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NEIGHBOURHOODS OF A CERTAIN SUBCLASS OF UNIFORMLY STARLIKE FUNCTIONS

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Abstract. In this paper, we introduced new subclasses $UCV_S(\alpha,\beta)$ and $SP_S(\alpha,\beta)$ which are sub classes of $UCV(\alpha,\beta)$ and $SP(\alpha,\beta)$ and studied the T - δ neighbourhoods of functions in these classes. The results obtained in this paper generalizes the recent results of Parvatham and Premabai [5], Ram Reddy and Thirupathi Reddy [6, 7], Padmanabhan [4], and Ronning [8].

1. Introduction

Let $\boldsymbol{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the unit disk $E = \{z : |z| < 1\}$. Further, let $\boldsymbol{S}$ be the subclass of $\boldsymbol{A}$ consisting of those functions that are univalent in E. Let CV and ST denote the subclasses of $\boldsymbol{S}$ consisting of convex and starlike functions respectively.

If $f(z) = z+\sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z+\sum_{n=2}^{\infty} b_n z^n$ then the convolution or Hadamard product of $f(z)$ and $g(z)$ denoted by $f * g$ is defined by

$$\left(f * g\right)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n.$$

Clearly

$$f(z) * \frac{z}{(1-z)^2} = zf'(z) \quad \text{and } f(z) * \frac{z}{\left(1-z^2\right)} = \left[\frac{f(z)-f(-z)}{2}\right].$$

Goodman [1, 2] defined the following subclasses of CV and ST.

Definition A. A function f is uniformly convex (Starlike) in E if f is in CV (ST) and has the property that for every circular arc γ contained in E with centre ξ also in E, the arc $f(\gamma)$ is convex (Starlike w.r.t $f(\xi)$).

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Goodman [1, 2] then gave the following two variable analytic characterizations of these classes, denoted by UCV and UST.

Theorem A. *A function f of the form (1) is in UCV if and only if*

$$\operatorname{Re}\left\{1+(z-\xi)\frac{f''(z)}{f'(z)}\right\} \geq 0,\ (z,\xi) \in E \times E. \tag{2}$$

and is in UST if any only if

$$\operatorname{Re}\left\{\frac{f(z)-f(\xi)}{(z-\xi)f'(z)}\right\} \geq 0,\ (z,\xi) \in E \times E. \tag{3}$$

The classical Alexander result that $f \in CV$ if and only if $zf' \in ST$ does not hold between the classes UCV and UST. Ronning [9] defined a subclass of starlike functions S_p with the property that a function $f \in UCV$ if and only if $zf' \in S_p$.

Definition B. Let $S_p = \{F \in ST \mid F(z) = zf'(z), f \in UCV\}$.

Ma and Minda [3] and Ronning [9] independently found a more applicable one variable characterization for UCV.

Theorem B. *A function f is in UCV if and only if*

$$\operatorname{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\} \geq \left|\frac{zf''(z)}{f'(z)}\right|,\quad z \in E. \tag{4}$$

Ronning [9] proved a one variable characterization for S_p as follows:

Theorem C. *A function f is in S_p if and only if*

$$\left|\frac{zf'(z)}{f(z)} - 1\right| \leq \operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\},\ z \in E. \tag{5}$$

A function $f \in A$ is uniformly convex of order α for $-1 \leq \alpha < 1$ if and only if $1 + \frac{zf''(z)}{f'(z)}$ lies in the parabolic region

$$\operatorname{Re}\{\omega - \alpha\} > |\omega - 1|. \tag{6}$$

In otherwords, the function f is uniformly convex of order α if

$$1+\frac{zf''(z)}{f'(z)} \prec 1 + \frac{2(1-\alpha)}{\pi^2}\left[\log\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right]^2,\ z \in E \tag{7}$$

where the symbol $\prec$ denotes subordination. This class was introduced by Ronning [8] and it is denoted by $UCV(\alpha)$. The class of all analytic functions $f(z) \in A$ for which $\frac{zf'(z)}{f(z)}$ lies in the parabolic region is denoted by $S_p(\alpha)$ and defined as follows.

