TAMKANG JOURNAL OF MATHEMATICS
Volume 38, Number 2, 139-151, Summer 2007

SP-CONVERGENCE IN L-TOPOLOGICAL SPACES

ZHEN-GUO XU AND FU-GUI SHI

Abstract. In this paper, SP-convergence theory of nets, ideals and filters are built by means of the concept of strongly preclosed L-sets. Their applications are presented.

1. Introduction and preliminaries

The convergence theory has some significant applications not only in topology and analysis but also in inference and some other aspects.

In [21], Pu and Liu introduced the concepts of the Q-neighborhood and established a systematic Moore-Smith convergence theory of fuzzy nets. Wang extended this theory to L-fuzzy set theory in terms of closed remote-neighborhoods of molecules [25]. Later on, all kinds of convergence theory were presented [3, 4, 5, 10, 11, 12, 14, 18, 16, 17, 23] etc..

In this paper, we shall establish the SP-convergence theory of nets, ideals and filters based on the idea of [25].

Throughout this paper $(L, \vee, \wedge, ')$ is a completely distributive de Morgan algebra, X a nonempty set. L^X is the set of all L-fuzzy sets (or L-sets for short) on X. The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$, respectively.

An element a in L is called prime if $a \geq b \wedge c$ implies $a \geq b$ or $a \geq c$. An element a in L is called co-prime if a' is a prime element [15]. The set of nonunit prime elements in L is denoted by $P(L)$. The set of nonzero co-prime elements in L is denoted by $M(L)$. The set of nonzero co-prime elements in L^X is denoted by $M(L^X)$. Each member in $M(L^X)$ is also called a point.

The binary relation $\prec$ in L is defined as follows : for $a, b \in L$, $a \prec b$ if and only if for every subset $D \subseteq L$, the relation $b \leq \sup D$ always implies the existence of $d \in D$ with $a \leq d$ [13]. In a completely distributive DeMorgan algebra L, each member b is a sup of $\{a \in L \mid a \prec b\}$. In the sense of [19, 25], $\{a \in L \mid a \prec b\}$ is the greatest minimal family of b, in symbol $\beta(G)$. Moreover for $b \in L$, define $\alpha(b) = \{a \in L \mid a' \prec b'\}$ and $\alpha^*(b) = \alpha(b) \cap P(L)$.

For an L-set $G \in L^X$, $\beta(G)$ denotes the greatest minimal family of G and $\beta^*(G) = \beta(G) \cap M(L^X)$.

Received August 16, 2005.
2000 *Mathematics Subject Classification*. 54A40.
Key words and phrases. L-topological space, strongly preclosed remote sets, SP-convergence.

An L-topological space (or L-space for short) is a pair $(X, \mathcal{T})$, where $\mathcal{T}$ is a subfamily of L^X which contains $\underline{0}$, $\underline{1}$ and is closed for any suprema and finite infima. $\mathcal{T}$ is called an L-topology on X. Each member of $\mathcal{T}$ is called an open L-set and its complement is called a closed L-set.

Definition 1.1.([1]) Let $(X, \mathcal{T})$ be an L-space, $G \in L^X$. Then G is called semiopen if $G \leq cl(int(G))$; G is called semiclosed if G' is semiopen.

Definition 1.2.([1]) Let $(X, \mathcal{T})$ be an L-space and $G \in L^X$. We define:

(1) $int_s(G) = \bigvee\{C \in L^X \mid C \leq G, C \text{ is semiopen}\}$;
(2) $cl_s(G) = \bigwedge\{C \in L^X \mid C \geq G, C \text{ is semiclosed}\}$.

$int_s(G)$ and $cl_s(G)$ are called semiinterior and semiclosure of G, respectively.

Definition 1.3.([5]) Let $(X, \mathcal{T})$ be an L-space and $G \in L^X$. Then G is called pre-semiopen if $G \leq int_s(cl(G))$; G is called pre-semiclosed if G' is pre-semiopen.

PSO(X) and **PSC**(X) will always denote the family of pre-semiopen L-sets and the family of pre-semiclosed L-sets in $(X, \mathcal{T})$, respectively.

Definition 1.4.([2, 6, 24]) Let $(X, \mathcal{T})$ be an L-space and $G \in L^X$. Then G is called strongly semiopen (or α-open) if $G \leq int(cl(int(G)))$; G is called strongly semiclosed if G' is strongly semiopen.

SSO(X) and **SSC**(X) will always denote the family of strongly semiopen L-sets and the family of strongly semiclosed L-sets in $(X, \mathcal{T})$, respectively.

In [7] and [8] , the concepts of strongly preopen sets, strongly preclosed sets and SP-irresolute mapping were introduced in [0,1]-fuzzy set theory by Biljana Krateska. They can easily be extended to L-sets as follows:

Definition 1.5. Let $(X, \mathcal{T})$ be an L-space and $G \in L^X$. Then G is called strongly preopen if $G \leq int(cl_p(G))$; G is called strongly preclosed if G' is strongly preopen.

SPO(X) and **SPC**(X) will always denote the family of strongly preopen L-sets and the family of strongly preclosed L-sets in $(X, \mathcal{T})$, respectively.

Definition 1.6.([8]) Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two L-spaces and $f : X \to Y$ be a mapping. f is called SP-irresolute if $f_L^{\leftarrow}(B)$ is strongly preopen in $(X, \mathcal{T}_1)$ for each strongly preopen L-set B in $(Y, \mathcal{T}_2)$.

Definition 1.7. Let $(X, \mathcal{T})$ be an L-space and $G \in L^X$. We define:

(1) $int_{sp}(G) = \bigvee\{D \in L^X \mid D \leq G, D \text{ is strongly preopen}\}$;
(2) $cl_{sp}(G) = \bigwedge\{D \in L^X \mid D \geq G, D \text{ is strongly preclosed}\}$.

$int_{sp}(G)$ and $cl_{sp}(G)$ are called strong preinterior and strong preclosure of G, respectively.

Theorem 1.8. *Let $(X, \mathcal{T})$ be an L-space and $G \in L^X$. Then*

(1) *G is strongly preopen if and only if $G = int_{sp}(G)$;*
(2) *G is strongly preclosed if and only if $G = cl_{sp}(G)$.*

Definition 1.9.([27]). A family $\mathcal{P} \subset L^X$ is called a filter on X if

(1) $P_1 \in \mathcal{P}$ and $P_2 \geq P_1$ implies $P_2 \in \mathcal{P}$;
(2) $P_1, P_2 \in \mathcal{P}$ implies that $P_1 \wedge P_2 \in \mathcal{P}$.

