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ALMOST p_s-CONTINUOUS FUNCTIONS

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Abstract. The purpose of this paper is to introduce a new class of functions called almost p_s-continuous function by using p_s-open sets in topological spaces. Some properties and characterizations of this function are given.

1. Introduction

Throughout this paper, a space X mean a topological space with out any separation axiom. We recall the following definitions, notations and terminology. The closure (resp. interior) of a subset A of X is denoted by ClA (resp. $IntA$). A subset A of X is said to be preopen [19] (resp. semi-open [17], α-open [22], β-open [1], regular open [31] and regular semi-open [5]) if $A \subseteq IntClA$ (resp. $A \subseteq ClIntA$, $A \subseteq IntClIntA$, $A \subseteq ClIntClA$, $A = IntClA$ and $A = sIntsClA$). The complement of a preopen (resp. semi-open, α-open, β-open, regular open and regular semi-open) set is said to be preclosed (resp. semi-closed, α-closed, β-closed, regular closed and regular semi-open). The family of all preopen (resp. semi-open, α-open, regular open, regular semi-open and regular closed) subsets of a topological space X is denoted by $PO(X)$ (resp. $SO(X)$, αO(X), $RO(X)$, $RSO(X)$ and $RC(X)$). A function $f : X \to Y$ is said to be precontinuous [19] (resp. super continuous [20]) if the inverse image of each open subset of Y is preopen (resp. δ-open) in X. A function $f : X \to Y$ is said to be almost precontinuous [21] (resp. almost continuous in the sense of Singal and Singal [30], almost α-continuous[23], R-map[6], almost strongly θ-continuous [27], almost s-continuous [14], weakly θ-irresolute [10] and θ-irresolute [16]) if the inverse image of each regular open subset of Y is preopen (resp., open, α-open, regular open, θ-open, closed, semi-closed and intersection of regular open sets) in X. A function $f : X \to Y$ is said to be δ-continuous [24] (resp., almost strongly θ-continuous [27]) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an open set U of X containing x such that $f(IntClU) \subseteq IntClV$ (resp., $f(ClU) \subseteq sClV$). A function $f : X \to Y$ is said to be irresolute [7] if the inverse image of each semi-open subset of Y is semi-open in X. A function $f : X \to Y$ is said to be weakly quasi-continuous [25] (resp. S-continuous [33]) if for every $F \in RC(Y)$, $f^{-1}(F) \in SO(X)$ (resp. $f^{-1}(F)$

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is the union of regular closed sets of X). A function $f: X \to Y$ is said to be preopen [19] (resp., semi-open [26]) if the image of each open set of X is preopen (resp., semi-open) in Y.

Definition 1.1 ([15]). A preopen subset A of a space X is called p_s-open if for each $x \in A$, there exists a semi-closed set F such that $x \in F \subseteq A$.

The family of all p_s-open subsets of a topological space X is denoted by $P_sO(X)$.

Definition 1.2 ([32]). A subset A of a space X is called δ-open (resp., θ-open) if for each $x \in A$, there exists an open set G such that $x \in G \subseteq IntClG \subseteq A$ (resp., $x \in G \subseteq ClG \subseteq A$).

The intersection of all p_s-closed (resp. preclosed, semi-closed, α-closed and δ-closed) sets of X containing A is called the p_s-closure (resp. preclosure, semi-closure, α-closure and δ-closure) of A and is denoted by P_sClA (resp. $pClA$, $sClA$, αClA and $Cl_\delta A$). The union of all p_s-open (resp. preopen, semi-open, α-open and δ-open) sets of X contained in A is called the p_s-interior (resp. preinterior, semi-interior, α-interior and δ-interior) of A and is denoted by P_sIntA (resp. $pIntA$, $sIntA$, $\alpha IntA$ and $Int_\delta A$).

Proposition 1.3 ([15]). *A subset A of a space X is p_s-open if and only if A is preopen and it is a union of semi-closed sets.*

Definition 1.4 ([13]). A subset A of a space X is called θ-semi-open if for each $x \in A$, there exists a semi-open set G such that $x \in G \subseteq ClG \subseteq A$. The family of all θ-semi-open subsets of a topological space X is denoted by $\theta SO(X)$.

Definition 1.5. A space X is s-regular[3] (resp., semi-regular[28]) if for each $x \in X$ and each open set G containing x, there exists a semi-open (resp., regular open) set H such that $x \in H \subseteq sClH \subseteq G$ (resp.,$x \in H \subseteq G$).

Definition 1.6. A space X is said to be:

(1) extremally disconnected [8] if ClU is open for each open set U.

(2) hyperconnected [9] if every nonempty open subset of X is dense in X.

(3) locally indiscrete [9] if every open subset of X is closed.

(4) semi-T_1 [18] if to each pair of distinct points x, y of X, there exists a pair of semi-open sets, one containing x but not y and the other containing y but not x.

Proposition 1.7. *The following statements are true:*

(1) *A space X is semi-T_1 if and only if for any point $x \in X$, the singleton set $\{x\}$ is semi-closed.[18].*

(2) *A space X is extremally disconnected if and only if $RO(X) = RC(X)$. [11].*

(3) *If a space X is semi-T_1, then $P_sO(X) = PO(X)$. [15].*

(4) *If a topological space (X,τ) is locally indiscrete, then $P_sO(X) = \tau$. [15].*

(5) *If a topological space (X,τ) is s-regular, then $\tau \subseteq P_sO(X)$. [15].*

Proposition 1.8 ([15])**.** *For any subset A of a space X. The following are equivalent:*

(1) *A is clopen.*

(2) *A is p_s-open and closed.*

(3) *A is α-open and closed.*

(4) *A is preopen and closed.*

Proposition 1.9 ([15])**.** *For any subset A of a space X. The following are equivalent:*

(1) *A is regular open.*

(2) *A is p_s-open and semi-closed.*

(3) *A is open and semi-closed.*

(4) *A is α-open and semi-closed.*

(5) *A is preopen and semi-closed.*

Lemma 1.10 ([15])**.** *The following properties are true:*

(1) *For any subset A of a space X. If $A \in \theta SO(X)$ and $A \in PO(X)$, then $A \in P_sO(X)$.*

(2) *If (X,τ) is extremally disconnected space and if $A \in \theta SO(X)$, then $A \in P_sO(X)$.*

