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DOUBLE-DIFFUSIVE CONVECTION IN A VISCOELASTIC FLUID

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Abstract. The double-diffusive convection in an Oldroydian viscoelastic fluid is mathematical investigated under the simultaneous effects of magnetic field and suspended particles through porous medium. A sufficient condition for the invalidity of the 'principle of exchange of stabilities' is derived, in the context, which states that the exchange principle is not valid provided the thermal Rayleigh number R, solutal Rayleigh number R_S, the medium permeability P_1 and the suspended particles parameter B are restricted by the inequality $\frac{BP_1}{\pi^2}(R+R_S)<1$.

1. Introduction

The study of a layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering discipline one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous medium. When a fluid permeates an isotropic and homogeneous porous medium, the gross effect is represented by the Darcy's law. A great number of applications in geophysics may be found in the books by Phillips [1], Ingham and Pop [2], and Nield and Bejan [3].

The theoretical and experimental results on thermal convection in a fluid layer, in the absence and presence of rotation and magnetic field have been given by Chandrasekhar [4]. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis' [5]. The buoyancy force can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration. Double-diffusive convection problems arise in oceanography, limnology and engineering. Examples of particular interest are provided by ponds built to trap solar heat (Tabor and Matz [6]) and some Antarctic lakes (Shirtcliffe [7]). The physics

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is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries.

The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust (Lister [8]). Generally it is accepted that comets consists of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice- versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context (McDonnel [9]). The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region.

In geophysical situations, more often than not, the fluid is not pure but may instead be permeated with suspended (or dust) particles. The effect of suspended particles on the stability of superposed fluids might be of industrial and chemical engineering importance. Further, motivation for this study is the fact that knowledge concerning fluid-particle mixtures is not commensurate with their industrial and scientific importance. Scanlon and Segel [10] have considered the effect of suspended particles on the onset of Benard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. The effect of suspended particles was found to destabilize the layer whereas the effect of a magnetic field was stabilizing. Palaniswamy and Purushotham [11] have studied the stability of shear flow of stratified fluids with fine dust and found the effect of fine dust to increase the region of instability. Alloui et al. [12] have studied the onset of double-diffusive convection in a horizontal Brinkman cavity and analysis made on the linear stability of the quiescent state within a horizontal porous cavity subject to vertical gradients of temperature and solute. Recently spacecraft observations have confirmed that the dust particle play an important role in the dynamics of atmosphere as well as in the diurnal and surface variations in the temperature of the Martian weather. It is, therefore, of interest to study the presence of suspended particles in astrophysical situations. The fluid has been considered to be Newtonian in all the above studies.

With the growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry, the investigations on such fluids are desirable. The stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below has been investigated by Vest and Arpaci [13]. The nature of the instability and some factors may have different effects on viscoelastic fluids as compared to the Newtonian fluids. For example, Bhatia and Steiner [14] have considered the effect of a uniform rotation on the thermal instability of a Maxwell fluid and have found that rotation has a destabilizing effect in contrast to the stabilizing effect on Newtonian fluid. Experimental demonstration by Toms and Strawbridge [15] has revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of the Oldroyd fluid. The thermal instability of an Oldroydian viscoelastic fluid has been considered in the presence of rotation (Eltayeb [16], Sharma [17]) and magnetic field (Sharma [18]). The thermosolutal instability of an Oldroydian viscoelastic fluid in porous medium has been considered by Sharma and Bhardwaj [19]. Kumar et al. [20] have considered the instability of the plane interface between two Oldroydian viscoelastic superposed fluids in the presence of uniform rotation and variable magnetic field in porous medium. It is found that the magnetic field succeeds in stabilizing certain wave-number range, which were unstable in the absence of magnetic field and rotation for the potentially unstable configuration. Linear stability analysis of Maxwell fluid in the Beńard problem for a double-diffusive mixture in a porous medium based on the Darcy Maxwell model has been studied by Wang and Tan [21]. Sekhar and Jayalatha [22] have considered the linear stability analysis of convection in viscoelastic liquids with temperature-dependent viscosity using normal modes and Galerkin method. It is found that the stationary convection be the preferred mode of instability when the ratio of strain retardation parameter to stress relaxation parameter is greater than unity while the possibility of oscillatory convection arise when this ratio is less than unity. The heat transport in Rayleigh-Beńard convection in viscoelastic liquid with/without gravity modulation using a most minimal representation of Fourier series and a representation with higher modes is studied by Siddeshwar et al. [23] and shown that the effect of gravity modulation is stabilizing thereby leading to a situation of reduced heat transfer. The problem of double-diffusive convection and cross-diffusion in a Maxwell fluid in a horizontal layer in porous media by using the modified Darcy-Brinkman model has been considered by Awad et al. [24] and analytical expression of the critical Darcy-Rayleigh numbers for the onset of stationary and oscillatory convection are derived. Recently, Wang and Tan [25] have studied the double-diffusive convection of viscoelastic fluid with Soret effect in a porous medium by using a modified-Maxwell-Darcy model and have shown that for oscillatory convection the system is destabilizing in the presence of Soret effect. The relaxation time also enhances the instability of the system.