A filter $\mathcal{P}$ is called a proper filter if $P \neq \underline{0}$.

For $\alpha \in M(L^X)$, a filter $\mathcal{P}$ is called an α-filter if $\bigvee_{x \in X} P(x) \geq \alpha$ for every $P \in \mathcal{P}$.

2. SP-adherence points and SP-accumulation points

Definition 2.1. Let $(X, \mathcal{T})$ be an L-space, $x_\lambda \in M(L^X)$ and $P \in L^X$. P is called a remote set of x_λ if $x_\lambda \not\leq P$. A remot set P of x_λ is called a strongly preclosed (strongly semiclosed, pre-semiclosed) remote set of x_λ if P is strongly preclosed (strongly semiclosed, pre-semiclosed respectively).

The set of all strong preclosed (strongly semiclosed, pre-semiclosed) remote sets of x_λ is denoted by $\eta_{sp}(x_\lambda)$ ($\eta_{ss}(x_\lambda)$, $\eta_{ps}(x_\lambda)$ respectively).

Remark 2.2. By Definition 2.1, we can see that $\eta_{ss}(x_\lambda) \subset \eta_{sp}(x_\lambda) \subset \eta_{ps}(x_\lambda)$, where $x_\lambda \in M(L^X)$. But each inverse is not true, these can be seen from the following example.

Example 2.3. Let $X = \{x_1, x_2\}$, $L = [0, 1]$ and A, B, C, $D \in L^X$, we define:

$$A(x_1) = 0.2,\ A(x_2) = 0.5,\ B(x_1) = 0.8,\ B(x_2) = 0.6;$$

$$C(x_1) = 0.8,\ C(x_2) = 0.4,\ D(x_1) = 0.7,\ D(x_2) = 0.6.$$

Let $(X, \mathcal{T})$ be an L-space, where $\tau = \{\underline{0}, A, B, \underline{1}\}$. Then C is strongly preclosed, but it is not strongly semiclosed, also D is pre-semiclosed, but it is not strongly preclosed. We can take $x_{0.5}$ and $x_{0.7}$, where $x = x_2 \in X$, then $x_{0.5}$ and $x_{0.7}$ are two points and $x_{0.5} \not\leq C$, $x_{0.7} \not\leq D$, thus $C \in \eta_{sp}(x_{0.5})$, but $C \notin \eta_{ss}(x_{0.5})$ and $D \in \eta_{ps}(x_{0.7})$, but $D \notin \eta_{sp}(x_{0.7})$.

Definition 2.4. Let $(X, \mathcal{T})$ be an L-space, $G \in L^X$ and $x_\lambda, x_\mu \in M(L^X)$. Then x_λ is called an SP-adherence point of G if $G \not\leq P$ for each $P \in \eta_{sp}(x_\lambda)$.

An SP-adherence point x_λ of G is called an SP-accumulation point of G if $x_\lambda \not\leq G$ or $x_\lambda \leq G$ implies that for each point x_μ satisfying $x_\lambda \leq x_\mu \leq G$, it follows that $G \not\leq x_\mu \vee P$. The union of all SP-accumulation points of G is called the SP-derived set of G and denoted by $G^{d_{sp}}$.

Theorem 2.5. *Let $(X, \mathcal{T})$ be an L-space, $G \in L^X$ and $x_\lambda \in M(L^X)$. Then*

(1) *x_λ is an SP-adherence point of G if and only if $x_\lambda \le cl_{sp}(G)$;*
(2) *$cl_{sp}(G)$ equals the union of all SP-adherence points of G;*
(3) *$cl_{sp}(G) = G \vee G^{d_{sp}}$;*
(4) *$cl_{sp}(G^{d_{sp}}) \le cl_{sp}(G)$.*

Proof.

(1) ($\Rightarrow$). Suppose that $x_\lambda \not\le cl_{sp}(G)$, then $cl_{sp}(G) \in \eta_{sp}(x_\lambda)$, by $G \le cl_{sp}(G)$, we know that x_λ is not an SP-adherence point of G, a contradiction.
($\Leftarrow$). Suppose that $x_\lambda \le cl_{sp}(G)$ and x_λ is not an SP-adherence point of G, then there exists a $P \in \eta_{sp}(x_\lambda)$ such that $G \le P$, this imples that $cl_{sp}(G) \le P$ since P is strongly preclosed. Thus $x_\lambda \not\le cl_{sp}(G)$, a contradiction.
(2) We need only consider the case $G \ne \underline{0}$. Since $cl_{sp}(G) = \bigvee\{x_\lambda \mid x_\lambda \le cl_{sp}(G)\}$ and by (1), we have that $cl_{sp}(G)$ is the union of all its SP-adherence points.
(3) We need only prove that $cl_{sp}(G) \le G \vee G^{d_{sp}}$. In fact, if for some point $x_\lambda \le cl_{sp}(G)$, it follows that $x_\lambda \not\le G$, then by (1) and Definition 2.4 we know that $x_\lambda \le G^{d_{sp}}$.
(4) If $x_\lambda \le cl_{sp}(G^{d_{sp}})$, then by (1) and Definition 2.4 we have that $G^{d_{sp}} \not\le P$ for each $P \in \eta_{sp}(x_\lambda)$. Hence there exists an SP-accumulation point e of G such that $e \not\le P$, which means $P \in \eta_{sp}(e)$. But e is an SP-adherence point of G, hence $G \not\le P$. Form above statement, we know that $G \not\le P$ for each $P \in \eta_{sp}(x_\lambda)$, so x_λ is SP-adherence point of G. Thus by (1) we have $x_\lambda \le cl_{sp}(G)$.

Theorem 2.6. *Let $(X, \mathcal{T})$ be an L-space and $G \in L^X$. Then G is strongly preclosed if and only if for each point $x_\lambda \not\le G$, there exists $P \in \eta_{sp}(x_\lambda)$ such that $G \le P$.*

Proof. The necessity is obvious. Now we prove the sufficiency. Suppose that for each point $x_\lambda \not\le G$, there exists $P \in \eta_{sp}(x_\lambda)$ such that $G \le P$, i.e., there exists $P \in \eta_{sp}(x_\lambda)$ such that $cl_{sp}(G) \le P$. Then $x_\lambda \not\le cl_{sp}(G)$. Hence above statement implies that $x_\lambda \not\le G \Rightarrow x_\lambda \not\le cl_{sp}(G)$. Thus $G \ge cl_{sp}(G)$. Therefore G is strongly preclosed.