(3) *If (Y,τ_Y) is a subspace of a space (X,τ), if $A \in P_sO(Y,\tau_Y)$ and $Y \in RO(X,\tau)$, then $A \in P_sO(X)$.*

(4) *If either $B \in RSO(X)$ or B is an open subspace of a space X and $A \in P_sO(X)$, then $A \cap B \in P_sO(B)$.*

Lemma 1.11. *The following statements are true:*

(1) *If $R \in RO(X)$ and $P \in PO(X)$, then $R \cap P \in RO(P)$. [9].*

(2) *Let A be a subset of a space (X,τ). Then $A \in PO(X,\tau)$ if and only if $sClA = IntClA$. [12].*

(3) *Let Y be a dense subspace of X. If O is regular open in Y, then $O = Y \cap IntClO$. [29].*

(4) *A subset A of a space (X,τ) is β-open if and only if ClA is regular closed. [4].*

Lemma 1.12. *Let A be a subset of a topological space (X,τ), then the following statement are true:*

(1) *If $A \in SO(X)$, then $Cl_\delta A = ClA = P_sClA = pClA = \alpha ClA$. [15].*

(2) *If $A \in \beta O(X)$, then $\alpha ClA = ClA$. [2].*

Definition 1.13 ([15]). A function $f : X \to Y$ is called p_s-continuous at a point $x \in X$ if for each open set V of Y containing $f(x)$, there exists a p_s-open set U of X containing x such that $f(U) \subseteq V$. Equivalently,a function $f : X \to Y$ is p_s-continuous if and only if $f^{-1}(V)$ is p_s-open set in X for each open set V in Y.

Proposition 1.14 ([21]). *A function $f : X \to Y$ is almost precontinuous if and only if $f^{-1}(V)$ is preopen set in X, for every δ-open set V in Y.*

Lemma 1.15. *The following results can be proved easily:*

(1) *If $f : X \to Y$ is almost precontinuous and Y is semi-regular, then f is precontinuous.*

(2) *If $f : X \to Y$ is almost continuous and Y is semi-regular, then f is continuous.*

Theorem 1.16 ([15]). . *If $f : X \to Y$ is a continuous and open function and V is a p_s-open set of Y, then $f^{-1}(V)$ is a p_s-open set of X.*

Theorem 1.17 ([12]). . *A function $f : X \to Y$ is preopen if and only if $f^{-1}(ClV) \subseteq Clf^{-1}(V)$, for each semi-open set V of Y.*

2. Almost p_s-Continuous Functions

In this section, we introduce the concept of almost p_s-continuous functions by using p_s-open sets. Some properties and characterizations are given.

Definition 2.1. A function $f : X \to Y$ is called almost p_s-continuous at a point $x \in X$ if for each open set V of Y containing $f(x)$, there exists a p_s-open set U of X containing x such that $f(U) \subseteq IntClV$. If f is almost p_s-continuous at every point of X, then it is called almost p_s-continuous.

Lemma 2.2. *The following results follows directly from their definitions:*

(1) *Every p_s-continuous function is almost p_s-continuous.*

(2) *Every almost p_s-continuous function is almost precontinuous.*

Proposition 2.3. *If a function $f : X \to Y$ is δ-continuous, then f is almost p_s-continuous.*

Proof. Let $x \in X$ and V be any open set of Y containing $f(x)$. Since f is δ-continuous, there exists an open set U of X containing x such that $f(IntClU) \subseteq IntClV$. Since $IntClU$ is a regular open set, hence it is p_s-open set of X containing x. Therefore, f is almost p_s-continuous. □

From Lemma 2.2, Proposition 2.3 and Diagram 3.1 in [15], we obtain the following diagram:

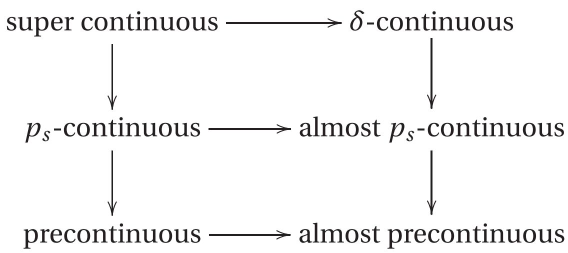

Diagram 2.1

In the sequel, we shall show that none of the implications that concerning almost p_s-continuity in Diagram 2.1 is reversible.

Example 2.4. Let $X = \{a,b,c,d\}$ with the two topologies $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{\phi, X, \{c\}, \{a,d\}, \{a,c,d\}\}$; then the family of p_s-open subsets of X with respect to τ is: $P_sO(X) = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}\}$. Let $f : (X,\tau) \to (X,\sigma)$ be the identity function. Then f is almost p_s-continuous, but it is not p_s-continuous, because $\{a,d\}$ is an open set in (X,τ) containing $f(d) = d$, there exist no p_s-open set U in (X,τ) containing d such that $d \in f(U) \subseteq \{a,d\}$.

Example 2.5. Let $X = \{a,b,c,d\}$ with the two topologies $\tau = \{\phi, X, \{c\}, \{a,d\}, \{a,c,d\}\}$ and $\sigma = \{\phi, X, \{a\}, \{b,c\}, \{a,b,c\}\}$; then the family of p_s-open subsets of X with respect to τ is: $P_sO(X) = \{\phi, X, \{c\}, \{a,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$. Let $f : (X,\tau) \to (X,\sigma)$ be a function defined as follows: $f(a) = f(b) = f(c) = a$ and $f(d) = b$. Then f is almost precontinuous (see Example 4.5 [21]) . However f is not almost p_s-continuous since $\{b,c\}$ is an open set in (X,τ) containing $f(d) = b$, there exist no p_s-open set U in (X,τ) containing d such that $f(\{d\}) = b \in f(U) \subseteq IntCl\{b,c\} = \{b,c\}$.

Example 2.6. Let $X = \{a,b,c,d\}$ with the two topologies $\tau = \{\phi, X, \{c\}, \{d\}, \{a,c\}, \{c,d\}, \{a,c,d\}\}$ and let $Y = \{x,y,z\}$ with the topology $\sigma = \{\phi, Y, \{x\}, \{y\}, \{x,y\}\}$; then the family of p_s-open subset of X with respect to τ is: $P_sO(X) = \{\phi, X, \{c\}, \{d\}, \{a,c\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$. Let $f : (X,\tau) \to (X,\sigma)$ be a function defined as follows: $f(a) = z$ and $f(b) = f(c) = f(d) = y$. Then f is almost p_s-continuous. But f is not almost continuous [21] Example 4.2 and hence it is not δ-continuous.

