TAMKANG JOURNAL OF MATHEMATICS
Volume 43, Number 4, 533-539, Winter 2012
doi:10.5556/j.tkjm.43.2012.533-539
Available online at http://journals.math.tku.edu.tw/

ON PAIRWISE b-LOCALLY OPEN AND PAIRWISE b-LOCALLY CLOSED FUNCTIONS IN BITOPOLOGICAL SPACES

BINOD CHANDRA TRIPATHY AND DIGANTA JYOTI SARMA

Abstract. The aim of this paper is to introduce and study pairwise b-locally open and pairwise b-locally closed functions in bitopological spaces and some characterization and several properties concerning these concepts are investigated.

1. Introduction and preliminaries

The study of bitopological spaces was first initiated by Kelly [4] and thereafter a large numbers of papers have been done to generalize the topological concepts to bitopological setting. Andrijevic [1] defined the notion of b-open sets in topological spaces. A set A is said to be b-open if $A \subset \mathrm{cl}(\mathrm{int}(A)) \cup \mathrm{int}(\mathrm{cl}(A))$. The complement of b-open set is called b-closed.

The notion of locally closedness was first introduced by Kurotowski and Sierpienski [5]. There after Nasef [6] introduced and studied b-locally closed sets in topological spaces. Rajesh [8] generalized the concepts of b-locally closed sets in bitopological setting as follows.

Definition 1.1. A subset A of a bitopological space (X, τ_1, τ_2) is called (τ_1, τ_2)-b-locally closed (in short (τ_1, τ_2)-bLC) if $A = P \cap Q$ where P is τ_1-b-open and Q is τ_2-b-closed in (X, τ_1, τ_2).

Recently Tripathy and Sarma [9] have introduced the notion of b-locally open sets in bitopological spaces as follows.

Definition 1.2. A subset A of a bitopological space (X, τ_1, τ_2) is called (τ_1, τ_2)-b-locally open (in short (τ_1, τ_2)-bLO) if $A = P \cup Q$, where P is τ_1-b-closed and Q is τ_2-b-open in (X, τ_1, τ_2).

Tripathy and Sarma [10] have introduced the following definitions.

Definition 1.3. The (τ_1, τ_2)-b-locally interior of a set A is denoted by (τ_1, τ_2)-$bL\,\mathrm{int}(A)$ and defined as (τ_1, τ_2)-$bL\,\mathrm{int}(A) = \cup\{B : B \text{ is } (\tau_1, \tau_2)\text{-}b\text{-locally open set and } B \subset A\}$.

Corresponding author: Binod Chandra Tripathy.
2010 *Mathematics Subject Classification*. 54A05, 54A10, 54D20, 54C05, 54C08, 54C10.
Key words and phrases. Bitopological spaces, b-open sets, b-closed sets, pairwise b-locally open, pairwise b-locally closed.

Definition 1.4. The (τ_1,τ_2)-b-locally closure of a set A is denoted by (τ_1,τ_2)-$bL\,\mathrm{cl}(A)$ and defined as (τ_1,τ_2)-$bL\,\mathrm{cl}(A) = \cap\{B : B$ is (τ_1,τ_2)-b-locally closed set and $A \subset B\}$.

In view of the above definitions we have the following results.

(i) (τ_1,τ_2)-$bL\,\mathrm{int}(A) \subset A$ and (τ_1,τ_2)-$bL\,\mathrm{cl}(A) \subset A$.

(ii) $A \subset B \Longrightarrow (\tau_1,\tau_2)$-$bL\,\mathrm{int}(A) \subset (\tau_1,\tau_2)$-$bL\,\mathrm{int}(B)$ and (τ_1,τ_2)-$bL\,\mathrm{cl}(A) \subset (\tau_1,\tau_2)$-$bL\,\mathrm{cl}(B)$.

Remark 1.1 (Tripathy and Sarma [10] Remark 2.1)**.** A set A is (τ_1,τ_2)-b-locally open if (τ_1,τ_2)-$bL\,\mathrm{int}(A) = A$ and (τ_1,τ_2)-b-locally closed if (τ_1,τ_2)-$bL\,\mathrm{cl}(A) = A$.

The following definition is due to Pervin [7].

Definition 1.5. A function $f : (X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ is said to be pairwise continuous if the induced functions $f : (X,\tau_1) \to (Y,\sigma_1)$ and $f : (X,\tau_2) \to (Y,\sigma_2)$ are both continuous.

