operatorname{cl}(B)$$

 $f[\operatorname{sgcl}(f^{-1}(B))] \le \operatorname{cl}(B)$

Hence sgc1[$f^{-1}(B)$] $\leq f^{-1}[c1(B)]$.

Theorem 3.20. Let $f : X \to Y$ be a function and $g : X \to X \times Y$ the graph of the function f. Then f is intuitionistic fuzzy sg-continuous if g is so.

Proof. Let *B* be an IFOS in *Y*. Then by **Lemma 2.11**, $f^{-1}(B) = f^{-1}(1_{\sim} \times B) = 1_{\sim} \cap f^{-1}(B) = g^{-1}(1_{\sim} \times B)$. Since *B* is an IFOS *Y*, $1_{\sim} \times B$ is an IFOS in *X* × *Y*. Also since *g* is intuitionistic fuzzy *sg*-continuous implies that $g^{-1}(1_{\sim} \times B)$ is an IFSGOS in *X*. Therefore $f^{-1}(B)$ is an IFSGOS in *X*. Hence *f* is intuitionistic fuzzy *sg*-continuous mapping.

Theorem 3.21. Let $f : X \to Y$ is a mapping from an IFTS X into an IFTS Y. If any union of IFSGCS is IFSGCS, then the following statements are equivalent.

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- (i) *f* is intuitionistic fuzzy sg-continuous mapping.
- (ii) For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFSGCS B such that $p_{(\alpha,\beta)} \in B \le f^{-1}(A)$.
- (iii) For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFSGCS B such that $p_{(\alpha,\beta)} \in B$ and $f(B) \leq A$.

Proof. (i) \Rightarrow (ii): Assume that *f* is intuitionistic fuzzy *sg*-continuous. Let $p_{(\alpha,\beta)}$ be an IFP in *X* and *A* be an IFN of $f(p_{(\alpha,\beta)})$. Then by Definition of IFN, there exists an IFCS *C* in *Y*, such that $f(p_{(\alpha,\beta)}) \in C \leq A$. Taking $B = f^{-1}(C) \in X$, since *f* is intuitionistic fuzzy *sg*-continuous, $f^{-1}(C)$ is IFSGCS and

$$p_{(\alpha,\beta)} \in B \le f^{-1}[f(p_{(\alpha,\beta)})] \le f^{-1}(C) = B \le f^{-1}(A).$$

Hence $p_{(\alpha,\beta)} \in B \leq f^{-1}(A)$.

(ii) \Rightarrow (iii): Let $p_{(\alpha,\beta)}$ be an IFP in *X* and *A* be an IFN of $f(p_{(\alpha,\beta)})$, such that there exists an IFSGCS *B* with $p_{(\alpha,\beta)} \in B \leq f^{-1}(A)$. From this we have $p_{(\alpha,\beta)} \in B$ and $B \leq f^{-1}(A)$. This implies $f(B) \leq f(f^{-1}(A)) = A$. Hence (iii) holds.

(iii) \Rightarrow (i): Assume that (iii) holds. Let *B* be an IFCS in *Y* and take $p_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(p_{(\alpha,\beta)}) \in f(f^{-1}(B)) \leq B$. Since *B* is IFCS in *Y*, it follows that *B* is an IFN of $f(p_{(\alpha,\beta)})$. Then from (iii), there exists an IFSGCS *A* such that $p_{(\alpha,\beta)} \in A$ and $f(A) \leq B$. This shows that $p_{(\alpha,\beta)} \in A \leq f^{-1}(f(A)) \leq f^{-1}(B)$. (i.e) $p_{(\alpha,\beta)} \in A \subseteq f^{-1}(B)$. Since $p(\alpha,\beta)$ is an arbitrary IFP and $f^{-1}(B)$ is union of all IFP contained in $f^{-1}(B)$, by assumption $f^{-1}(B)$ is an IFSGCS. Hence *f* is intuitionistic fuzzy *sg*-continuous mapping.