Theorem 2.7. *For a function $f : X \to Y$, the following statements are equivalent:*

(1) *f is almost p_s-continuous.*

(2) *For each $x \in X$ and each open set V of Y containing $f(x)$, there exists a p_s-open set U in X containing x such that $f(U) \subseteq sClV$.*

(3) *For each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists a p_s-open set U in X containing x such that $f(U) \subseteq V$.*

(4) *For each $x \in X$ and each δ-open set V of Y containing $f(x)$, there exists a p_s-open set U in X containing x such that $f(U) \subseteq V$.*

Proof. (1) ⇒ (2). Let $x \in X$ and let V be any open set of Y containing $f(x)$. By (1), there exists a p_s-open set U of X containing x such that $f(U) \subseteq IntClV$. Since V is open, hence V is preopen set. Therefore, by Lemma 1.11(2), $f(U) \subseteq sClV$.
(2) ⇒ (3). Let $x \in X$ and let V be any regular open set of Y containing $f(x)$. Then V is an open set of Y containing $f(x)$. By (2), there exists a p_s-open set U in X containing x such that $f(U) \subseteq sClV$. Since V is regular open and hence is preopen set. Therefore, by Lemma 1.11(2), $f(U) \subseteq IntClV$. Since V is regular open, then $f(U) \subseteq V$.
(3) ⇒ (4). Let $x \in X$ and let V be any δ-open set of Y containing $f(x)$. Then for each $f(x) \in V$, there exists an open set G containing $f(x)$ such that $G \subseteq IntClG \subseteq V$. Since $IntClG$ is a regular open set of Y containing $f(x)$, by (3), there exists a p_s-open set U in X containing x such that $f(U) \subseteq IntClG \subseteq V$. This completes the proof.
(4) ⇒ (1). Let $x \in X$ and let V be any open set of Y containing $f(x)$. Then $IntClV$ is δ-open set of Y containing $f(x)$. By (4), there exists a p_s-open set U in X containing x such that $f(U) \subseteq IntClV$. Therefore, f is almost p_s-continuous. □

Theorem 2.8. *For a function $f : X \to Y$, the following statements are equivalent:*

(1) *f is almost p_s-continuous.*

(2) *$f^{-1}(IntClV)$ is p_s-open set in X, for each open set V in Y.*

(3) *$f^{-1}(ClIntF)$ is p_s-closed set in X, for each closed set F in Y.*

(4) *$f^{-1}(F)$ is p_s-closed set in X, for each regular closed set F of Y.*

(5) *$f^{-1}(V)$ is p_s-open set in X, for each regular open set V of Y.*

Proof. (1) ⇒ (2). Let V be any open set in Y. We have to show that $f^{-1}(IntClV)$ is p_s-open set in X. Let $x \in f^{-1}(IntClV)$. Then $f(x) \in IntClV$ and $IntClV$ is a regular open set in Y. Since f is almost p_s-continuous, by Theorem 2.7, there exists a p_s-open set U of X containing x such that $f(U) \subseteq IntClV$. Which implies that $x \in U \subseteq f^{-1}(IntClV)$. Therefore, $f^{-1}(IntClV)$ is p_s-open set in X.
(2) ⇒ (3). Let F be any closed set of Y. Then $Y - F$ is an open set of Y. By (2), $f^{-1}(IntCl(Y \setminus F))$ is p_s-open set in X and $f^{-1}(IntCl(Y \setminus F)) = f^{-1}(Int(Y \setminus IntF)) = f^{-1}(Y \setminus ClIntF) = X \setminus f^{-1}(ClIntF)$ is p_s-open set in X and hence $f^{-1}(ClIntF)$ is p_s-closed set in X.
(3) ⇒ (4). Let F be any regular closed set of Y. Then F is a closed set of Y. By (3), $f^{-1}(ClIntF)$ is p_s-closed set in X. Since F is regular closed set, then $f^{-1}(ClIntF) = f^{-1}(F)$. Therefore,

$f^{-1}(F)$ is p_s-closed set in X.
(4) ⇒ (5). Let V be any regular open set of Y. Then $Y \setminus V$ is regular closed set of Y and by (4), we have $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is p_s-closed set in X and hence $f^{-1}(V)$ is p_s-open set in X.
(5) ⇒ (1). Let $x \in X$ and let V be any regular open set of Y containing $f(x)$. Then $x \in f^{-1}(V)$. By (5), we have $f^{-1}(V)$ is p_s-open set in X. Therefore, we obtain $f(f^{-1}(V)) \subseteq V$. Hence by Theorem 2.7, f is almost p_s-continuous. □

The following result can be proved easily from the above theorem.

Proposition 2.9. *Let $f : X \to Y$ be a function. Let B be any basis for τ_S in Y. Then f is almost p_s-continuous if and only if for each $B \in B$, $f^{-1}(B)$ is a p_s-open subset of X.*

Theorem 2.10. *For a function $f : X \to Y$, the following statements are equivalent:*

(1) *f is almost p_s-continuous.*
(2) *$f(P_sCl A) \subseteq Cl_\delta f(A)$, for each $A \subseteq X$.*
(3) *$P_sClf^{-1}(B) \subseteq f^{-1}Cl_\delta(B)$, for each $B \subseteq Y$.*
(4) *$f^{-1}(F)$ is p_s-closed set in X, for each δ-closed set F of Y.*
(5) *$f^{-1}(V)$ is p_s-open set in X, for each δ-open set V of Y.*
(6) *$f^{-1}(Int_\delta B) \subseteq P_sIntf^{-1}(B)$, for each $B \subseteq Y$.*