Keeping in mind the importance and applications in chemical engineering, biomechanics and various applications mentioned above, the effects of magnetic field and suspended particles on double-diffusive convection in an Oldroydian viscoelastic fluid through porous medium has been considered in the present paper.

2. Formulation of the problem and perturbation equations

Let T_{ij}, τ_{ij}, e_{ij}, μ, λ, λ_0 ($<\lambda$), p, δ_{ij}, v_i, x_i and $\frac{d}{dt}$ denote respectively the total stress tensor, the shear stress tensor, the rate-of-strain tensor, the viscosity, the stress relaxation time, the strain retardation time, the isotropic pressure, the Kronecker delta, the velocity vector, the position vector and the mobile operator. Then the Oldroydian viscoelastic fluid is described by the constitutive relations

$$\left.\begin{aligned} &T_{ij} = -p\delta_{ij} + \tau_{ij}, \\ &\left(1+\lambda\tfrac{d}{dt}\right)\tau_{ij} = 2\mu\left(1+\lambda_0\tfrac{d}{dt}\right)e_{ij}, \\ &e_{ij} = \tfrac{1}{2}\left(\tfrac{\partial v_i}{\partial x_j} + \tfrac{\partial v_j}{\partial x_i}\right). \end{aligned}\right\} \tag{1}$$

Relations of the type (1) were proposed and studied by Oldroyd [26]. Oldroyd showed that many rheological equations of general validity reduce to (1) when linearized. $\lambda_0 = 0$ yields the Maxwellian fluid, whereas $\lambda = \lambda_0 = 0$ gives the Newtonian viscous fluid.

Here we consider an infinite horizontal layer of an electrically conducting incompressible Oldroydian viscoelastic fluid-particle layer of depth d in porous medium which is acted on by a uniform vertical magnetic field $\vec{H}(0,0,H)$ and gravity field $\vec{g}(0,0,-g)$.

Let $\delta\rho$, δp, θ, γ, $\vec{q}(u,v,w)$, $\vec{q}_d(l,r,s)$ and $\vec{h}(h_x,h_y,h_z)$ denote respectively the perturbations in density ρ, pressure p, temperature T, solute concentration C, fluid velocity (initially zero), particle velocity (initially zero) and magnetic field $\vec{H}$. Let κ, κ', α, α', $\beta(=|\frac{dT}{dz}|)$, $\beta'(|\frac{dC}{dz}|)$ stand for thermal diffusivity, solute diffusivity, thermal coefficient of expansion, an analogous solvent expansion, uniform temperature gradient and uniform solute gradient respectively. The linearized thermosolutal hydromagnetic perturbation equations through porous medium containing suspended particles, following Boussinesq approximation, are

$$\begin{aligned} \frac{\rho_0}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial\vec{q}}{\partial t} &= \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta p + \vec{g}\delta\rho + \frac{\mu_e}{4\pi}\{(\nabla\times\vec{h})\times\vec{H}\} + \frac{KN}{\varepsilon}(\vec{q}_d - \vec{q})\right] \\ &\quad -\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\mu}{k_1}\vec{q}, \end{aligned} \tag{2}$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\vec{q}_d = \vec{q}, \tag{3}$$