Theorem 3.22. Let $f : X \to Y$ is a mapping from an IFTS X into an IFTS Y. Then the following statements are equivalent.

- (i) *f* is intuitionistic fuzzy sg-continuous mapping.
- (ii) $f^{-1}(B)$ is an IFSGOS in X, for every IFOS B in X.
- (iii) $f(\operatorname{sgc} 1(A)) \leq c1(f(A))$, for every fuzzy set A in X.
- (iv) $sgc1(f^{-1}(B)) \le f^{-1}(c1(B))$ for every fuzzy set B in Y.
- (v) For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFSGCS B such that $p_{(\alpha,\beta)} \in B \le f^{-1}(A)$.
- (vi) For each IFP $p_{(\alpha,\beta)} \in X$ and every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFSGCS B such that $p_{(\alpha,\beta)} \in B$ and $f(B) \leq A$.

Proof. Follows form the Theorems.3.18, 3.19 and 3.22.

Theorem 3.23. If $f : X \to Y$ is intuitionistic fuzzy sg-continuous and $g : Y \to Z$ is intuitionistic fuzzy completely continuous, then $g \circ f : X \to Z$ is intuitionistic fuzzy sg-continuous.

Proof. Let *B* be any IFCS in *Z*. Since *g* is intuitionistic fuzzy completely continuous, $g^{-1}(B)$ is an IFRCS in *Y*. In [6], it has been proved that every IFRCS is an IFCS. Therefore $g^{-1}(B)$ is an IFCS in *Y*. Also since *f* is intuitionistic fuzzy *sg*-continuous mapping $f^{-1}[g^{-1}(B)]$ is an IFSGCS in *X*.

We have $(g \circ f)^{-1}[B] = f^{-1}[g^{-1}(B)]$ is IFSGCS in *X*, for every IFCS *B* in *Z*. Hence $g \circ f$ is intuitionistic fuzzy *sg*-continuous mapping.

Theorem 3.24. If $f : X \to Y$ is intuitionistic fuzzy sg-continuous and $g : Y \to Z$ is intuitionistic fuzzy continuous, then $g \circ f : X \to Z$ is intuitionistic fuzzy sg-continuous.

Proof. Let *B* be any intuitionistic fuzzy closed set in *Z*. Since *g* is intuitionistic fuzzy continuous, $g^{-1}(B)$ is intuitionistic fuzzy closed set in *Y*. Since *f* is intuitionistic fuzzy *sg*-continuous mapping $f^{-1}[g^{-1}(B)]$ is an intuitionistic fuzzy *sg*-closed set in *X*.

 $(g \circ f)^{-1}[B] = f^{-1}[g^{-1}(B)]$ is intuitionistic fuzzy *sg*-closed set, for every intuitionistic fuzzy closed *B* in *Z*.

Hence $g \circ f$ is intuitionistic fuzzy *sg*-continuous mapping.

Theorem 3.25. Let $f : X \to Y$ is a mapping from an IFTS X into an IFTS Y. If X is intuitionistic fuzzy semi- $T_{1/2}$ space, then f is intuitionistic fuzzy sg-continuous iff it is intuitionistic fuzzy semi-continuous.

Proof. Let *f* be intuitionistic fuzzy *sg*-continuous mapping and let *A* be an intuitionistic fuzzy closed set in *Y*. Then by definition of intuitionistic fuzzy semi-generalized continuous $f^{-1}(A)$ is intuitionistic fuzzy *sg*-closed in *X*. Since *X* is intuitionistic fuzzy semi- $T_{1/2}$ space, $f^{-1}(A)$ is intuitionistic fuzzy semi-closed set.

Hence f is intuitionistic fuzzy semi-continuous.

Conversely assume that f is intuitionistic fuzzy semi-continuous. Then by **Theorem 3.11** f is intuitionistic fuzzy sg-continuous mapping.