Proof. (1) ⇒ (2). Let A be a subset of X. Since $Cl_\delta f(A)$ is δ-closed set in Y, so $Cl_\delta f(A) = \cap\{F_\alpha : F_\alpha \in RC(Y), \alpha \in \Lambda\}$, where Λ is an index set. Then $A \subseteq f^{-1}(Cl_\delta f(A)) = f^{-1}(\cap\{F_\alpha : \alpha \in \Lambda\})$ $=\cap\{f^{-1}(F_\alpha) : \alpha \in \Lambda\}$. By (1) and Theorem 2.8, $f^{-1}(Cl_\delta f(A))$ is p_s-closed set of X. Hence $P_sClA \subseteq f^{-1}(Cl_\delta f(A))$. Therefore, we obtain that $f(P_sClA) \subseteq Cl_\delta f(A)$.
(2) ⇒ (3). Let B be any subset of Y. Then $f^{-1}(B)$ is a subset of X. By (2), we have $f(P_sClf^{-1}(B)) \subseteq Cl_\delta f(f^{-1}(B)) = Cl_\delta B$. Hence $P_sClf^{-1}(B) \subseteq f^{-1}(Cl_\delta B)$.
(3) ⇒ (4). Let F be any δ-closed set of Y. By (3), we have $P_sClf^{-1}(F)) \subseteq f^{-1}(Cl_\delta F)$=$f^{-1}(F)$ and hence $f^{-1}(F)$ is p_s-closed set in X.
(4) ⇒ (5). Let V be any δ-open set of Y. Then $Y \setminus V$ is δ-closed set of Y and by (4), we have $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is p_s-closed set in X. Hence $f^{-1}(V)$ is p_s-open set in X.
(5) ⇒ (6). For each subset B of Y. We have $Int_\delta B \subseteq B$. Then $f^{-1}(Int_\delta B) \subseteq f^{-1}(B)$. By (5), $f^{-1}(Int_\delta B)$ is p_s-open set in X. Then $f^{-1}(Int_\delta B) \subseteq P_sIntf^{-1}(B)$.
(6) ⇒ (1). Let $x \in X$ and V be any regular open set of Y containing $f(x)$. Since V is a regular open set, hence it is δ-open and by (6), $f^{-1}(Int_\delta V) \subseteq P_sIntf^{-1}(V)$. Therefore, $f^{-1}(V) \subseteq P_sIntf^{-1}(V)$, so $f^{-1}(V)$ is a p_s-open set in X which contains x and clearly $f(f^{-1}(V)) \subseteq V$. Hence, by Theorem 2.7, f is almost p_s-continuous. □

Theorem 2.11. *For a function $f : X \to Y$, the following statements are equivalent:*

(1) *f is almost p_s-continuous.*
(2) $P_sClf^{-1}(V) \subseteq f^{-1}(ClV)$, *for each β-open set V of Y.*
(3) $f^{-1}(IntF) \subseteq P_sIntf^{-1}(F)$, *for each β-closed set F of Y.*
(4) $f^{-1}(IntF) \subseteq P_sIntf^{-1}(F)$, *for each semi-closed set F of Y.*
(5) $P_sClf^{-1}(V) \subseteq f^{-1}(ClV)$, *for each semi-open set V of Y.*

Proof. (1) ⇒ (2). Let V be any β-open set of Y. It follows from Lemma 1.11(4) that ClV is regular closed set in Y. Since f is almost p_s-continuous, by Theorem 2.8, $f^{-1}(ClV)$ is p_s-closed set in X. Therefore, we obtain $P_sClf^{-1}(V) \subseteq f^{-1}(ClV)$.
(2) ⇒ (3). Let F be any β-closed set of Y. Then $Y \setminus F$ is β-open set of Y and by (2), we have $P_sClf^{-1}(Y \setminus F) \subseteq f^{-1}(Cl(Y \setminus F))$ and $P_sCl(X \setminus f^{-1}(F)) \subseteq f^{-1}(Y \setminus IntF)$ and hence, $X \setminus P_sIntf^{-1}(F) \subseteq X \setminus f^{-1}(IntF)$. Therefore, $f^{-1}(IntF) \subseteq P_sIntf^{-1}(F)$.
(3) ⇒ (4). Obvious since every semi-closed set is β-closed.
(4) ⇒ (5). Let V be any semi-open set of Y. Then $Y \setminus V$ is semi-closed set in Y and by (4), we have $f^{-1}(Int(Y \setminus V)) \subseteq P_sIntf^{-1}(Y \setminus V)$ and $f^{-1}(Y \setminus ClV) \subseteq P_sInt(X \setminus f^{-1}(V))$ and hence, $X \setminus f^{-1}(ClV) \subseteq X \setminus P_sClf^{-1}(V)$. Therefore, $P_sClf^{-1}(V) \subseteq f^{-1}(ClV)$.
(5) ⇒ (1). Let F be any regular closed set of Y. Then F is a semi-open set of Y. By (5), we have $P_sClf^{-1}(F) \subseteq f^{-1}(ClF) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is a p_s-closed set in X. Therefore, by Theorem 2.8, f is almost p_s-continuous. □

Corollary 2.12. *For a function $f : X \to Y$, the following statements are equivalent:*

(1) *f is almost p_s-continuous.*
(2) $P_sClf^{-1}(V) \subseteq f^{-1}(\alpha ClV)$, *for each β-open set V of Y.*
(3) $P_sClf^{-1}(V) \subseteq f^{-1}(Cl_\delta V)$, *for each β-open set V of Y*
(4) $P_sClf^{-1}(V) \subseteq f^{-1}(P_sClV)$, *for each semi-open set V of Y.*
(5) $P_sClf^{-1}(V) \subseteq f^{-1}(pClV)$, *for each semi-open set V of Y.*

Proof. (1) ⇒ (2). Follows from Theorem 2.11 and Lemma 1.12(2).
(2) ⇒ (3). Follows from the fact that $\alpha ClV \subseteq Cl_\delta V$.
(3) ⇒ (4) and (4) ⇒ (5). Follows from Theorem 2.11 and Lemma 1.12(1).
(5) ⇒ (1). Follows from Theorem 2.11 and Lemma 1.12(1).
□

The following result also can be concluded directly.

Corollary 2.13. *For a function $f : X \to Y$, the following statements are equivalent:*

(1) *f is almost p_s-continuous.*

(2) $f^{-1}(\alpha IntF) \subseteq P_s Intf^{-1}(F)$, *for each* β*-closed set* F *of* Y.

(3) $f^{-1}(Int_\delta F) \subseteq P_s Intf^{-1}(F)$, *for each* β*-closed set* F *of* Y.

(4) $f^{-1}(P_s IntF) \subseteq P_s Intf^{-1}(F)$, *for each semi-closed set* F *of* Y.