In this paper, we introduce the notion of pairwise b-locally open function and pairwise b-locally closed function in bitopological spaces and investigate the properties of these functions.

Throughout (X,τ_1,τ_2) will denote a bitopological space on which no separation axioms are assumed. For a subset A of a bitopological space (X,τ_1,τ_2), τ_i-$\mathrm{cl}(A)$(resp. τ_i-$\mathrm{int}(A)$) denotes the closure (resp. interior) of A with respect to τ_i for $i = 1,2$.

The collection of all (τ_1,τ_2)-bLO sets [resp. (τ_1,τ_2)-bLC sets] of (X,τ_1,τ_2) will be denoted by (τ_1,τ_2)-$bLO(X)$ [resp. (τ_1,τ_2)-$bLC(X)$].

2. Pairwise b-locally open mapping

Definition 2.1. Let (X,τ_1,τ_2) and (Y,σ_1,σ_2) be two bitopological spaces. A function $f : (X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ is called pairwise b-locally open (in short, pairwise bLO) mapping if the image of each (τ_1,τ_2)-b-locally open set in X is σ_i-open set in Y, where $i = 1,2$.

Example 2.1. Let $X = Y = \{a,b,c\}$, $\tau_1 = \{\emptyset,\{b\},X\}$, $\tau_2 = \{\emptyset,\{a\},X\}$, $\sigma_1 = \{\emptyset,\{b\},\{c\},\{b,c\},Y\}$, $\sigma_2 = \{\emptyset,\{c\},\{b,c\},Y\}$. Consider the function $f : (X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ defined by $f(a) = b$, $f(b) = c$, $f(c) = c$. Here (τ_1,τ_2)-bLO sets are $\{\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},X\}$ and image of each (τ_1,τ_2)-bLO sets are σ_1-open. Also (τ_2,τ_1)-bLO sets are $\{\emptyset,\{b\},\{c\},\{a,b\},\{b,c\},X\}$ and image of each (τ_2,τ_1)-bLO sets are σ_2-open. Hence f is pairwise b-locally open mapping.

Theorem 2.1. *Let $f : (X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ be a mapping between two bitopological spaces. Then the following are equivalent :*

(a) *f is pairwise b-locally open mapping.*

(b) $f((\tau_1,\tau_2)\text{-}bL\operatorname{int}(A)) \subset \sigma_i\text{-int}(f(A))$, *for every subset A of X, where $i = 1,2$.*

Proof. (a) $\Longrightarrow$ (b) Since $(\tau_1,\tau_2)\text{-}bL\operatorname{int}(A)$ is a $(\tau_1,\tau_2)\text{-}bLO$ set in X for any subset A of X and $(\tau_1,\tau_2)\text{-}bL\operatorname{int}(A) \subset A$. Then we have $f((\tau_1,\tau_2)\text{-}bL\operatorname{int}(A)) \subset f(A)$.

Since f is pairwise bLO-mapping, therefore $f((\tau_1,\tau_2)\text{-}bL\operatorname{int}(A))$ is σ_i-open.

Hence $f((\tau_1,\tau_2)\text{-}bL\operatorname{int}(A)) \subset \sigma_i\text{-int}(f(A))$.

(b) $\Longrightarrow$ (a) Let A be a $(\tau_1,\tau_2)\text{-}bLO$ set in X.

$$\text{We have} \quad \sigma_i\text{-int}(f(A)) \subset f(A). \tag{1}$$

$$\text{By hypothesis} \quad f((\tau_1,\tau_2)\text{-}bL\operatorname{int}(A)) \subset \sigma_i\text{-int}(f(A)) \Longrightarrow f(A) \subset \sigma_i\text{-int}(f(A)) \tag{2}$$

Therefore from (1) and (2), we get $f(A)$ is σ_i-open in Y. Hence f is pairwise b-locally open map.

Definition 2.2. A subset S is called an (τ_1,τ_2)-b-locally open neighbourhood of a point x of (X,τ_1,τ_2) if there exists a (τ_1,τ_2)-b-locally open set V such that $x \in V \subset S$.