3. SP-convergence of nets

In this section, we shall discuss SP-convergence of nets.

Definition 3.1. Let $(X, \mathcal{T})$ be an L-space, $x_\lambda \in M(L^X)$ and $S = \{S(n) \mid n \in D\}$ a net in L^X. Then

(1) x_λ is said to be an SP-limit point of S, in symbols, $S \xrightarrow{SP} x_\lambda$ if for each $P \in \eta_{sp}(x_\lambda)$, $S(n) \not\le P$ is eventually true;
(2) x_λ is said to be an SP-cluster point of S, in symbols, $S \overset{SP}{\propto} x_\lambda$ if for each $P \in \eta_{sp}(x_\lambda)$, $S(n) \not\le P$ is frequently true.

The union of all SP-limit points and the union of all SP-cluster points of S will be denoted by $\lim_{sp} S$ and $\mathrm{ad}_{sp}S$, respectively. Obviously $\lim_{sp} S \le \mathrm{ad}_{sp}S$.

Remark 3.2. From Definition 3.1 and Definition 2.1 in [3] and Definition 5.1 in [4] we easily know that PS-ad$S \le \mathrm{ad}_{sp}S \le$ Q-ad$S$ and PS-$\lim S \le \lim_{sp} S \le$ Q-$\lim S$.

Theorem 3.3. *Let $(X, \mathcal{T})$ be an L-space, $x_\lambda, x_\mu \in M(L^X)$ and S be a net in L^X. Then the following statements are true.*

(1) *Let $T = \{T(n) \mid n \in D\}$ be a net with the same domain as S and for each $n \in D, T(n) \ge S(n)$. If $S \xrightarrow{SP} x_\lambda$, then $T \xrightarrow{SP} x_\lambda$;*
(2) *Let $T = \{T(n) \mid n \in D\}$ be a net with the same domain as S and for each $n \in D, T(n) \ge S(n)$. If $S \overset{SP}{\propto} x_\lambda$, then $T \overset{SP}{\propto} x_\lambda$;*
(3) *If $S \xrightarrow{SP} x_\lambda$ and $x_\mu \le x_\lambda$, then $S \xrightarrow{SP} x_\mu$;*
(4) *If $S \overset{SP}{\propto} x_\lambda$ and $x_\mu \le x_\lambda$, then $S \overset{SP}{\propto} x_\mu$.*

Proof. It is simple and omitted.

Theorem 3.4. *Let $(X, \mathcal{T})$ be an L-space, $x_\lambda \in M(L^X)$ and S be a net in L^X. Then*

(1) *$S \xrightarrow{SP} x_\lambda$ if and only if $x_\lambda \le \lim_{sp} S$;*
(2) *$S \overset{SP}{\propto} x_\lambda$ if and only if $x_\lambda \le \mathrm{ad}_{sp}S$.*

Proof.

(1) The necessity is obvious. We prove the sufficiency.
Suppose that $x_\lambda \le \lim_{sp} S$ and $P \in \eta_{sp}(x_\lambda)$. Then $\lim_{sp} S \not\le P$. By the definition of $\lim_{sp} S$, there exists an SP-limit point e of S such that $e \not\le P$, i.e., $P \in \eta_{sp}(e)$. By e is an SP-limit point of S, we know that S is eventually not in P, therefore $S \xrightarrow{SP} x_\lambda$.
(2) This is analogous to proof of (1).

Theorem 3.5. *Let $(X, \mathcal{T})$ be an L-space, $x_\lambda \in M(L^X)$ and $S = \{S(n) \mid n \in D\}$ be a net in L^X. If S has a subnet T such that $T \xrightarrow{SP} x_\lambda$, then $S \overset{SP}{\propto} x_\lambda$.*

Proof. Suppose that $T(m) = \{T(m) \mid m \in E\}$ is a subnet of $S, T \xrightarrow{SP} x_\lambda$, $P \in \eta_{sp}(x_\lambda)$ and $n_0 \in D$. By the definition of subnet, there exists a mapping $N : E \to D$ and $m_0 \in E$ such that $N(m) \ge n_0 (N(m) \in D)$ when $m \ge m_0 (m \in E)$. Since T SP-converges to x_λ, there is $m_1 \in E$ such that $T(m) \not\le P$ when $m \ge m_1 (m \in E)$. Because E is a directed set, there exists $m_2 \in E$ such that $m_2 \ge m_0$ and $m_2 \ge m_1$. Hence $T(m_2) \not\le P$ and $N(m_2) \ge n_0$. Let $n = N(m_2)$. Then $S(n) = S(N(m_2)) = T(m_2) \not\le P$ and $n \ge n_0$. This implies that $S(n) \not\le P$ is frequently true. Thus $S \overset{SP}{\propto} x_\lambda$.

Theorem 3.6. *Let $(X, \mathcal{T})$ be an L-space and S be a net in L^X. Then $\lim_{sp} S$ and $\mathrm{ad}_{sp}S$ are strongly preclosed.*

Proof. Let $x_\lambda \leq cl_{sp}(\lim_{sp} S)$. Then $\lim_{sp} S \not\leq P$ for each $P \in \eta_{sp}(x_\lambda)$. Hence there exists a point e such that $e \leq \lim_{sp} S$ and $e \not\leq P$. Then $P \in \eta_{sp}(e)$. By Theorem 3.4 $S \xrightarrow{SP} e$. Hence $S \not\leq P$ is eventually true. Thus $x_\lambda \leq \lim_{sp} S$. This implies that $\lim_{sp} S$ is strongly preclosed.

Similarly $\mathrm{ad}_{sp}S$ is strongly preclosed.