(5) $f^{-1}(pIntF) \subseteq P_s Intf^{-1}(F)$, *for each semi-closed set* F *of* Y.

Theorem 2.14. *A function* $f : X \to Y$ *is almost* p_s*-continuous if and only if* $f^{-1}(V) \subseteq P_s Intf^{-1}(IntClV)$ *for each preopen set* V *of* Y.

Proof. Necessity. Let V be any preopen set of Y. Then $V \subseteq IntClV$ and $IntClV$ is a regular open set in Y. Since f is almost p_s-continuous, by Theorem 2.8, $f^{-1}(IntClV)$ is p_s-open set in X and hence we obtain that $f^{-1}(V) \subseteq f^{-1}(IntClV)$=$p_sIntf^{-1}(IntClV)$.
Sufficiency. Let V be any regular open set of Y. Then V is a preopen set of Y. By hypothesis, we have $f^{-1}(V) \subseteq P_s Intf^{-1}(IntClV) = P_s Intf^{-1}(V)$. Therefore, $f^{-1}(V)$ is p_s-open set in X and hence by Theorem 2.8, f is almost p_s-continuous. □

We obtain the following corollary.

Corollary 2.15. *The following statements are equivalent for a function* $f : X \to Y$ *:*

(1) f *is almost* p_s*-continuous.*

(2) $f^{-1}(V) \subseteq P_s Intf^{-1}(sClV)$ *for each preopen set* V *of* Y.

(3) $P_s Clf^{-1}(ClIntF) \subseteq f^{-1}(F)$ *for each preclosed set* F *of* Y.

(4) $P_s Clf^{-1}(sIntF) \subseteq f^{-1}(F)$ *for each preclosed set* F *of* Y.

Corollary 2.16. *For a function* $f : X \to Y$*, the following statements are equivalent:*

(1) f *is almost* p_s*-continuous.*

(2) *For each neighborhood* V *of* $f(x)$, $x \in P_s Intf^{-1}(sClV)$.

(3) *For each neighborhood* V *of* $f(x)$, $x \in P_s Intf^{-1}(IntClV)$.

Proof. Follows from Theorem 2.14 and Corollary 2.15. □

Theorem 2.17. *Let* $f : X \to Y$ *be an almost* p_s*-continuous function and let* V *be any open subset of* Y. *If* $x \in P_s Clf^{-1}(V) \setminus f^{-1}(V)$, *then* $f(x) \in P_s ClV$.

Proof. Let $x \in X$ be such that $x \in P_s Clf^{-1}(V) \setminus f^{-1}(V)$ and suppose $f(x) \notin P_s ClV$. Then there exists a p_s-open set H containing $f(x)$ such that $H \cap V = \phi$. Then $ClH \cap V = \phi$ implies $IntClH \cap V = \phi$ and $IntClH$ is a regular open set. Since f is almost p_s-continuous, by Theorem 2.7, there exists a p_s-open set U in X containing x such that $f(U) \subseteq IntClH$. Therefore, $f(U) \cap V = \phi$. However, since $x \in P_s Clf^{-1}(V)$, $U \cap f^{-1}(V) \neq \phi$ for every p_s-open set U in X containing x, so that $f(U) \cap V \neq \phi$. We have a contradiction. It follows that $f(x) \in P_s ClV$. □

Theorem 2.18. *If a function $f : X \to Y$ is almost precontinuous. Then the following statements are equivalent:*

(1) *f is almost p_s-continuous.*
(2) *For each $x \in X$ and each open set V of Y containing $f(x)$, there exists a semi-closed set F in X containing x such that $f(F) \subseteq IntClV$.*
(3) *For each $x \in X$ and each open set V of Y containing $f(x)$, there exists a semi-closed set F in X containing x such that $f(F) \subseteq sClV$.*
(4) *For each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists a semi-closed set F in X containing x such that $f(F) \subseteq V$.*
(5) *For each $x \in X$ and each δ-open set V of Y containing $f(x)$, there exists a semi-closed set F in X containing x such that $f(F) \subseteq V$.*

Proof. (1) $\Rightarrow$ (2). Let $x \in X$ and let V be any open set of Y containing $f(x)$. By (1), there exists a p_s-open set U of X containing x such that $f(U) \subseteq IntClV$. Since U is p_s-open set, so for each $x \in U$ there exists a semi-closed set F in X such that $x \in F \subseteq U$. Therefore, we have $f(F) \subseteq IntClV$.
(2) $\Rightarrow$ (3). Obvious.
(3) $\Rightarrow$ (4). Let $x \in X$ and let V be any regular open set of Y containing $f(x)$. Then V is an open set of Y containing $f(x)$. By (3), there exists a semi-closed set F in X containing x such that $f(F) \subseteq sClV$. Since V is regular open and hence is preopen. Therefore, by Lemma 1.11(2), $f(F) \subseteq IntClV$. Since V is regular open, then $f(F) \subseteq V$.
(4) $\Rightarrow$ (5). Let $x \in X$ and let V be any δ-open set of Y containing $f(x)$. Then for each $f(x) \in V$, there exists an open set G containing $f(x)$ such that $G \subseteq IntClG \subseteq V$. Since $IntClG$ is a regular open set of Y containing $f(x)$, by (4), there exists a semi-closed set F in X containing x such that $f(F) \subseteq IntClG \subseteq V$. This completes the proof.
(5) $\Rightarrow$ (1). Let V be any δ-open set of Y. We have to show that $f^{-1}(V)$ is p_s-open set in X. Since f is almost precontinuous, by Proposition 1.14, $f^{-1}(V)$ is preopen set in X. Let $x \in f^{-1}(V)$, then $f(x) \in V$. By hypothesis, there exists a semi-closed set F of X containing x such that $f(F) \subseteq V$. Which implies that $x \in F \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V)$ is p_s-open set in X. Hence by Theorem 2.10, f is almost p_s-continuous. □

Theorem 2.19. *A function $f : (X,\tau) \to (Y,\sigma)$ is almost p_s-continuous if and only if $f : (X,\tau) \to (Y,\sigma_S)$ is p_s-continuous.*

Proof. Necessity. Let $H \in \sigma_S$, then H is a regular open set in (Y,σ). Since $f : (X,\tau) \to (Y,\sigma)$ is almost p_s-continuous, by Theorem 2.8, $f^{-1}(H)$ is p_s-open set in X. Therefore, $f : (X,\tau) \to (Y,\sigma_S)$ is p_s-continuous.