Theorem 2.2. *Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a function. Then the following are equivalent.*

(a) *f is pairwise b-locally open mapping.*

(b) *For each $x \in X$ and each (τ_1,τ_2)-b-locally open neighbourhood A of x in X, there exists a σ_i-neighbourhood C of $f(x)$ such that $C \subset f(A)$, where $i = 1,2$.*

Proof. (a) $\Longrightarrow$ (b) Let A be a (τ_1,τ_2)-b-locally open neighbourhood of x and $x \in X$. Therefore there exists a (τ_1,τ_2)-b-locally open set B in X such that $x \in B \subset A$. Since f is pairwise b-locally open map, so by Theorem 2.1., we have

$$f((\tau_1,\tau_2)\text{-}bL\operatorname{int}(B)) \subset \sigma_i\text{-int}(f(B)) \Longrightarrow f(B) \subset \sigma_i\text{-int}(f(B)).$$

Hence $f(B)$ is σ_i-open set such that $f(x) \in f(B) \subset f(A)$.

Putting $f(B) = C$, we get C is a σ_i-open set such that $C \subset f(A)$.

(b) $\Longrightarrow$ (a) Let A be a (τ_1,τ_2)-b-locally open set in X and $x \in X$. By hypothesis, for each $f(x) \in f(A)$, there exists a σ_i-neighbourhood $B_{f(x)}$ of $f(x)$ such that $B_{f(x)} \subset f(A)$.

Since $B_{f(x)}$ is σ_i-neighbourhood of $f(x)$, there exists a σ_i-open set $C_{f(x)}$ such that $f(x) \in C_{f(x)} \subset B_{f(x)}$.

Now $f(A) = \cup\{C_{f(x)} : f(x) \in f(A)\}$. It is clear that $f(A)$ is σ_i-open.

Hence f is pairwise b-locally open mapping.

Theorem 2.3. *Let, $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\theta_1,\theta_2)$ be two mappings. If $g_0 f: X\to Z$ is pairwise b-locally open map and g is pairwise continuous injection, then f is pairwise b-locally open map.*

Proof. Let A be an (τ_1,τ_2)-b-locally open set in X. Since $g_0 f$ is pairwise b-locally open map, therefore $(g_0 f)(A)$ is θ_i-open.

Further g is pairwise continuous and injective, therefore $g^{-1}(g(f(A)) = f(A)$ is σ_i-open. Hence f is pairwise b-locally open mapping.

Theorem 2.4. *A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise b-locally open if and only if for any subset A of Y and for any (τ_1,τ_2)-b-locally closed set B in X such that $f^{-1}(A)\subset B$, then there exists a σ_i-closed set C containing A such that $f^{-1}(C)\subset B$, where $i=1,2$.*

Proof. Let G be a (τ_1,τ_2)-b-locally open set in (X,τ_1,τ_2).

$$\text{Put}\qquad A=Y-f(G)\Longrightarrow f^{-1}(A)=f^{-1}(Y)-G=X-G. \tag{3}$$

Then $X-G$ is a (τ_1,τ_2)-b-locally closed set in X such that $f^{-1}(A)\subset X-G$. By hypothesis, there exists a σ_i-closed set C containing A such that

$$\begin{aligned} f^{-1}(C)\subset X-G &\Longrightarrow C\subset f(X)-f(G)=Y-f(G)\\ &\Longrightarrow f(G)\subset Y-C. \end{aligned}\tag{4}$$

Since $A\subset C$, we have $Y-C\subset Y-A=f(G)$, by (3)

$$\Longrightarrow Y-C\subset f(G). \tag{5}$$

Therefore from (4) and (5), we get $f(G)=Y-C$ and so $f(G)$ is σ_i-open since C is σ_i-closed.

Hence f is pairwise b-locally open mapping.

Conversely, let A be any subset of Y and B be a (τ_1,τ_2)-b-locally closed set of X such that $f^{-1}(A)\subset B$.

Suppose that f is pairwise b-locally open map. Let $C=Y-f(X-B)$.

Since B is (τ_1,τ_2)-b-locally closed set in X, so $X-B$ is (τ_1,τ_2)-b-locally open set in X.