$$\nabla\cdot\vec{q} = 0, \tag{4}$$

$$\nabla\cdot\vec{h} = 0, \tag{5}$$

$$(E+b\varepsilon)\frac{\partial\theta}{\partial t} = \beta(w+bs) + \kappa\nabla^2\theta, \tag{6}$$

$$(E' + b\varepsilon)\frac{\partial \gamma}{\partial t} = \beta'(w + bs) + \kappa' \nabla^2 \gamma, \tag{7}$$

$$\varepsilon \frac{\partial \vec{h}}{\partial t} = (\vec{H} \cdot \nabla)\vec{q} + \varepsilon\eta\nabla^2 \vec{h}, \tag{8}$$

where $N(\bar{x}, t)$, denote the number density of the suspended particles, ε is the medium porosity and $K = 6\pi\mu\eta'$ is the Stokes' drag coefficient, η' being the particle radius.

Here $\bar{x}(x, y, z)$, $E = \varepsilon + (1-\varepsilon)\frac{\rho_s C_s}{\rho_0 C}$ and ρ_0, C_f; ρ_s, C_s stand for density and heat capacity of fluid and solid matrix, respectively. E' is an analogous solute parameter, $b = \frac{mNC_p t}{\rho_0 C_f}$, m is the mass of particles per unit volume. C, η and e stand for speed of light, electrical resistivity and charge of an electron.

The equation of state is

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(C - C_0)], \tag{9}$$

where the suffix zero refers to values at the reference level $z = 0$, i.e. ρ_0, T_0 and C_0 stand for density, temperature and solute concentration at the lower boundary $z = 0$.

The change in density $\delta\rho$, caused by the perturbation θ, γ, in temperature and solute concentration, respectively, is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma). \tag{10}$$

3. Dispersion relation

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, h_z, \zeta, \xi, \theta, \gamma] = [W(z), K(z), Z(z), X(z), \Theta(z), \Gamma(z)] \exp(ik_x x + ik_y y + nt), \tag{11}$$

where k_x, k_y are horizontal wave numbers, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and n is, in general, a complex constant. $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ are the z-components of the vorticity and current density, respectively.

Expressing the coordinates (x, y, z) in the new unit of length 'd' and letting

$$a = kd,\ \sigma = \frac{nd^2}{\nu},\ p_1 = \frac{\nu}{\kappa},\ p_2 = \frac{\nu}{\eta},\ P_1 = \frac{k_1}{d^2},\ M = \frac{mN}{\rho_0},\ B = b + 1,\ E_1 = E + b\varepsilon$$

$$E_2 = E' + b\varepsilon,\ q = \frac{\nu}{\kappa'},\ \tau_1 = \frac{\tau\nu}{d^2},\ F = \frac{\lambda\nu}{d^2},\ F_0 = \frac{\lambda_0\nu}{d^2},\ D^* = d \cdot \frac{d}{dz} = dD$$

and suppressing the superscript. The physical significance of suspended particles parameter B is that it does not depend on the model under consideration, however, it does depend upon the porosity of the medium.

Equations (2)-(9) with the help of equations (10) and (11), in non-dimensional form become

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F_0\sigma}{(1+F\sigma)P_1}\right](D^2-a^2)W=\frac{\mu_e Hd}{4\pi\rho_0\nu}(D^2-a^2)DK-\frac{gd^2a^2}{\nu}(\alpha\Theta-\alpha'\Gamma), \quad (12)$$

$$(D^2-a^2-\sigma p_2)K=-\frac{Hd}{\varepsilon\eta}DW, \quad (13)$$

$$(D^2-a^2-\sigma E_1 p_1)\Theta=-\frac{\beta d^2}{\kappa}\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \quad (14)$$

$$(D^2-a^2-\sigma E_2 q_1)\Gamma=-\frac{\beta' d^2}{\kappa'}\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W. \quad (15)$$