Sufficiency. Let G be any regular open set in (Y,σ). Then $G \in \sigma_S$. Since $f:(X,\tau)\to(Y,\sigma_S)$ is p_s-continuous, by Definition 1.13, $f^{-1}(G)$ is p_s-open set in X. Therefore, by Theorem 2.8, $f:(X,\tau)\to(Y,\sigma)$ is almost p_s-continuous. □

Theorem 2.20. *Let X be a locally indiscrete space. Then the function $f:(X,\tau)\to(Y,\sigma)$ is almost p_s-continuous if and only if $f:(X,\tau)\to(Y,\sigma_S)$ is continuous.*

Proof. Necessity. Let $H \in \sigma_S$, then H is a regular open set in (Y,σ). Since $f:(X,\tau)\to(Y,\sigma)$ is almost p_s-continuous, by Theorem 2.8, $f^{-1}(H)$ is p_s-open set in X. Since X is locally indiscrete space, by Proposition 1.7(4), $f^{-1}(H)$ is open set in X. Therefore, $f:(X,\tau)\to(Y,\sigma_S)$ is continuous.
Sufficiency. Let G be any regular open set in (Y,σ). Then $G\in\sigma_S$. Since $f:(X,\tau)\to(Y,\sigma_S)$ is continuous, so $f^{-1}(G)$ is open set in X. Since X is locally indiscrete space, by Proposition 1.7(4), $f^{-1}(G)$ is p_s-open set in X. Therefore, by Theorem 2.8, $f:(X,\tau)\to(Y,\sigma)$ is almost p_s-continuous. □

3. Properties and Comparisons

In this section, we give some properties of almost p_s-continuous functions and compare it with other types of continuous functions.

Proposition 3.1. *Let $f:X\to Y$ be an almost p_s-continuous function. If A is either open or regular semi-open subset of X, then $f|A:A\to Y$ is almost p_s-continuous in the subspace A.*

Proof. Let V be any regular open set of Y. Since f is almost p_s-continuous, by Theorem 2.8, $f^{-1}(V)$ is p_s-open set in X. Since A is either open or regular semi-open subset of X. By Lemma 1.10(4), $(f|A)^{-1}(V)=f^{-1}(V)\cap A$ is a p_s-open subset of A. This shows that $f|A:A\to Y$ is almost p_s-continuous. □

Corollary 3.2. *Let $f:X\to Y$ be almost p_s-continuous function. If A is regular open subset of X, then $f|A:A\to Y$ is almost p_s-continuous in the subspace A.*

Proof. Follows from Proposition 3.1 □

Theorem 3.3. *A function $f:X\to Y$ is almost p_s-continuous. If for each $x\in X$, there exists a regular open set A of X containing x such that $f|A:A\to Y$ is almost p_s-continuous.*

Proof. Let $x\in X$, then by hypothesis, there exists a regular open set A containing x such that $f|A:A\to Y$ is almost p_s-continuous. Let V be any open set of Y containing f (x), there exists a p_s-open set U in A containing x such that $(f|A)(U)\subseteq IntClV$. Since A is regular open, by Lemma 1.10(3), U is p_s-open set in X and hence $f(U)\subseteq IntClV$. This shows that f is almost p_s-continuous. □

Corollary 3.4. *Let $\{U_\gamma : \gamma \in \Delta\}$ be a regular open cover of a topological space X. A function $f : X \to Y$ is almost p_s-continuous if and only if $f|U_\gamma : U_\gamma \to Y$ is almost p_s-continuous for each $\gamma \in \Delta$.*

Proof. This is an immediate consequence of Corollary 3.2 and Theorem 3.3. □

Theorem 3.5. *If $X = R \cup S$, where R and S are regular open sets and $f : X \to Y$ is a function such that both $f|R$ and $f|S$ are almost p_s-continuous, then f is almost p_s-continuous.*

Proof. Let V be any regular open set of Y. Then $f^{-1}(V) = (f|R)^{-1}(V) \cup (f|S)^{-1}(V)$. Since $f|R$ and $f|S$ are almost p_s-continuous, by Theorem 2.8, $(f|R)^{-1}(V)$ and $(f|S)^{-1}(V)$ are p_s-open sets in R and S, respectively. Since R and S are regular open sets in X, then by Lemma 1.10(3), $(f|R)^{-1}(V)$ and $(f|S)^{-1}(V)$ are p_s-open sets in X. Since a union of two p_s-open sets is p_s-open, hence $f^{-1}(V)$ is p_s-open set in X. Therefore, by Theorem 2.8, f is almost p_s-continuous. □

In general, if $X = \cup\{K_\gamma : \gamma \in \Delta\}$, where each K_γ is a regular open set and $f : X \to Y$ is a function such that $f|K_\gamma$ is almost p_s-continuous for each γ, then f is almost p_s-continuous.

Theorem 3.6. *Let $f : X \to Y$ be almost p_s-continuous and let A be either open or regular semi-open subset of X such that $f(A)$ is dense in Y. Then $f|A : A \to f(A)$ is almost p_s-continuous.*

Proof. Let O be a regular open set of $f(A)$. Then by Lemma 1.11(3), $O = f(A) \cap IntClO$. Thus, $(f|A)^{-1}(O) = (f|A)^{-1}(f(A) \cap IntClO) = (f|A)^{-1}(f(A)) \cap (f|A)^{-1}(IntClO) = A \cap (f|A)^{-1}(IntClO)$ $= A \cap f^{-1}(IntClO) = A \cap f^{-1}(O)$. Since f is almost p_s-continuous, by Theorem 2.8, $f^{-1}(O) = f^{-1}(IntClO)$ is p_s-open in X. Since A is either open or regular semi-open subset of X. Then by Lemma 1.10(4), $(f|A)^{-1}(O)$ is p_s-open set in the subspace A. Therefore, by Theorem 2.8, $f|A : A \to f(A)$ is almost p_s-continuous. □

Theorem 3.7. *Let $X = R_1 \cup R_2$, where R_1 and R_2 are regular open sets in X. Let $f : R_1 \to Y$ and $g : R_2 \to Y$ be almost p_s-continuous. If $f(x) = g(x)$ for each $x \in R_1 \cap R_2$. Then $h : R_1 \cup R_2 \to Y$ such that $h(x) = f(x)$ for $x \in R_1$ and $h(x) = g(x)$ for $x \in R_2$ is almost p_s-continuous.*