Theorem 3.7. *Let $(X, \mathcal{T})$ be an L-space, $G \in L^X$, $x_\lambda \in M(L^X)$. If there exists a net $S = \{S(n) \mid n \in D\}$ in G such that $S \overset{SP}{\propto} x_\lambda$, then $x_\lambda \leq cl_{sp}G$.*

Proof. Suppose that $S = \{S(n) \mid n \in D\}$ is a net in G and $S \overset{SP}{\propto} x_\lambda$. Let $P \in \eta_{sp}(x_\lambda)$, then S is not frequently in P, hence there is $n \in D$ such that $S(n) \not\leq P$, but $S(n) \leq G$, so $G \not\leq P$. Thus x_λ is an SP-adherence point of G, i.e., $x_\lambda \leq cl_{sp}(G)$.

Now we give characterization of SP-accumulation point of L-set G by means of net. Let $G \in L^X$, $x \in X$, we define $G - x_1$ follow as:

$$(G - x_1)(t) = \begin{cases} G(t), & \text{if } x \neq t, \\ 0, & \text{if } x = t. \end{cases} \tag{1}$$

Then $G - x_1 = G \wedge x_1' = \Big\{t_{G(t)} \mid t \in \mathrm{supp}G - \{x\}\Big\}$.

Theorem 3.8. *Let $(X, \mathcal{T})$ be an $[0,1]$-space, $G \in I^X$ and $x_\lambda \in M(I^X)$ in G. If there exists a net S in $G - x_1$ such that $S \xrightarrow{SP} x_\lambda$, then x_λ is an SP-accumulation point of G.*

Proof. Suppose that $x_\lambda \leq G$ and there exists a net $S = \{S(n) \mid n \in D\}$ in $G - x_1$ such that $S \xrightarrow{SP} x_\lambda$. Let $P \in \eta_{sp}(x_\lambda)$ and x_μ is a point satisfying $x_\lambda \leq x_\mu \leq G$. Hence there exists $n \in D$ such that $S(n) = y_\gamma \not\leq P$ and by $y_\gamma \leq G - x_1$, we know $y \neq x$, so $y_\gamma \not\leq x_\mu$. Hence $y_\gamma \not\leq P \vee x_\mu$, therefore $G \not\leq P \vee x_\mu$. By Definition 2.4, we have that x_λ is an SP-accumulation point of G.

4. SP-convergence of ideals

Definition 4.1. Let $(X, \mathcal{T})$ be an L-space, I be an ideal in L^X and $x_\lambda \in M(L^X)$. Then

(1) x_λ is said to be SP-limit point of I, in symbols, $I \xrightarrow{SP} x_\lambda$ if $\eta_{sp}(x_\lambda) \subset I$;

(2) x_λ is said to be SP-cluster point of I, in symbols, $I \overset{SP}{\propto} x_\lambda$ if for each $G \in I$ and each $P \in \eta_{sp}(x_\lambda)$, it follows that $G \vee P \neq \underline{1}$.

The union of all SP-limit points and union of all SP-cluster points of I are denoted by $\lim_{sp} I$ and $\mathrm{ad}_{sp}I$, respectively. Obviously, $\lim_{sp} I \leq \mathrm{ad}_{sp}I$.

Theorem 4.2. *Let both I and J be ideals in L^X, $I \subset J$ and $x_\lambda, x_\mu \in M(L^X)$. Then*

(1) *$I \xrightarrow{SP} x_\lambda$ implies $J \xrightarrow{SP} x_\lambda$;*
(2) *$J \overset{SP}{\propto} x_\lambda$ implies $I \overset{SP}{\propto} x_\lambda$;*
(3) *If $I \xrightarrow{SP} x_\lambda$ and $x_\mu \leq x_\lambda$, then $I \xrightarrow{SP} x_\mu$;*
(4) *If $I \overset{SP}{\propto} x_\lambda$ and $x_\mu \leq x_\lambda$, then $I \overset{SP}{\propto} x_\mu$.*

Proof. It is simple and omitted.

Theorem 4.3. *Let $(X, \mathcal{T})$ be an L-space, $G \in L^X$ and $x_\lambda \in M(L^X)$. If there exists an ideal I in L^X such that $G \notin I$ and $I \xrightarrow{SP} x_\lambda$, then $x_\lambda \leq cl_{sp}(G)$.*

Proof. Suppose that $I \xrightarrow{SP} x_\lambda$ and $G \notin I$. Let $P \in \eta_{sp}(x_\lambda)$, then by the fact that $\eta_{sp}(x_\lambda) \subset I$ and I is a lower set, we know that $G \not\leq P$, so x_λ is an SP-adherence point of G, therefore $x_\lambda \leq cl_{sp}(G)$.

Theorem 4.4. *Let $(X, \mathcal{T})$ be a $[0,1]$-space, $G \in L^X$, $x_\lambda \in M(L^X)$, and $x_\lambda \leq G$. If there exists an ideal I in L^X such that $G - x_1 \notin I$ and $I \xrightarrow{SP} x_\lambda$, then x_λ is an SP-accumulation point of G.*

Proof. Suppose that there exists an ideal I in L^X such that $G - x_1 \notin I$ and $I \xrightarrow{SP} x_\lambda$. Let $P \in \eta_{sp}(x_\lambda)$, then by $\eta_{sp}(x_\lambda) \subset I$, we have $P \in I$. Since I is lower set, we know that $G - x_1 \not\leq P$, so $G \not\leq P \vee x_{G(x)}$. In particular, for each point $x_\mu \in M(L^X)$ with $x_\lambda \leq x_\mu \leq G$, we have $G \not\leq P \vee x_\mu$. Hence x_λ is an SP-accumulation point of G.

Theorem 4.5. *Let $(X, \mathcal{T})$ be an L-space, I be an ideal in L^X and $x_\lambda \in M(L^X)$. Then*

(1) *$I \xrightarrow{SP} x_\lambda$ if and only if $x_\lambda \leq \lim_{sp} I$;*
(2) *$I \overset{SP}{\propto} x_\lambda$ if and only if $x_\lambda \leq \mathrm{ad}_{sp} I$.*

Proof. We prove only the sufficiency of (1). Suppose that $x_\lambda \leq \lim_{sp} I$, $P \in \eta_{sp}(x_\lambda)$. Then $x_\lambda \not\leq P$, so $\lim_{sp} I \not\leq P$. By definition of $\lim_{sp} I$, we know that I has an SP-limit point e such that $e \not\leq P$, i.e., $P \in \eta_{sp}(e) \subset I$, thus $P \in I$, therefore $\eta_{sp}(x_\lambda) \subset I$. Hence $I \xrightarrow{SP} x_\lambda$.