Theorem 3.26. Let X, X_1, X_2 are IFTS's and $p_i : X_1 \times X_2 \rightarrow X_i$ (i = 1, 2) are projections of $X_1 \times X_2$ onto X_i . If $f : X \rightarrow X_1 \times X_2$ is intuitionistic fuzzy sg-continuous, then $p_i \circ f$ (i = 1, 2) is intuitionistic fuzzy sg-continuous mapping.

Proof. It follows from the facts that projections are intuitionistic fuzzy continuous mappings. $\hfill \Box$

4. Intuitionistic fuzzy semi-generalized irresolute mappings

Definition 4.1. A mapping $f : X \to Y$ from an IFTS *X* into an IFTS *Y* is said to be intuitionistic fuzzy semi-generalized irresolute (intuitionistic fuzzy *sg*-irresolute) if $f^{-1}(B)$ is an IFSGCS in *X* for every IFSGCS *B* in *Y*.

Theorem 4.2. Let $f : X \to Y$ is a mapping from an IFTS X into an IFTS Y. Then every intu*itionistic fuzzy* sg*-irresolute mapping is intuitionistic fuzzy* sg*-continuous*.

Proof. Assume that $f: X \to Y$ is an intuitionistic fuzzy *sg*-irresolute mapping and let *A* be an IFCS in *Y*. In [12], it has been proved that every intuitionistic fuzzy closed set is an intuitionistic fuzzy *sg*-closed. Therefore *A* is an IFSGCS in *Y*. Since *f* is intuitionistic fuzzy *sg*-irresolute, by definition $f^{-1}(A)$ is IFSGCS in *X*. Hence *f* is intuitionistic fuzzy *sg*-continuous.

Example 4.3. Let $X = \{a, b, c\}, Y = \{u, v, w\}.$

Let
$$A = \left\langle x, \left(\begin{array}{ccc} \underline{a} & , \ \underline{b} & , \ \underline{c} \\ 0.8 & 0.4 & 0.4 \end{array} \right), \left(\begin{array}{ccc} \underline{a} & , \ \underline{b} & , \ \underline{c} \\ 0.1 & 0.6 & 0.6 \end{array} \right) \right\rangle$$

$$B = \left\langle y, \left(\begin{array}{ccc} \underline{u} & , \ \underline{v} & , \ \underline{w} \\ 1 & 0.4 & 0.4 \end{array} \right), \left(\begin{array}{ccc} \underline{u} & , \ \underline{v} & , \ \underline{w} \\ 0 & 0.6 & 0.6 \end{array} \right) \right\rangle.$$

Then $\tau = \{0_{\sim}, 1_{\sim}, A\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, B\}$ are IFTS on *X* and *Y* respectively. Define a mapping *h* : $(X, \tau) \rightarrow (Y, \kappa)$ by h(a) = u, h(b) = v, h(c) = w. Clearly *h* is intuitionistic fuzzy *sg*-continuous map. Infact we have

$$C = \left\langle y, \begin{pmatrix} \underline{u} & \underline{v} & \underline{w} \\ 0.0 & 0.4 & 0.2 \end{pmatrix}, \begin{pmatrix} \underline{u} & \underline{v} & \underline{w} \\ 0 & 0.6 & 0.6 \end{pmatrix} \right\rangle \text{ be an IFSGCS in } Y.$$

 $h^{-1}(C) = \left\langle x, \begin{pmatrix} \underline{a} & , \underline{b} & , \underline{c} \\ 0.0 & 0.4 & 0.2 \end{pmatrix}, \begin{pmatrix} \underline{a} & , \underline{b} & , \underline{c} \\ 0.0 & 0.6 & 0.6 \end{pmatrix} \right\rangle.$

 $scl(h^{-1}(C)) = 1_{\sim} = 1_{\sim}$. $h^{-1}(C) \subset A$, but $scl(h^{-1}(C)) \not\subseteq A$, which shows that $h^{-1}(C)$ is not an IFSGCS in *X*. Therefore *f* is not an intuitionistic fuzzy *sg*-irresolute map.