Proof. Let O be a regular open set of Y. Now $h^{-1}(O) = f^{-1}(O) \cup g^{-1}(O)$. Since f and g are almost p_s-continuous, by Theorem 2.8, $f^{-1}(O)$ and $g^{-1}(O)$ are p_s-open set in R_1 and R_2 respectively. But R_1 and R_2 are both regular open sets in X. Then by Lemma 1.10(3), $f^{-1}(O)$ and $g^{-1}(O)$ are p_s-open sets in X. Since union of two p_s-open sets is p_s-open, so $h^{-1}(O)$ is a p_s-open set in X. Hence by Theorem 2.8, h is almost p_s-continuous. □

Theorem 3.8. *Let $f : X \to Y$ be almost p_s-continuous surjection and A be an open subset of X. If f is an open function, then the function $g : A \to f(A)$, defined by $g(x) = f(x)$ for each $x \in A$, is almost p_s-continuous.*

Proof. Suppose that $H = f(A)$. Let $x \in A$ and V be any open set in H containing $g(x)$. Since H is open in Y and V is open in H, so V is open in Y. Since f is almost p_s-continuous, hence there exists a p_s-open set U in X containing x such that $f(U) \subseteq IntClV$. Taking $W = U \cap A$, since A is either open or regular semi-open subset of X, by Lemma 1.10(3), W is a p_s-open set in A containing x and $g(W) \subseteq Int_Y Cl_Y V \cap H = Int_Y Cl_H V$. Then $g(W) \subseteq Int_H Cl_H V$. This shows that g is almost p_s-continuous. □

Theorem 3.9. *Let $f : X \to Y$ be almost p_s-continuous. If Y is a preopen subset of Z, then $f : X \to Z$ is almost p_s-continuous.*

Proof. Let V be any regular open set of Z. Since Y is preopen, by Lemma 1.11(1), $V \cap Y$ is a regular open set in Y. Since $f : X \to Y$ is almost p_s-continuous, by Theorem 2.8, $f^{-1}(V \cap Y)$ is a p_s-open set in X. But $f(x) \in Y$ for each $x \in X$. Thus $f^{-1}(V) = f^{-1}(V \cap Y)$ is a p_s-open set of X. Therefore, by Theorem 2.8, $f : X \to Z$ is almost p_s-continuous. □

Theorem 3.10. *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then the composition function $gof : X \to Z$ is almost p_s-continuous if f and g satisfy one of the following conditions:*

(1) *f is p_s-continuous and g is almost continuous.*

(2) *f is almost p_s-continuous and g is δ-continuous.*

(3) *f is continuous and open and g is almost p_s-continuous.*

(4) *f is almost p_s-continuous and g is continuous and open.*

Proof. (1). Let W be any regular open subset of Z. Since g is almost continuous, so $g^{-1}(W)$ is open subset of Y. Since f is p_s-continuous, by Definition 1.13, $(gof)^{-1}(W) = f^{-1}(g^{-1}(W))$ is p_s-open subset in X. Therefore, by Theorem 2.8, gof is almost p_s-continuous.
(2). Let W be any δ-open subset of Z. Since g is δ-continuous, so $g^{-1}(W)$ is δ-open subset of Y. Since f is almost p_s-continuous, by Theorem 2.10, $(gof)^{-1}(W) = f^{-1}(g^{-1}(W))$ is p_s-open subset in X. Therefore, by Theorem 2.10, gof is almost p_s-continuous.
(3). Let W be any regular open subset of Z. Since g is almost p_s-continuous, by Theorem 2.8, $g^{-1}(W)$ is p_s-open subset of Y. Since f is continuous and open, by Theorem 1.16, $f^{-1}(g^{-1}(W)) = (gof)^{-1}(W)$ is a p_s-open set in X. Hence by Theorem 2.8, gof is almost p_s-continuous.
(4). Let $x \in X$ and W be an open set of Z containing $g(f(x))$. Since g is continuous, then $g^{-1}(W)$ is an open set of Y containing $f(x)$. Since f is almost p_s-continuous, there exists a p_s-open set U of X containing x such that $f(U) \subseteq IntClg^{-1}(W)$. Also, since g is continuous, then we obtain $(gof)(U) \subseteq g(Intg^{-1}(ClW))$. Since g is open, we obtain $(gof)(U) \subseteq IntClW$. Therefore, gof is almost p_s-continuous. □

Theorem 3.11. *If $f : X \to Y$ is almost p_s-continuous and $g : Y \to Z$ is super continuous functions. Then the composition function $gof : X \to Z$ is p_s-continuous.*

Proof. Let W be any open subset of Z. Since g is super continuous, then $g^{-1}(W)$ is δ-open subset of Y. Since f is almost p_s-continuous, by Theorem 2.10, $(gof)^{-1}(W) = f^{-1}(g^{-1}(W))$ is p_s-open subset in X. Therefore, by Definition 1.13, gof is p_s-continuous. □

Theorem 3.12. *If $f : X \to Y$ is an almost p_s-continuous function and Y is semi-regular. Then f is p_s-continuous.*

Proof. Let $x \in X$ and let V be any open set of Y containing $f(x)$. By the semi-regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subseteq V$. Since f is almost p_s-continuous, by Theorem 2.7, there exists a p_s-open set U of X containing x such that $f(U) \subseteq G \subseteq V$. Therefore, f is p_s-continuous. □

Corollary 3.13. *Let $f : X \to Y$ be a function and X is locally indiscrete space. Then f is almost p_s-continuous if and only if f is almost continuous.*

Proof. Follows from the definition and Proposition 1.7(4). □

Corollary 3.14. *If X is a locally indiscrete space and Y is semi-regular space, then the following statements are equivalent for a function $f : X \to Y$:*

(1) *f is p_s-continuous.*

(2) *f is almost p_s-continuous.*

(3) *f is almost continuous.*

(4) *f is continuous.*

Proof. Follows from Theorem 3.12, Corollary 3.13 and Proposition 1.7(4). □

Corollary 3.15. *Let $f : X \to Y$ be a function and X is s-regular space. If f is almost continuous, then f is almost p_s-continuous.*