Theorem 4.6. *Let $(X, \mathcal{T})$ be an L-space, I be an ideal in L^X. Then $\lim_{sp} I$ and $\mathrm{ad}_{sp} I$ are strongly preclosed.*

Proof. The proof is analogous to the proof of The Theorem 3.6.

5. SP-convergence of filters

In this section, we first introduce the concept of SP-convergence of filters and then discuss its some properties.

Definition 5.1. Let $(X,\mathcal{T})$ be an L-space, $x_\lambda \in M(L^X)$ and $P \in L^X$. P is called a quasi set of x_λ if $x_\lambda \not\leq P'$, in this case, we also say that x_λ quasi-coincides with P and it is denote by $x_\lambda \hat{q} P$. A quasi set P of x_λ is called a strongly preopen quasi set of x_λ if P is strongly preopen.

The set of all strong preopen quasi sets of x_λ is denoted by $\mathcal{Q}_{sp}(x_\lambda)$.

Remark 5.2. From the above definition , we can see that if $A, B \in L^X$, $A \leq B$, $x_\lambda \in M(L^X)$ and $x_\lambda \hat{q} A$, then $x_\lambda \hat{q} B$.

Definition 5.3. Let $(X,\mathcal{T})$ be an L-space, $\mathcal{P}$ be a proper filter in L^X and $e \in M(L^X)$.

(1) e is called an SP-cluster point of $\mathcal{P}$, in symbol, $\mathcal{P} \overset{SP}{\propto} e$ if for every $U \in \mathcal{Q}_{sp}(e)$ and every $A \in \mathcal{P}$, it follows that $U \vee A \neq \underline{0}$, in this case, we also say that $\mathcal{P}$ SP-accumulates to e.
(2) e is called an SP-limit point of $\mathcal{P}$, in symbol, $\mathcal{P} \xrightarrow{SP} e$ if $\mathcal{Q}_{sp}(e) \subset \mathcal{P}$.

The union of all SP-cluster points of $\mathcal{P}$ is denoted by $\mathrm{ad}_{sp}\mathcal{P}$ and the union of all SP-milit points of $\mathcal{P}$ is denoted by $\lim_{sp} \mathcal{P}$.

Theorem 5.4. *Let $(X,\mathcal{T})$ be an L-space, $\mathcal{P}$ be a proper filter and $e \in M(L^X)$. Then*

(1) *If $\mathcal{P} \xrightarrow{SP} e$, then $\mathcal{P} \overset{SP}{\propto} e$;*
(2) *$\lim_{sp} \mathcal{P} \leq \mathrm{ad}_{sp}\mathcal{P}$;*
(3) *If $\mathcal{P} \overset{SP}{\propto} e$ and $d \leq e$, then $\mathcal{P} \overset{SP}{\propto} d$;*
(4) *If $\mathcal{P} \xrightarrow{SP} e$ and $d \leq e$, then $\mathcal{P} \xrightarrow{SP} d$;*
(5) *$\mathcal{P} \xrightarrow{SP} e$ if and only if $e \leq \lim_{sp} \mathcal{P}$;*
(6) *$\mathcal{P} \overset{SP}{\propto} e$ if and only if $e \leq \mathrm{ad}_{sp}\mathcal{P}$.*

Proof. It is simple and omitted.

Definition 5.5. Let $(X,\mathcal{T})$ be an L-space, $\mathcal{P}$,$\mathcal{G}$ be proper filters in L^X. Say $\mathcal{G}$ is finer than $\mathcal{P}$, or say $\mathcal{P}$ is coarser than $\mathcal{G}$, if $\mathcal{P} \subset \mathcal{G}$.

Theorem 5.6 *Let $(X,\mathcal{T})$ be an L-space, $\mathcal{P}$,$\mathcal{G}$ be proper filters in L^X, $\mathcal{P}$ be coarser than $\mathcal{G}$ and $e \in M(L^X)$. Then*

(1) *$\mathrm{ad}_{sp}\mathcal{G} \leq \mathrm{ad}_{sp}\mathcal{P}$;*
(2) *$\lim_{sp} \mathcal{P} \leq \lim_{sp} \mathcal{G}$;*
(3) *If $\mathcal{G} \overset{SP}{\propto} e$, then $\mathcal{P} \overset{SP}{\propto} e$;*
(4) *$\mathcal{P} \xrightarrow{SP} e$, then $\mathcal{G} \xrightarrow{SP} e$.*

Proof. It is simple and omitted.

6. Relations among nets, ideals, filters

In this section, we discuss relations among nets, ideals and filters.

Definition 6.1.([26]) Let $(X, \mathcal{T})$ be an L-space.

(1) Let I be an ideal in L^X and $D(I) = \{(e, G) \mid e \in M(L^X), G \in I \text{and } e \not\leq G\}$. For every pair of elements (e_1, G_1) and (e_2, G_2) in $D(I)$, we define that $(e_1, G_1) \leq (e_2, G_2)$ if and only if $G_1 \leq G_2$. Then $(D(I), \leq)$ is a directed set. Clearly, $S(I) = \{S(I)(e, G) = e \mid (e, G) \in D(I)\}$ is a net in L^X and is called the net induced by I.

(2) Let S be a net in L^X. Then $I(S) = \{G \in L^X \mid S$ in not eventually in $G\}$ is an ideal in L^X and is called the ideal induced by S.

Theorem 6.2. *Let $(X, \mathcal{T})$ be an L-space and I be an ideal in L^X. Then*

(1) $\lim_{sp} I = \lim_{sp} S(I)$;

(2) $\text{ad}_{sp} I = \text{ad}_{sp} S(I)$.

Proof. We prove only (1). Let $e \leq \lim_{sp} I$. Then $I \stackrel{SP}{\longrightarrow} e$, so $P \in I$ for each $P \in \eta_{sp}(x_\lambda)$. Hence $(e, P) \in D(I)$. If $(a, G) \in D(I)$ and $(a, G) \geq (e, P)$, then we have $S(I)(a, G) = a \not\leq G \geq P$. This implies that $S(I)$ is not eventually in P for each $P \in \eta_{sp}(x_\lambda)$, i.e., $S(I) \stackrel{SP}{\longrightarrow} x_\lambda$.

