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FINITE SUMMATION FORMULAE FOR MULTIVARIABLE H-FUNCTION

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Abstract. In the present paper, an attempt has been made to derive finite summation formulae for the H-function introduced by H.M. Srivastava and R. Panda [6]. Since the multivariable H- Function includes a large number of a special functions of one and more variables as its particular cases. Therefore, the results established here serve as key formulas giving as a large number of new and interesting results by specializing the parameter involved.

1. Introduction

For the multivariable H-Function, which was introduced by Srivastava and Panda ([6], [7]), is an extension of the Multivariable G-function. This multivariable H-Function includes Fox's H- and Meijer's G-functions of One and Two Variables, the generalized Lauricella function of Srivastava and Daoust [4], Apell function, the Whittaker functions etc. Therefore, the results established in this paper are of general character and hence encompass several cases of interest.

The object of this paper is to establish four finite summation formulae for the multivariable H-Function. These formulae will yield a number of new and known results including the results of Gupta and Garg ([2], [3]).

The multivariable H-Function define by the Srivastava and Panda ([6], [7]) and represented in the manner already detailed by Srivastava, Gupta and Goyal ([5], p.251). Since only the parameters with subscript 1 in the definition of the of the multivariable H-function ([5], p.251) undergo changes in our summation formulae that following, to simplify notational problems, we specify only these parameters in them. Thus $H[(a_1-r;h,k),(b_1-r;\beta_h,\beta_k)]$ would represent the multivariable H-function defined by Srivastava, Gupta and Goyal ([5], p.251) but having a_1 replaced by a_1-r, $\alpha_1^{(i)}$ replaced by $h^{(i)}$ $(i=1,\dots,r)$, b_1 replaced by b_1-r, $\beta_1^{(i)}$ replaced by $\beta_h^{(i)}$ $(i=1,\dots,r)$ the rest of the parameters remaining unchanged and so on.

We shall given below three -term contiguous relation for the multivariable H-function and use them later on.

(i) $\quad [\Gamma(1-a_1+r+b_1/\beta)]^{-1}H[(a_1+1-r;h^{(1)},\dots,h^{(r)}),(b_1+1;\beta_{h^{(1)}},\dots,\beta_{h^{(r)}})]$

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$$= \beta[\Gamma(1-a_1+r+b_1/\beta)]^{-1}H[(a_1-r;h^{(1)},\ldots,h^{(r)}),(b_1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$$
$$-\beta[\Gamma(-a_1+r+b_1/\beta)]^{-1}H[(a_1-r+1;h^{(1)},\ldots,h^{(r)}),(b_1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})], \tag{1.1}$$

(ii) $\beta(-1)^r[(b_1+r-a_1\beta)]^{-1}H[(a_1+1;h^{(1)},\ldots,h^{(r)}),(b_1+r-1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$
$$= (-1)^r[(b_1+r-a_1\beta)]^{-1}H[(a_1+1;h^{(1)},\ldots,h^{(r)}),(b_1+r;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$$
$$+(-1)^r[(b_1+r-1-a_1\beta)]^{-1}H[(a_1+1;h^{(1)},\ldots,h^{(r)}),(b_1+r-1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})], \tag{1.2}$$

(iii) $\begin{vmatrix} r-b_1 & 1-a_1+r \\ \beta & 1 \end{vmatrix} H[(a_1-r;h^{(1)},\ldots,h^{(r)}),(b_1-r;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$
$$= H[(a_1-r;h^{(1)},\ldots,h^{(r)}),(b_1-r+1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$$
$$-\beta H[(a_1-r-1;h^{(1)},\ldots,h^{(r)}),(b_1-r;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})], \tag{1.3}$$

(iv) $(b_1-a_1+1)H[(a_1;h^{(1)},\ldots,h^{(r)}),(b_1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$
$$= H[(a_1-1;h^{(1)},\ldots,h^{(r)}),(b_1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]. \tag{1.4}$$

The contiguous relations (1.1), (1.2), (1.3) and (1.4) can be developed on lines similar to those given by Bushman and Gupta [1].

2. Finite summation formula

The finite summation formulae to be established are

(i) $\sum_{r=1}^{n}[\Gamma(1-a_1+r+b_1/\beta)]^{-1}H[(a_1+1-r;h^{(1)},\ldots,h^{(r)}),(b_1+1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$
$$= \beta[(1-a_1+n+b_1/\beta)]^{-1}H[(a_1-n;h^{(1)},\ldots,h^{(r)}),(b_1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$$
$$[(-a_1+b_1/\beta)]^{-1}H[(a_1;h^{(1)},\ldots,h^{(r)}),(b_1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})], \tag{2.1}$$

(ii) $\beta\sum_{r=1}^{n}(-1)^r[(b_1+r-a_1\beta)]^{-1}H[(a_1;h^{(1)},\ldots,h^{(r)}),(b_1+r-1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$
$$= (-1)^n[(b_1+n-a_1\beta)]^{-1}H[(a_1+1;h^{(1)},\ldots,h^{(r)}),(b_1+n;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$$
$$-[(b_1-a_1\beta)]^{-1}H[(a_1+1;h^{(1)},\ldots,h^{(r)}),(b_1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})] \tag{2.2}$$

(iii) $\sum_{r=1}^{n}\begin{vmatrix} r-b_1 & 1-a_1+r \\ \beta & 1 \end{vmatrix}\beta^{r-1}H[(a_1+r;h^{(1)},\ldots,h^{(r)}),(b_1-r;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$
$$= H[(a_1-1;h^{(1)},\ldots,h^{(r)}),(b_1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$$
$$-\beta n H[(a_1-n-1;h^{(1)},\ldots,h^{(r)}),(b_1-n;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})], \tag{2.3}$$

(iv) $\sum_{r=1}^{n}(-1)^r\binom{n}{r}H[(a_n-n+r;h^{(1)},\ldots,h^{(r)}),(b_1+r;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]$
$$= (b_1-a_1+1)_n H[(a_1-1;h^{(1)},\ldots,h^{(r)}),(b_1;\beta_{h^{(1)}},\ldots,\beta_{h^{(r)}})]. \tag{2.4}$$

Provided that the series involved in all the above formulae is absolutely convergent.

Proof. To prove (2.1), putting $r = 1, 2, \ldots, n$ in (1.1) in succession and after taking the sum we see that in the resulting series on the right hand side the alternate terms cancel out , and we arrive at the require results (2.1).

Similarly, (2.2) and (2.3) can be established by using the results (1.2) and (1.3) respectively in place of (1.1). [Multiplying by the quantities $1, \beta^1, \beta^2, \ldots, \beta^{n-1}$ respectively only for (2.3)]. To prove (2.4), if we iterate by expanding term on the right hand side of (1.4) by the use of this varies relation and do not write the repeated parameters $h_1, \ldots, h_r$, again and again continuing this process of iteration, we finally arrive at the require result (2.4).

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