Theorem 4.4. Let $f : X \to Y$ be a mapping from a IFTS X into an IFTS Y. Then the following statements are equivalent.

- (i) f is intuitionistic fuzzy sg-irresolute mapping.
- (ii) $f^{-1}(B)$ is an IFSGOS in X, for every IFSGOS B in X.

Proof. Similar to Theorem 3.18.

Theorem 4.5. Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then the following statements are equivalent.

- (i) *f* is an intuitionistic fuzzy semi-generalized irresolute mapping.
- (ii) $f^{-1}(B)$ is an IFSGOS in X for each IFSGOS B in Y.
- (iii) $sgcl(f^{-1}(B)) \le f^{-1}(sgcl(B))$, for each IFS B of Y.
- (iv) $f^{-1}(sgintB) \leq sgint[f^{-1}(B)]$, for each IFS B of Y.

Proof. (i) \Rightarrow (ii) It can be proved by using the complement and **Definition 4.1**.

(ii)⇒(iii) Let *B* be an IFS in *Y*. Since $B \leq \operatorname{sgcl}(B)$, $f^{-1}(B) = f^{-1}(\operatorname{sgcl}(B))$. Since $\operatorname{sgcl}(B)$ is an IFSGCS in *Y*, by our assumption, $f^{-1}(\operatorname{sgcl}(B))$ is an IFSGCS in *X*. Therefore $\operatorname{sgcl}(f^{-1}(B)] \leq f^{-1}(\operatorname{sgcl}(B))$.

(iii) \Rightarrow (iv) By taking complement we get the result.

(iv)⇒(i) Let *B* be any IFSGOS in *Y*. Then sgint(*B*) = *B*. By our assumption we have $f^{-1}(B) = f^{-1}(\text{sgint}(B)) \leq \text{sgint}[f^{-1}(B)]$, so $f^{-1}(B)$ is an IFSGOS in *X*. Hence *f* is intuitionistic fuzzy *sg*-irresolute mapping.

Theorem 4.6. Let $f: X \to Y$ be intuitionistic fuzzy sg-irresolute mapping. Then f is intuitionistic fuzzy irresolute mapping if (X, τ) is intuitionistic fuzzy semi- $T_{1/2}$ space.

Proof. Let *B* be an IFSCS in *Y*. Then *B* is an IFSGCS in *Y*. Since *f* is intuitionistic fuzzy *sg*-irresolute, $f^{-1}(B)$ is an IFSGCS in *X*. But (X, τ) is intuitionistic fuzzy semi- $T_{1/2}$ space implies $f^{-1}(B)$ is an IFSCS in *X*. Hence *f* is intuitionistic fuzzy irresolute.

Theorem 4.7. If a mapping $f : X \to Y$ is intuitionistic fuzzy sg-irresolute mapping, then $f(sgcl(B)) \leq scl(f(B))$ for every IFS B of X.

Proof. Let *B* be an IFS of *X*. Since scl(f(B)) is an IFSGCS in *Y*, by our assumption $f^{-1}[scl(f(B))]$ is an IFSGCS in *X*. Furthermore $B \le f^{-1}(f(B)) \le f^{-1}(scl(f(B)))$ and hence $sgcl(B) \le f^{-1}[scl(f(B))]$ and consequently $f[sgcl(B)] \le f[f^{-1}[scl(f(B))]] \le scl(f(B))$.

Theorem 4.8. Let (Y, κ) be an IFTS such that every IFSCS in Y is an IFCS. If $f : (X, \tau) \to (Y, \kappa)$ is bijective and intuitionistic fuzzy sg-continuous then f is intuitionistic fuzzy sg-irresolute.