Conversely, let $e \leq \lim_{sp} S(I)$. Then $S(I) \stackrel{SP}{\longrightarrow} e$. Therefore for each $P \in \eta_{sp}(e)$ there exists $(a, G) \in D(I)$ such that $S(I)(b, H) = b \not\leq P$ whenever $(b, H) \geq (a, G)$ and $(b, H) \in D(I)$. In particular, take $H = G$, we know that $b \not\leq G$ implies $b \not\leq P$, or equivalently $b \leq P$ implies $b \leq G$. Hence $P \leq G$ follows from Theorem 1.5.29 in [25]. Note that I is a lower set and $G \in I$, so $P \in I$. This shows that $\eta_{sp}(e) \subset I$. Hence $I \stackrel{SP}{\longrightarrow} e$. From Theorem 4.5 we have $e \leq \lim_{sp} I$. Thus (1) holds.

Theorem 6.3. *Let $(X, \mathcal{T})$ be an L-space and S be a net in L^X. Then*

(1) $\lim_{sp} S = \lim_{sp} I(S)$;

(2) $\text{ad}_{sp} S \leq \text{ad}_{sp} I(S)$.

Proof. We prove only (2). In accordance with Theorems 4.5 and 3.4, we need only prove that $S \stackrel{SP}{\propto} x_\lambda$ implies $I(S) \stackrel{SP}{\propto} x_\lambda$. Let $S \stackrel{SP}{\propto} x_\lambda$. Then S is not frequently in P for each $P \in \eta_{sp}(x_\lambda)$. On the other hand, S is not eventually in G for each $G \in I(S)$. Hence S is not frequently in $P \vee G$ for each $P \in \eta_{sp}(x_\lambda)$ and each $G \in I(S)$. This means that $P \vee G \neq \underline{1}$. Thus $I(S) \stackrel{SP}{\propto} x_\lambda$.

Now we give relations between nets and filters.

Definition 6.4. Let $(X, \mathcal{T})$ be an L-space, $\mathcal{P}$ be a filter in L^X and S be a net in L^X. For S, define the filter associated with the net S as the family $\mathcal{P}(S)$ of all the L-subsets on X which the net S eventually quasi-coincides with.

For $\mathcal{P}$, let

$$D(\mathcal{P}) = \{(e, A) \mid e \in M(L^X), e\hat{q}A \in \mathcal{P}\}$$

and equip it with a relation $\le$ on it as

$$\forall (e, A), (d, B) \in D(\mathcal{P}), (e, A) \le (d, B) \Leftrightarrow A \ge B.$$

Define the net associated with the filter $\mathcal{P}$ as the mapping

$$S(\mathcal{P}) : D(\mathcal{P}) \to M(L^X), S(\mathcal{P})(e, A) = e, \forall (e, A) \in D(\mathcal{P}).$$

Then the filter $\mathcal{P}(S)$ associated with S is a proper filter in L^X, $D(\mathcal{P})$ equipped with $\le$ is a directed set and the $S(\mathcal{P})$ associated with $\mathcal{P}$ is a net in L^X.

Theorem 6.5. *Let $(X, \mathcal{T})$ be an L-space, S a net in L^X, $\mathcal{P}$ a proper filter in L^X and $e \in M(L^X)$. Then*

(1) $S \xrightarrow{SP} e$ *if and only if* $\mathcal{P}(S) \xrightarrow{SP} e$;
(2) $\mathcal{P} \xrightarrow{SP} e$ *if and only if* $S(\mathcal{P}) \xrightarrow{SP} e$;
(3) $\mathcal{P} \overset{SP}{\propto} e$ *if and only if* $S(\mathcal{P}) \overset{SP}{\propto} e$;
(4) $S \overset{SP}{\propto} e$ *implies* $\mathcal{P}(S) \overset{SP}{\propto} e$

Proof.

(1) $(\Leftrightarrow)$ By the relative definitions.

(2) $(\Rightarrow)$ Suppose $\mathcal{P} \xrightarrow{SP} e = x_a \in M(L^X), U \in \eta_{sp}(e)$, then $x_a \not\le U$. Take $x_\lambda \in M(L^X)$ such that $x_\lambda \le x_a, x_\lambda \not\le U$, so $x_\lambda \hat{q} U'$. By Theorem 5.4(4) $\mathcal{P} \xrightarrow{SP} x_\lambda \le x_a$, $U' \in \mathcal{Q}_{sp}(x_\lambda) \subset \mathcal{P}$. So $(x_\lambda, U') \in D(\mathcal{P})$. $\forall (d, A) \in D(\mathcal{P})$ such that $(d, A) \ge (x_\lambda, U')$, then $d\hat{q}A \le U'$. By Remark 5.2 $S(\mathcal{P})(d, A) = d\hat{q}U'$, $S(\mathcal{P})$ eventually quasi-coincides with U', i.e., $S(\mathcal{P}) \not\le (U')' = U$ eventually is true. By the arbitrariness of $U \in \eta_{sp}(e)$, $S(\mathcal{P}) \xrightarrow{SP} e$.

$(\Leftarrow)$ Suppose $S(\mathcal{P}) \xrightarrow{SP} e$, $U \in \mathcal{Q}_{sp}(e)$, then $U' \in \eta_{sp}(e)$. So $S(\mathcal{P}) \not\le U'$ eventually is true. $\exists (d_0, A_0) \in D(\mathcal{P})$ such that $\forall (d, A) \ge (d_0, A_0)$, $d = S(\mathcal{P})(d, A) \not\le U'$, i.e., $d\hat{q}U$. So $\forall d \in M(L^X)$ such that $d\hat{q}A_0$, we have $(d, A_0) \in D(\mathcal{P})$, $(d, A_0) \ge (d_0, A_0)$ and hence $d\hat{q}U$. That is to say $\forall d \in M(L^X)$, $d\hat{q}A_0$ implies that $d\hat{q}U$, i.e., $d \le U'$ implies that $d \le A_0$. So $U' \le A_0'$, $U \ge A_0$. Since $A_0 \in \mathcal{P}$, $\mathcal{P}$ is a filter, so $U \in \mathcal{P}$. By the arbitrariness of $U \in \mathcal{Q}_{sp}(e)$, $\mathcal{Q}_{sp}(e) \subset P$. $\mathcal{P} \xrightarrow{SP} e$.