Since f is pairwise b-locally open, therefore $f(X-B)$ is σ_i-open.

$$\begin{aligned} &\Longrightarrow Y-f(X-B)\text{ is }\sigma_i\text{-closed}\\ &\Longrightarrow C\text{ is }\sigma_i\text{-closed.}\end{aligned}$$

Now, $C = Y - f(X-B)$

$$\Longrightarrow f^{-1}(C) = X - (X-B) \subset B$$
$$\Longrightarrow f^{-1}(C) \subset B.$$

Since $f^{-1}(A) \subset B$

$$\Longrightarrow X - f^{-1}(A) \supset X - B$$
$$. \Longrightarrow Y - A \supset f(X-B)$$
$$\Longrightarrow A \subset Y - f(X-B) = C$$
$$\Longrightarrow A \subset C$$

Thus there exists a σ_i-closed set C such that $f^{-1}(C) \subset B$.

Theorem 2.5. *Let $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a mapping. Then the following are equivalent.*

(a) *f is pairwise b-locally open mapping.*

(b) *$f^{-1}(\sigma_i\text{-cl}(A)) \subset (\tau_1,\tau_2)\text{-}bL\,\text{cl}(f^{-1}(A))$ for every subset A of Y, where $i = 1,2$.*

Proof. (a) $\Longrightarrow$ (b) Let A be a subset of Y. Therefore we have $f^{-1}(A) \subset (\tau_1,\tau_2)\text{-}bL\,\text{cl}(f^{-1}(A))$ and $(\tau_1,\tau_2)\text{-}bL\text{cl}(f^{-1}(A))$ is (τ_1,τ_2)-b-locally closed set in X. Since f is pairwise b-locally open mapping, so by Theorem 2.4, there exists a σ_i-closed set B such that $A \subset B$ and $f^{-1}(B) \subset (\tau_1,\tau_2)\text{-}bL\text{cl}(f^{-1}(A))$.

Also $A \subset B$

$\Longrightarrow f^{-1}(A) \subset f^{-1}(B) \subset (\tau_1,\tau_2)\text{-}bL\text{cl}(f^{-1}(A)) \Longrightarrow f^{-1}(\sigma_i\text{-cl}(A)) \subset (\tau_1,\tau_2)\text{-}bL\text{cl}(f^{-1}(A))$.

(b) $\Longrightarrow$ (a) Let A be a subset of Y and B be a (τ_1,τ_2)-b-locally closed set in X such that $f^{-1}(A) \subset B$. We have $A \subset \sigma_i\text{-cl}(A)$ and $\sigma_i\text{-cl}(A)$ is σ_i-closed.

Therefore by hypothesis, $f^{-1}(\sigma_i\text{-cl}(A)) \subset (\tau_1,\tau_2)\text{-}bL\,\text{cl}(f^{-1}(A)) \subset (\tau_1,\tau_2)\text{-}bL\,\text{cl}(B) = B \Longrightarrow f^{-1}(\sigma_i\text{-cl}(A)) \subset B$.

Hence by Theorem 2.4, we have f is pairwise b-locally open mapping.

3. Pairwise b-locally closed mapping

Definition 3.1. Let (X,τ_1,τ_2) and (Y,σ_1,σ_2) be two bitopological spaces. A function f: $(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is called pairwise b-locally closed (in short, pairwise bLC) mapping if the image of each (τ_1,τ_2)-b-locally closed set in X is σ_i-closed set in Y, where $i = 1,2$.

Example 3.1. Let $X = Y = \{a,b,c\}$, $\tau_1 = \{\emptyset,\{b\},X\}$, $\tau_2 = \{\emptyset,\{a\},X\}$, $\sigma_1 = \{\emptyset,\{a\},\{a,b\},Y\}$, $\sigma_2 = \{\emptyset,\{a\},\{a,b\},\{a,c\},Y\}$. Consider the function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ defined by $f(a) = b$,

$f(b) = c$, $f(c) = c$. Here (τ_1,τ_2)-bLC sets are $\{\emptyset,\{b\},\{c\},\{a,b\},\{b,c\},X\}$ and image of each (τ_1,τ_2)-bLC sets are σ_1-closed. Also (τ_2,τ_1)-bLC sets are $\{\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},X\}$ and image of each (τ_2,τ_1)-bLC sets are σ_2-closed. Hence f is pairwise b-locally closed mapping.