Here we consider the case where both the boundaries are free and the medium adjoining the fluid is non-conducting. The case of two free boundaries is slightly artificial, except in stellar atmospheres (Spiegel [27]) and in certain geophysical situations where it is most appropriate. However, the case of two free boundaries allows us to obtain analytical solution without affecting the essential features of the problem. The appropriate boundary conditions for this case are

$$W=D^2W=0, \quad \Theta=0, \quad \Gamma=0, \quad DZ=0, \quad X=0, \qquad \text{at } z=0 \text{ and } z=1. \quad (16)$$

Further $K=0$ on both the boundaries if the regions outside the fluid are perfectly conducting or $DK=\mp aK$ on both the boundaries if the region outside the fluid are insulating. (17)

4. Mathematical analysis

We first prove the following lemma:

Lemma. If $[\sigma=\sigma_r+i\sigma_i, W, \Theta, \Gamma, K]$ is a non-trivial solution of the double eigen value problem for σ_r and σ_i described by the equations (12)-(15) with the boundary conditions (16) and (17). Then a necessary condition for $\sigma=0$ (i.e. $\sigma_r=\sigma_i=0$) to be an eigen value is that

$$\int_0^1(|D\Theta|^2+a^2|\Theta|^2)dz\le\frac{\beta^2d^4B^2}{\kappa^2\pi^2}\int_0^1|W|^2dz,$$

Proof of Lemma. Since $\sigma=0$ is an eigen value, we have from equation (14)

$$(D^2-a^2)\Theta=-\frac{\beta d^2B}{\kappa}W. \quad (18)$$

Multiplying both sides of equation (18) by Θ^* (the complex conjugate of Θ), integrating the resulting equation by parts for sufficient number of times over the vertical range of z by making the use of boundary condition (16) and separating the real parts of both sides of the equation so obtained, we get

$$\int_0^1(|D\Theta|^2+a^2|\Theta|^2)dz=Re\frac{\beta d^2B}{\kappa}\int_0^1\Theta^*W dz=\frac{\beta d^2B}{\kappa}Re\int_0^1\Theta^*W dz. \quad (19)$$

Now

$$Re\int_0^1 \Theta^* W dz \leq \left|\int_0^1 \Theta^* W dz\right| \leq \int_0^1 |\Theta^* W| dz \leq \int_0^1 |\Theta||W| dz$$
$$\leq \sqrt{\int_0^1 |\Theta|^2 dz}\sqrt{\int_0^1 |W|^2 dz}, \qquad \text{(by Schwartz inequality).}$$

Equation (18) and inequality (19) implies that

$$\int_0^1 (|D\Theta|^2 + a^2|\Theta|^2) dz \leq \frac{\beta d^2 B}{\kappa}\sqrt{\int_0^1 |\Theta|^2 dz}\sqrt{\int_0^1 |W|^2 dz}, \tag{20}$$

which in turn implies that

$$\int_0^1 |D\Theta|^2 dz \leq \frac{\beta d^2 B}{\kappa}\sqrt{\int_0^1 |\Theta|^2 dz}\sqrt{\int_0^1 |W|^2 dz}, \tag{21}$$

whence we derive from inequality (21) using Rayleigh-Ritz inequality

$$\int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz, \qquad \text{(since } \Theta = 0 \text{ at } z = 0 \text{ and } z = 1) \tag{22}$$

$$\sqrt{\int_0^1 |\Theta|^2 dz} \leq \frac{\beta d^2 B}{\pi^2 \kappa}\sqrt{\int_0^1 |W|^2 dz}. \tag{23}$$

Inequalities (20) and (23) lead to

$$\int_0^1 (|D\Theta|^2 + a^2|\Theta|^2) dz \leq \left(\frac{\beta d^2 B}{\pi\kappa}\right)^2 \int_0^1 |W|^2 dz, \tag{24}$$

and hence the lemma.