(3) $(\Rightarrow)$ Suppose $\mathcal{P} \overset{SP}{\propto} e$, $U \in \eta_{sp}(e)$, i.e., $U' \in \mathcal{Q}_{sp}(e)$, $(d_0, A_0) \in D(\mathcal{P})$. We need to find a $(d, A) \in D(\mathcal{P})$ such that $(d, A) \ge (d_0, A_0)$, $S(d, A) \not\le U$. Since $\mathcal{P} \overset{SP}{\propto} e$, $U' \in \mathcal{Q}_{sp}(e)$ and $A_0 \in \mathcal{P}$, $A_0 \wedge U' \ne \underline{0}$, $A_0' \vee U \ne \underline{1}$. So $\exists d \in M(L^X)$ such that $d \not\le A_0' \vee U$, so $d\hat{q}(A_0 \wedge U')$. Therefore $d\hat{q}A_0$, by $(d_0, A_0) \in D(\mathcal{P})$, $A_0 \in \mathcal{P}$, so $(d, A_0) \in D(\mathcal{P})$, $(d, A_0) \ge (d_0, A_0)$. By $d\hat{q}(A_0 \wedge U')$ and Remark 5.2, $S(d, A_0) = d\hat{q}U'$, i.e., $S(d, A_0) \not\le U$, this is that we need to prove.

($\Leftarrow$) Suppose $S(\mathcal{P}) \overset{SP}{\propto} e$, $A \in \mathcal{P}$, $U \in \mathcal{Q}_{sp}(e)$(so $U' \in \eta_{sp}(e)$), we need to show $A \wedge U \neq \underline{0}$. Since $A \in \mathcal{P}$ and $\mathcal{P}$ is a proper filter in L^X, $A \neq \underline{0}$, $A' \neq \underline{1}$. So $\exists d \in M(L^X)$ sucht that $d \not\leq A'$, i.e., $d\hat{q}A$, so $(d, A) \in D(\mathcal{P})$. Since $S(\mathcal{P}) \overset{SP}{\propto} e$, $\exists (d_0, A_0) \in D(\mathcal{P})$ such that $(d_0, A_0) \geq (d, A)$, $d_0 = S(d_0, A_0) \not\leq U'$. So $d_0 \not\leq A_0'$, $d_0 \not\leq U'$. By $d_0 \in M(L^X)$, $d_0 \not\leq A_0' \vee U' = (A_0 \wedge U)'$, $(A_0 \wedge U)' \neq \underline{1}$, $A_0 \wedge U \neq \underline{0}$. Since $(d_0, A_0) \geq (d, A)$, $A_0 \leq A$. so $A \wedge U \neq \underline{0}$.

(4) Suppose $S = \{S(n), n \in D\}$, $A \in \mathcal{P}(S)$, $U \in \mathcal{Q}_{sp}(e)$, going to show $A \wedge U \neq \underline{0}$. Since $A \in \mathcal{P}(S)$, $\exists n_0 \in D$ such that $\forall n \geq n_0$, $S(n) \not\leq A'$. Since $U \in \mathcal{Q}_{sp}(e)$, i.e., $U' \in \eta_{sp}(e)$, $S \overset{SP}{\propto} e$, $\exists n_1 \in D$, $n_1 \geq n_0$ sucht that $S(n_1) \not\leq U'$. So $S(n_1) \not\leq A', U'$. But $S(n_1) \in M(L^X)$, so $S(n_1) \not\leq A' \vee U' = (A \wedge U)'$,$(A \wedge U)' \neq \underline{1}$, $A \wedge U \neq \underline{0}$.

7. Applications of SP-convergence theory of nets

Theorem 7.1. *Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two L-spaces. A mapping $f : X \to Y$ is SP-irresolute if and only if $cl_{sp}(f_L^{\leftarrow}(P)) \in c(x_\lambda)$ for each $P \in \eta_{sp}(f_L^{\rightarrow}(x_\lambda))$, where $x_\lambda \in M(L^X)$.*

Proof. Suppose that f is SP-irresolute and $x_\lambda \in M(L^X)$. Then $f_L^{\leftarrow}(P)$ is strongly preclosed for each $P \in \eta_{sp}(f_L^{\rightarrow}(x_\lambda))$. Clearly $x_\lambda \not\leq f_L^{\leftarrow}(P)$. Hence $f_L^{\leftarrow}(P) = cl_{sp}(f_L^{\leftarrow}(P))$ $\in \eta_{sp}(x_\lambda)$.

Conversely, let P be strongly preclosed in $(Y, \mathcal{T}_2)$. We may assume that $f_L^{\leftarrow}(P) \neq \underline{1}$ and $x_\lambda \not\leq f_L^{\leftarrow}(P)$. Then $f_L^{\rightarrow}(x_\lambda) \not\leq P$, i.e., $P \in \eta_{sp}(f_L^{\rightarrow}(x_\lambda))$. Hence $cl_{sp}(f_L^{\leftarrow}(P)) \in \eta_{sp}(x_\lambda)$, i.e., $x_\lambda \not\leq f_L^{\leftarrow}(P)$ implies that $x_\lambda \not\leq cl_{sp}(f_L^{\leftarrow}(P))$. So $cl_{sp}(f_L^{\leftarrow}(P)) \leq f_L^{\leftarrow}(P)$. Thus $f_L^{\leftarrow}(P)$ is strongly preclosed in $(X, \mathcal{T}_1)$. This shows that f is SP-irresolute.

Theorem 7.2. *Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two L-spaces, $x_\lambda \in M(L^X)$ and $f : X \to Y$ is SP-irresolute. If a net $S \xrightarrow{SP} x_\lambda$ in L^X, then $f_L^{\rightarrow}(S) \xrightarrow{SP} f_L^{\rightarrow}(x_\lambda)$ in L^Y.*

Proof. Suppose that f is SP-irresolute and $S \xrightarrow{SP} x_\lambda$. Let $P \in \eta_{sp}(f_L^{\rightarrow}(x_\lambda))$. Then $f_L^{\leftarrow}(P) \leq cl_{sp}(f_L^{\leftarrow}(P)) \in \eta_{sp}(x_\lambda)$ from f is SP-irresolute and so $S(n) \not\leq f_L^{\leftarrow}(P)$ is eventually true from $S \xrightarrow{SP} x_\lambda$. Therefore $f_L^{\rightarrow}(S) \not\leq P$ is eventually true. Thus $f_L^{\rightarrow}(S) \xrightarrow{SP} f_L^{\rightarrow}(x_\lambda)$.