Proof. Proof. Follows from Proposition 1.7(5). □

Corollary 3.16. *Let $f : X \to Y$ be a function and X is semi-T_1 space. Then f is almost p_s-continuous if and only if f is almost precontinuous.*

Proof. Proof. Follows from Proposition 1.7(1). □

Corollary 3.17. *If X is a semi-T_1 space and Y is semi-regular space, then the following statements are equivalent for a function $f : X \to Y$:*

(1) *f is p_s-continuous.*

(2) *f is almost p_s-continuous.*

(3) *f is almost precontinuous.*

(4) *f is precontinuous.*

Proof. Follows from Theorem 3.12, Corollary 3.16 and Proposition 1.7(1). □

Theorem 3.18. *If a function $f : X \to Y$ is almost strongly θ-continuous, then f is almost p_s-continuous.*

Proof. Let V be any regular open set of Y. Since f is almost strongly θ-continuous, so $f^{-1}(V)$ is θ-open set and hence it is p_s-open set. Therefore, by Theorem 2.8, f is almost p_s-continuous. □

Theorem 3.19. *The following statements are equivalent for a function $f : X \to Y$:*

(1) *f is R-map.*

(2) *f is almost p_s-continuous and either weakly θ-irresolute or weakly quasi-continuous.*

(3) *f is almost continuous and either weakly θ-irresolute or weakly quasi-continuous.*

(4) *f is almost α-continuous and either weakly θ-irresolute or weakly quasi-continuous.*

(5) *f is almost precontinuous and either weakly θ-irresolute or weakly quasi-continuous.*

Proof. Follows from their definitions and Proposition 1.9. □

Theorem 3.20. *If $f : X \to Y$ is almost θs-continuous and almost precontinuous, then f is almost p_s-continuous.*

Proof. Let V be a regular open set in Y. Since f is almost θs-continuous and almost precontinuous function, then $f^{-1}(V)$ is both θ-semi-open and preopen set in X. Therefore, $f^{-1}(V)$ is p_s-open set in X. Hence by Theorem 2.8, f is almost p_s-continuous. □

Theorem 3.21. *Let $f : X \to Y$ be a function and X be extremally disconnected space. If f is almost θs-continuous, then f is almost p_s-continuous.*

Proof. Let V be a regular open set in Y. Since f is almost θs-continuous, then $f^{-1}(V)$ is θ-semi-open set in X. Therefore, by Lemma 1.10, $f^{-1}(V)$ is p_s-open set in X. Hence by Theorem 2.8, f is almost p_s-continuous. □

Theorem 3.22. *Let Y be an extremally disconnected space. If $f : X \to Y$ is an almost precontinuous and either S-continuous or θ-irresolute function, then f is almost p_s-continuous.*

Proof. Let V be a regular open set in Y. Since f is almost precontinuous, then $f^{-1}(V)$ is preopen set of X. Since Y is extremally disconnected space, by Proposition 1.7(2), V is regular closed set of Y. Since f is either S-continuous or θ-irresolute, then $f^{-1}(V)$ is the union of regular closed sets of X and hence is the union of semi-closed sets of X. By Proposition 1.3, $f^{-1}(V)$ is p_s-open set of X. Therefore, by Theorem 2.8, f is almost p_s-continuous. □

Corollary 3.23. *Let $f : X \to Y$ be either S-continuous or θ-irresolute function and Y be an extremally disconnected space. Then f is almost p_s-continuous if and only if f is almost precontinuous.*

Theorem 3.24. *If Y is a hyperconnected space, then every function $f : X \to Y$ is almost p_s-continuous.*

Proof. Let $x \in X$ and V be any open set of Y containing $f(x)$. Since Y is a hyperconnected space, then $ClV = Y$ and hence $IntClV = Y$. Therefore, we have $f(U) \subseteq IntClV$, for any p_s-open set U in X. This shows that f is almost p_s-continuous. □

Theorem 3.25. *If a function $f : X \to Y$ is almost p_s-continuous and semi-open, then $f(P_sClV) \subseteq P_sClf(V)$ for each open set V of X.*

Proof. Let V be any open set of X. Since f is semi-open, then $f(V)$ is semi-open set in Y. Since f is almost p_s-continuous, then by Corollary 2.12, we obtain that $P_sClf^{-1}(f(V)) \subseteq f^{-1}(P_sClf(V))$ which implies that $f(P_sClV) \subseteq P_sClf(V)$. □

Corollary 3.26. *If a function $f : X \to Y$ is almost p_s-continuous and semi-open, then $P_sIntf(F) \subseteq f(P_sIntF)$ for each closed set F of X.*

Theorem 3.27. *If a function $f : X \to Y$ is irresolute and preopen. Then f is almost p_s-continuous if and only if $P_sClf^{-1}(V) = f^{-1}(P_sClV)$ for each semi-open set V of Y.*

Proof. Necessity. Let V be any semi-open set of Y. Since f is almost p_s-continuous, by Corollary 2.12, $P_sClf^{-1}(V) \subseteq f^{-1}(P_sClV)$. Since V is semi-open set of Y, by Lemma 1.12, $P_sClV = ClV$ which implies that $f^{-1}(P_sClV) \subseteq f^{-1}(ClV)$. Since V is semi-open set of Y and f is preopen, by Theorem 1.17, we have $f^{-1}(P_sClV) \subseteq f^{-1}(ClV) \subseteq Clf^{-1}(V)$ and hence $f^{-1}(P_sClV) \subseteq Clf^{-1}(V)$. Since V is semi-open set of Y and f is irresolute, so $f^{-1}(V)$ is semi-open set in X. Then by Lemma 1.12, we obtain that $f^{-1}(P_sClV) \subseteq P_sClf^{-1}(V)$. Therefore, we have $P_sClf^{-1}(V) = f^{-1}(P_sClV)$. Sufficiency. Follows from Theorem 2.12. □

From the above theorem and Lemma 1.12 we obtain the following results:

Corollary 3.28. *If a function $f : X \to Y$ is almost p_s-continuous, irresolute and preopen, then $P_sIntf^{-1}(F) = f^{-1}(P_sIntF)$ for each semi-closed set F of Y.*

Corollary 3.29. *If a function $f : X \to Y$ is almost p_s-continuous, irresolute and preopen, then $Clf^{-1}(V) = f^{-1}(ClV)$ for each semi-open set V of Y.*

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