The contents of the above lemma when presented otherwise from the point of view of theoretical hydrodynamics imply that

Lemma. *A necessary condition for the validity of the principle of exchange of stabilities in thermohaline convection configuration of an Oldroydian viscoelastic fluid in porous medium in the presence of magnetic field and suspended particles is that*

$$\int_0^1 (|D\Theta|^2 + a^2|\Theta|^2) dz \leq \left(\frac{\beta d^2 B}{\pi\kappa}\right)^2 \int_0^1 |W|^2 dz.$$

Similarly,

$$\int_0^1 (|D\Gamma|^2 + a^2|\Gamma|^2) dz \leq \left(\frac{\beta' d^2 B}{\pi\kappa'}\right)^2 \int_0^1 |W|^2 dz. \tag{25}$$

The essential contents of lemma are true for this case also.

We now prove the following theorem:

Theorem. *If* $[\sigma=\sigma_r+i\sigma_i, W, \Theta, \Gamma, K]$ *is a non-trivial solution of the double eigen value problem for* σ_r *and* σ_i *described by the equations* (12)-(15) *with the boundary conditions* (16) *and* (17) *for given values of other parameters, then a sufficiency condition for the invalidity of* $\sigma=0$ (*i.e.* $\sigma_r=\sigma_i=0$) *to be an eigen value is that* $\frac{BP_1}{\pi^2}(R+R_S)<1$.

Proof. Multiplying equation (12) by W^* (the complex conjugate of W) and integrating the resulting equation over the vertical range of z, we obtain

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1+\sigma}\right)+\frac{1+F_0\sigma}{(1+F\sigma)P_1}\right]\int_0^1 W^*(D^2-a^2)W\,dz-\frac{\mu_e Hd}{4\pi\rho_0\nu}\int_0^1 W^*(D^2-a^2)DK\,dz \\ +\frac{gd^2a^2}{\nu}\int_0^1 W^*(\alpha\Theta-\alpha'\Gamma)\,dz=0, \tag{26}$$

Integrating equation (26) by parts for sufficient times by making use of boundary conditions (16) and (17) and equations (13)-(15), it follows that

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F_0\sigma}{(1+F\sigma)P_1}\right]I_1+\frac{\mu_e\varepsilon d}{4\pi\rho_0\nu}[I_2+\sigma^* p_2\{I_3+a(|K|_1^2+|K|_0^2)\}] \\ -\frac{ga^2}{\nu}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)\left[\frac{\alpha\kappa}{\beta}(I_4+\sigma^* E_1p_1I_5)-\frac{\alpha'\kappa'}{\beta'}(I_6+\sigma^* E_2q_1I_7)\right]=0, \tag{27}$$

where σ^* is the complex conjugate of σ and

$$I_1=\int_0^1(|DW|^2+a^2|W|^2)\,dz,\qquad I_2=\int_0^1|(D^2-a^2)K|^2\,dz,\qquad I_3=\int_0^1(|DK|^2+a^2|K|^2)\,dz,$$
$$I_4=\int_0^1(|D\Theta|^2+a^2|\Theta|^2)\,dz,\quad I_5=\int_0^1|\Theta|^2\,dz,\quad I_6=\int_0^1(|D\Gamma|^2+a^2|\Gamma|^2)\,dz,\quad I_7=\int_0^1|\Gamma|^2\,dz,$$

where the integrals I_1-I_7. are all positive definite.

Putting $\sigma_r=0$ in equation (27) and separating the real and imaginary parts of the resulting equation, we derive

$$\left[\frac{M\tau_1\sigma_i^2}{\varepsilon(1+\sigma_i^2\tau_1^2)}+\frac{(1+FF_0\sigma_i^2)}{P_1(1+F^2\sigma_i^2)}\right]I_1+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}I_2-\frac{ga^2}{\nu}\left(\frac{B+\sigma_i^2\tau_i^2}{B^2+\sigma_i^2\tau_i^2}\right)\left[\frac{\alpha\kappa}{\beta}I_4-\frac{\alpha'\kappa'}{\beta'}I_6\right] \\ +\frac{ga^2}{\nu}\frac{(B-1)\tau_1\sigma_i^2}{B^2+\sigma_i^2\tau_1^2}\left[\frac{\alpha\kappa}{\beta}E_1p_1I_5-\frac{\alpha'\kappa'}{\beta'}E_2q_1I_7\right]=0, \tag{28}$$