Corollary 7.3. *Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two L-spaces. If a mapping $f : X \to Y$ is SP-irresolute, then*

(1) $f_L^{\rightarrow}(\lim_{sp} S) \leq \lim_{sp} f_L^{\rightarrow}(S)$ *for each net S in L^X;*
(2) $f_L^{\leftarrow}(\lim_{sp} T) \leq \lim_{sp} f_L^{\leftarrow}(T)$ *for each net T in L^Y.*

Proof.

(1) Suppose that $S = \{S(n) \mid n \in D\}$ is a net in L^X and $g \in f_L^{\rightarrow}(\lim_{sp} S)$. Then there exists $e \leq \lim_{sp} S$ with $g = f_L^{\rightarrow}(e)$. We prove that $g \leq \lim_{sp} f_L^{\rightarrow}(S)$. In fact, by

$e \le \lim_{sp} S$, we know that $S \xrightarrow{SP} e$ from Theorem 3.4. Since f is SP-irresolute, we obtain that $f_L^{\rightarrow}(S) \xrightarrow{SP} f_L^{\rightarrow}(e) = g$ from Theorem 7.2. And by Theorem 3.4, we have that $g \le \lim_{sp} f_L^{\rightarrow}(S)$. Thus

$$f_L^{\rightarrow}(\lim_{sp} S) \le \lim_{sp} f_L^{\rightarrow}(S).$$

(2) Let $T = \{T(n) \mid n \in D\}$ be a net in L^Y. Then

$$f_L^{\leftarrow}(T) = \{f_L^{\leftarrow}(T(n)) \mid n \in D\}$$

is a net in L^X. Since f is SP-irresolute, according to (1) we have

$$f_L^{\rightarrow}(\lim_{sp} f_L^{\leftarrow}(T)) \le \lim_{sp} f_L^{\rightarrow}(f_L^{\leftarrow}(T)) \le \lim_{sp} T.$$

Hence $\lim_{sp} f_L^{\leftarrow}(T) \le f_L^{\leftarrow}(\lim_{sp} T)$.

References

[1] K. K. Azad, *On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. and Appl. **82**(1981), 14–32.

[2] S.-Z. Bai, *Fuzzy strongly semiopen sets and fuzzy strong semicontinuity*, Fuzzy Sets and Systems **52**(1992), 345–351.

[3] S.-Z. Bai, *Q-convergence of fuzzy nets and weak separation axioms in fuzzy lattices*, Fuzzy Sets and Systems **88**(1997), 379–386.

[4] S.-Z. Bai, *Pre-semiclosed sets and PS-convergence in L-fuzzy topological space*, J. Fuzzy Math. **2**(2001), 497–509.

[5] S.-Z. Bai, *PS-convergence theory of fuzzy nets and its applications*, Information Sciences **153**(2003), 237–346.

[6] A. S. Bin Shahna, *On fuzzy strong semicontinuity and fuzzy precontinuity*, Fuzzy Sets and Systems **44**(1991), 303–308.

[7] B. Krateska, *Fuzzy strongly preopen L-sets and fuzzy strong precontinuity*, Math. Vesn. **50**(1998), 111-123.

[8] B. Krateska, *Fuzzy strongly preopen L-sets and fuzzy strong precontinuity*, Math. Vesn. **51**(1999), 39-51.

[9] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24**(1968), 182–190.

[10] S.-L. Chen and J. S. Cheng, *θ-convergence of nets of L-fuzzy sets and its applications*, Fuzzy Sets and Systems **86**(1997), 235–240.

[11] S.-L. Chen and J. R. Wu, *SR-convergence theory in fuzzy lattices*, Information Sciences **125**(2000), 233–247.

[12] S.-L. Chen and S. T. Chen and X. G. Wang, *σ-convergence theory and its applications in fuzzy lattices*, Information Sciences **165**(2004), 45–58.

[13] P. Dwinger,*Characterizations of the complete homomorphic images of a completely distributive complete lattice*, I, Nederl. Akad. Wetensch. indag. Math. **44**(1982), 403–414.

[14] W. Gähler, *Convergence*, Fuzzy Set and Systems **73**(1995), 97–129.

[15] G. Gierz, et al., A Compendium of Continuous Lattices, Springer Verlag, Berlin, 1980.

[16] D. N. Georgiou and B. K. Papadopoulos, *On fuzzy θ-convergences*, Fuzzy Sets and Systems **116**(2000), 385–399.

[17] A. K. Katsaras, *Convergence in fuzzy topological spaces*, Fuzzy Math. **3**(1984), 35–44.

[18] Y.-J. Lee, *Generalized fuzzy filter and limit structure*, Fuzzy Sets and Systems **104**(1999), 415–422.

[19] Y.-M. Liu and M. K. Luo, Fuzzy Topology, World Scientific, Singapore, 1997.

[20] R. Lowen, *Convergence in fuzzy topology*, Topology Appl. **10**(1979), 147–160.

[21] B.-M. Pu and Y. M. Liu, *Fuzzy topology I. Neighborhood structure of a fuzzy point and Moore-Smith convergence*, J. Math. Anal. Appl. **76**(1980), 571–599.

[22] Sarma and N. Ajmal,*Fuzzy nets and their applications*, Fuzzy Sets and Systems **51**(1992), 41–52.

[23] F.-G. Shi and C.-Y. Zheng, *O-convergence of fuzzy nets and its applications*, Fuzzy Sets and Systems **140**(2003), 499–507.

[24] M. K. Singal, *Fuzzy alpha-sets and alpha-continuous maps*, Fuzzy Sets and Systems **48**(1992), 383–390.

[25] G.-J. Wang, Theory of L-Fuzzy Topological Spaces, Shaanxi Normal University Press, Xi'an, 1988 (in Chinese).

[26] Z.-Q. Yang, *Ideals in topological molecular lattices*, Acta Math. Sinica **29**(1986), 276–279.

[27] D.-S. Zhao,*The N-compactness in L-fuzzy topological spaces*, J. Math. Anal. Appl. **128**(1987), 64–79.

Department of Mathematics, Beijing Institute of Technology, Beijing 100081, P. R. China.
E-mail: zhenguoxu@eyou.com
E-mail: fuguishi@bit.edu.cn