Proof. Let *B* be an IFSGCS in *Y* and let $f^{-1}(B) \le A$, where *A* is an IFSOS in *X*. Then $B \le f(A)$. Since f(A) is an IFSOS in *Y* and *B* is an IFSGCS in *Y*, then $scl(B) \le f(A)$ and hence $f^{-1}(scl(A)) \le f^{-1}(f(A)) = A$. Since *f* is intuitionistic fuzzy *sg*-continuous and scl(B) is an IFCS in *Y*, then $f^{-1}(scl(B))$ is an IFSGCS in *X*. Therefore $scl[f^{-1}(scl(B))] \le A$ and so $scl(f^{-1}(B)) \le A$. Hence $f^{-1}(B)$ is an IFSGCS in *X*. Hence *f* is intuitionistic fuzzy *sg*-irresolute mapping.

Theorem 4.9. Let $f: X \to Y$ be an intuitionistic fuzzy sg-irresolute mappings. Then f is intuitionistic fuzzy irresolute, if (X, τ) is an intuitionistic fuzzy semi- $T_{1/2}$ space.

Proof. Let *A* be any IFSCS in *Y*. In [12], it has been proved that every IFSCS is an IFSGCS. Therefore *A* is an IFSGCS in *Y* and *f* is an intuitionistic fuzzy *sg*-irresolute. Then by definition $f^{-1}(A)$ is IFSGCS in *X*. But (X, τ) is an intuitionistic fuzzy semi- $T_{1/2}$ space, so $f^{-1}(A)$ is an IFSCS. Hence *f* is an intuitionistic fuzzy irresolute.

Theorem 4.10. If any union of IFSGCS is an IFSGCS, then a mapping $f : X \to Y$ from an IFTS X into an IFTS Y is intuitionistic fuzzy sg-irresolute if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IFSGCS B in Y such that $f(p_{(\alpha,\beta)}) \in B$, there exists an IFSGCS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \leq B$.

Proof. Let *f* be any intuitionistic fuzzy *sg*-irresolute mapping, $p_{(\alpha,\beta)}$ an IFP in *X* and *B* be any IFSGCS in *Y*, such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B) = \text{sgcl}[f^{-1}(B)]$. We take $A = \text{sgcl}[f^{-1}(B)]$. Then *A* is an IFSGCS in *X*, containing IFP $p_{(\alpha,\beta)}$ and $f(A) = f[\text{sgcl}(f^{-1}(B))] \leq f[f^{-1}(B)] \leq B$.

Conversely assume that *B* be any IFSGCS in *Y* and IFP $p_{(\alpha,\beta)}$ in *X*, such that $p_{(\alpha,\beta)} \in f^{-1}(B)$. By assumption there exists IFSGCS *A* in *X* such that $p_{(\alpha,\beta)} \in A$ and $f(A) \leq B$. Therefore $p_{(\alpha,\beta)} \in A \leq f^{-1}(B)$ and $p_{(\alpha,\beta)} \in A = \operatorname{sgcl}(A) \leq \operatorname{sgcl}[f^{-1}(B)]$. Since $p_{(\alpha,\beta)}$ is an arbitrary IFP and $f^{-1}(B)$ is union of all IFP contained in $f^{-1}(B)$, $f^{-1}(B)$ is an IFSGCS in *X*, so *f* is an intuitionistic fuzzy semi-generalized irresolute mapping.

Corollary 4.11. A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is intuitionistic fuzzy semi-generalized irresolute if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IFSGCS B in Y such that $f(p_{(\alpha,\beta)}) \in B$, there exists an IFSGCS A in X such that $p_{(\alpha,\beta)} \in A$ and $A \leq f^{-1}(B)$.

Proof. Follows from Theorem 4.10.

Theorem 4.12. Let $f : X \to Y$ and $g : Y \to Z$ are intuitionistic fuzzy sg-irresolute mappings, where X, Y, Z are IFTS. Then $g \circ f$ is an intuitionistic fuzzy sg-irresolute mapping.