and

$$\sigma_i\left(\begin{array}{l}\left[\frac{1}{\varepsilon}\left(1+\frac{M}{1+\sigma_i^2\tau_1^2}\right)-\frac{(F-F_0)}{1+F^2\sigma_i^2}\right]I_1-\frac{\mu_e\varepsilon\eta p_2}{4\pi\rho_0\nu}[I_3+a\{(|K|^2)_1+(|K|^2)_0\}] \\ -\frac{ga^2}{\nu}\left(\frac{\tau_1(1-B)}{B^2+\sigma_i^2\tau_1^2}\left[\frac{\alpha\kappa}{\beta}I_4-\frac{\alpha'\kappa'}{\beta'}I_6\right]-\frac{B+\tau_i^2\sigma_i^2}{B^2+\sigma_i^2\tau_i^2}\left[\frac{\alpha\kappa}{\beta}E_1p_1I_5-\frac{\alpha'\kappa'}{\beta'}E_2q_1I_7\right]\right)\end{array}\right)=0. \tag{29}$$

Equations (28) and (29) must be satisfied when $\sigma_r=0$. Further since σ_i is also zero as a necessary condition of the theorem, equation (29) is identically satisfied while equation (28) reduces to

$$\frac{1}{P_1}I_1+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}I_2-\frac{ga^2\alpha\kappa}{\nu\beta B}I_4+\frac{ga^2\alpha'\kappa'}{\nu\beta' B}I_6=0. \tag{30}$$

Now making use of inequalities (24), (25) and the inequality

$$\int_0^1 (|DW|^2 + a^2|W|^2)dz \geq a^2 \int_0^1 |W|^2 dz, \qquad \text{(which is always valid)},$$

we derive from the equation (30)

$$\left\{\frac{1}{P_1} I_1 + \frac{\mu_e \varepsilon \eta}{4\pi\rho_0 \nu} I_2 + \frac{g a^2 \alpha' \kappa'}{\nu \beta' B} I_6 + \frac{g a^2 \alpha \kappa}{\nu \beta B} I_4\right\}$$
$$> \left\{\frac{a^2}{P_1}\left[1 - \frac{BP_1}{\pi^2}(R + R_S)\right]\int_0^1 |W|^2 dz + \frac{\mu_e \varepsilon \eta}{4\pi\rho_0 \nu} I_2 + \frac{2g a^2 \alpha' \kappa'}{\nu \beta' B} I_6\right\}, \tag{31}$$

where $R = \frac{g\alpha\beta d^4}{\nu\kappa}$ and $R_S = \frac{g\alpha'\beta' d^4}{\nu\kappa'}$ are the thermal Rayleigh number and solutal Rayleigh number, respectively.

Now if $\frac{BP_1}{\pi^2}(R + R_S) < 1$, then the right hand side of inequality (31) is a positive definite which in turn implies that the left hand side of the inequality (31) must also be positive definite and therefore (30) can not be satisfied. Thus a sufficiency condition for the invalidity of zero being an eigen-value for σ is that $\frac{BP_1}{\pi^2}(R + R_S) < 1$.

It is clear from above that when regions outside the fluid are perfectly conducting

$$a\{(|K|^2)_1 + (|K|^2)_0\} = 0, \tag{32}$$

and hence the above analysis holds good for this case.

Presented otherwise from the point of view of theoretical hydrodynamics, we have the following theorem:

Theorem. *A sufficiency condition for the invalidity of principle of exchange of stabilities in a double-diffusive convection configuration of an Oldroydian viscoelastic fluid in porous medium in the presence of suspended particles and magnetic field is that the thermal Rayleigh number R, solutal Rayleigh number R_S, the medium permeability P_1 and suspended particles parameter B are restricted by the inequality $\frac{BP_1}{\pi^2}(R + R_S) < 1$,*

or in the context of overstability, we can state the above theorem as:

Theorem. *A sufficiency condition for the existence of overstability in a double-diffusive convection configuration of an Oldroydian viscoelastic fluid in porous medium in the presence of suspended particles is that the thermal Rayleigh number R, solutal Rayleigh number R_S, medium permeability P_1 and suspended particles parameter B are restricted by the inequality $\frac{BP_1}{\pi^2}(R + R_S) < 1$.*

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