Theorem 3.1. *Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a mapping between two bitopological spaces. Then the following are equivalent :*

(a) *f is pairwise b-locally closed mapping.*

(b) *σ_i-cl$(f(A))\subset f((\tau_1,\tau_2)$-$bLcl(A))$, for each subset A of X, where $i=1,2$.*

Proof. (a) $\Longrightarrow$ (b) Let f be pairwise b-locally closed mapping. Then for any subset A of X we have $A\subset(\tau_1,\tau_2)$-bLcl(A) and (τ_1,τ_2)-bLcl(A) is (τ_1,τ_2)-b-locally closed set in X. Thus $f(A)\subset f((\tau_1,\tau_2)$-$bLcl(A))$.

By assumption we obtain $f((\tau_1,\tau_2)$-bLcl$(A))$ is σ_i-closed.

Hence σ_i-cl$(f(A))\subset f((\tau_1,\tau_2)$-$bLcl(A))$.

(b) $\Longrightarrow$ (a) Let A be a (τ_1,τ_2)-b-locally closed set in X. We have $f(A)\subset\sigma_i$-cl$(f(A))$. By hypothesis, σ_i-cl$(f(A))\subset f((\tau_1,\tau_2)$-$bLcl(A))\Longrightarrow\sigma_i$-cl$(f(A))\subset f(A)$.

Hence $f(A)$ is σ_i-closed in Y and therefore f is pairwise b-locally closed mapping.

Theorem 3.2. *Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a mapping. Then the following are equivalent.*
(a) f be pairwise b-locally closed mapping.

(b) For any subset A of Y and for any (τ_1,τ_2)-b-locally open set B in X such that $f^{-1}(A)\subset B$, then there exists a σ_i-open set C containing A such that $f^{-1}(C)\subset B$, where $i=1,2$.

Proof. The proof is straightforward, therefore omitted. □

Definition 3.2 ([8]). A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise b-locally closed irresolute if $f^{-1}(A)\in(\tau_1,\tau_2)$-$bLC(X)$ for every $A\in(\sigma_1,\sigma_2)$-$bLC(Y)$.

Theorem 3.3. *Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\theta_1,\theta_2)$ be two functions such that $g_0f:X\to Z$ is pairwise b-locally closed mapping. If f is pairwise b-locally closed irresolute surjection, then g is pairwise b-locally closed mapping.*

Proof. Suppose that, A be (σ_1,σ_2)-b-locally closed set in Y. Since f is pairwise b-locally closed irresolute, therefore $f^{-1}(A)$ is (τ_1,τ_2)-b-locally closed set in X.

Since g_0f is pairwise b-locally closed map and f is surjective, we have $(g_0f)(f^{-1}(A))$ is θ_i-closed $\Longrightarrow g(A)$ is θ_i-closed.

Hence g is pairwise b-locally closed map.

Acknowledgement

The authors thank the reviewer for the comments those improved the presentation of the paper.

References

[1] D. Andrijevic, *On b-open sets*, Mat Vesnik, **48** (1996), 59–64.

[2] M. Jelic, *A decomposition of pairwise continuity*, J. Inst. Math. Comput. Sci.Math. Ser., **3**(1990), 25–29.

[3] M. Jelic, *On some mappings of bitopological spaces*, Suppl. Rend. Circ., Math. Palermo (2), **29**(1992), 483–494.

[4] J. C. Kelly, *Bitopological spaces*, Proc, London Math. Soc., **13** (1963), 71–89.

[5] C. Kuratowski and Sierpinski, *Surles difference edeux ensembles ferms*, Tohoku Math. Jour., **20**, (1921), 22–25.

[6] A. A. Nasef, *b-locally closed sets and related topics*, Chaous, Solitions & Fractals, **12**(2001), 1909–1915.

[7] W. J. Pervin, *Connectedness in bitopological spaces*, Indag. Math., **29**(1967), 369–372.

[8] N. Rajesh, *On b-locally closed sets in Bitopological spaces*, Jour. Ind. Acad. Math., **30**(2008), 551–556.

[9] B. C. Tripathy and D. J. Sarma, *On b-locally open sets in Bitopological spaces*, Kyungpook Math. Jour., **51**(2011), 429–433.

[10] B. C. Tripathy and D. J. Sarma, *On pairwise continuous functions in Bitopological spaces*, Acta Sci. Technol., **35**(2013), (in press).

Mathematical Sciences Division, Institute of Advanced Study in Science and Technology, Paschim Boragaon, Garchuk; Guwahati - 781035, Assam, India.

E-mail: tripathybc@yahoo.com; tripathybc@rediffmail.com

E-mail: djs_math@rediffmail.com