Proof. Let *A* be an intuitionistic fuzzy *sg*-closed set in *Z*. Since *g* is an intuitionistic fuzzy semi-generalized irresolute mapping $g^{-1}(A)$ is an intuitionistic fuzzy *sg*-closed set in *Y*. Also since *f* is intuitionistic fuzzy semi-generalized irresolute mapping, $f^{-1}[g^{-1}(A)]$ is an intuitionistic fuzzy *sg*-closed set in *X*.

 $(g \circ f)^{-1}(A) = f^{-1}[g^{-1}(A)]$ for each *A* in *Z*. Hence $(g \circ f)^{-1}(A)$ is an intuitionistic fuzzy *sg*-closed set in *X*. Therefore $g \circ f$ is an intuitionistic fuzzy semi-generalized irresolute mapping.

Theorem 4.13. Let $f : X \to Y$ and $g : Y \to Z$ are intuitionistic fuzzy semi-generalized irresolute and intuitionistic fuzzy continuous mappings respectively, where X, Y, Z are IFTS. Then $g \circ f$ is an intuitionistic fuzzy semi-generalized continuous mapping.

Proof. Let *A* be any IFCS in *Z*. Since *g* is intuitionistic fuzzy semi-generalized continuous, $g^{-1}(A)$ is an IFSGCS in *Y*. Also, since *f* is intuitionistic fuzzy semi-generalized irresolute, $f^{-1}[g^{-1}(A)]$ is an IFSGCS in *X*.

 $(g \circ f)^{-1}(A) = f^{-1}[g^{-1}(A)]$ is an IFSGCS in *X*. Hence $g \circ f$ is intuitionistic fuzzy semigeneralized continuous.

Theorem 4.14. Let (X,τ) , (Y,κ) , (Z,δ) be any intuitionistic fuzzy topological spaces. Let f: $(X,\tau) \rightarrow (Y,\kappa)$ be intuitionistic fuzzy semi-generalized irresolute and $g: (Y,\kappa) \rightarrow (Z,\sigma)$ is intuitionistic fuzzy continuous, then $g \circ f$ is intuitionistic fuzzy semi-generalized continuous.

Proof. Let *B* be any intuitionistic fuzzy closed set in *Z*. Since *g* is intuitionistic fuzzy continuous, $g^{-1}(B)$ is IFCS in *Y*. In paper [12], it has been proved that every IFCS is an IFSGCS. Therefore $f^{-1}(B)$ is an IFSGCS in *Y*. But since *f* is an intuitionistic fuzzy sgirresolute mapping $f^{-1}(g^{-1}(B))$ is an IFSGCS in *X*.

 $[g \circ f]^{-1}(B) = f^{-1}(g^{-1}(B))$ is IFSGCS in *X* for every IFCS '*B*' in *X*.

Hence $g \circ f$ is intuitionistic fuzzy *sg*-continuous.

Theorem 4.15. Let X, X_1, X_2 are IFTS's and $p_i : X_1 \times X_2 \to X_i$ (i = 1, 2) are projections of $X_1 \times X_2$ onto X_i . If $f : X \to X_1 \times X_2$ is intuitionistic fuzzy semi-generalized irresolute, then $p_i f$ is intuitionistic fuzzy semi-generalized continuous mapping.

Proof. $p_i f : X \to X_i$ (*i* = 1,2). It follows from the fact that p_i (*i* = 1,2) are intuitionistic fuzzy continuous mappings and by **Theorem 5.5**.

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Department of Mathematics, Nallamuthu Gounder Mahalingam College Pollachi - 642 001.

E-mail: santhifuzzy@yahoo.co.in

Department of Mathematics, Kongu Engineering College Perundurai - 638 052.

E-mail: arun.kannusamy@yahoo.co.in

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