Definition C. A function $f(z)$ is said to be in the class $S_p(\alpha)$ if for all $z \in E$,

$$\left|\frac{zf'(z)}{f(z)} - 1\right| \le \operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} - \alpha, \text{ for } -1 < \alpha < 1. \tag{8}$$

This implies $f \in S_p(\alpha)$ for $z \in E$ if and only if $\frac{zf'(z)}{f(z)}$ lies in the region Ω_α bounded by a parabola with vertex at $\left(\frac{1+\alpha}{2}, 0\right)$ and parameterized by

$$\frac{t^2 + 1 - \alpha^2 + 2it(1-\alpha)}{2(1-\alpha)} \quad \text{for any real } t.$$

It is known [8] that the function

$$P_\alpha(z) = 1 + \frac{2(1-\alpha)}{\pi^2}\left[\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right]^2 \tag{9}$$

maps the unit disk E on to the parabolic region Ω_α (The branch $\sqrt{z}$ is choosen in such a way that Im $\sqrt{z} \ge 0$). Then from the above definition $f \in A$ is in the class $S_p(\alpha)$ if and only if $\frac{zf'(z)}{f(z)} \prec P_\alpha(z)$.

The notion of δ-neighbourhood was first introduced by St. Ruscheweyh [11].

Definition D. For $\delta \ge 0$, the δ-neighbourhood of $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in A$ is defined by

$$N_\delta(f) = \left\{ g(z) = z + \sum_{n=2}^{\infty} b_n z^n : \sum_{n=2}^{\infty} n|a_n - b_n| \le \delta \right\}. \tag{10}$$

Recently Padmanabhan [4] has introduced the neighbourhoods of functions in the calss S_p and studied various properties.

In this paper we introduce a new class of functions and study the properties of neighbourhoods of functions in this class which generalizes the recent results of Padmanabhan [4] and Ronning [8] Ram Reddy and Thirupathi Reddy [6, 7].

First let us state a lemma which is needed to establish our results in the sequel.

Lemma A([10])**.** *If ϕ is a convex univalent function with $\phi(0) = 0 = \phi'(0) - 1$ in the unit disc E and g is starlike univalent in E, then for each analytic function F in E, the image of E under $\frac{(\phi * F_g)(z)}{(\phi * g)(z)}$ is a subset of the convex hull of $F(E)$.*

2. Main results

Definition 1. Let $SP_s(\alpha, \beta)$ be the class of all function $f \in S$ with the property that,

$$\left|\frac{2zf'(z)}{f(z) - f(-z)} - (\alpha + \beta)\right| \le \operatorname{Re}\left(\frac{2zf'(z)}{f(z) - f(-z)} + \alpha - \beta\right)$$

$z \in E$, $0 < \alpha < \infty$ and $0 \le \beta < 1$.

This implies that $f \in SP_S(\alpha,\beta)$ for z in E which $\frac{2zf'(z)}{f(z)-f(-z)}$ lies in $\Omega = \{\text{w}: |w-(\alpha+\beta)| \leq \text{Re}\,(w+\alpha-\beta)\}$.

That is that portion of the plane which contains $w = 1$ and is bouned by the parabola $y^2 = 4\alpha(x-\beta)$ whose vertex is the point $w = \beta$. Under the choice of $0 < \alpha < \infty$, $0 \leq \beta < 1$, $\Omega \subset \{w : \text{Re}\, w > \beta\}$ and hence $SP_s(\alpha,\beta) \subset ST(\beta)$ the class of starlike univalent functions of order β.

Definition 2. Let $UCV_s(\alpha,\beta)$ be the class of all function $f(z) \in S$ such that $\frac{2zf'(z)}{f(z)-f(-z)} \in SP_s(\alpha,\beta)$.

We now give a characterization of the class $SP_s(\alpha,\beta)$ in terms of convolution.

Definition 3. Let $SP'_s(\alpha,\beta)$ be the class of all function $H(z)$ in A of the form

$$H(z) = \frac{4\alpha}{4\alpha(1-\beta)-4\alpha i t - t^2}\left[\frac{z}{(1-z)^2} - \frac{t^2+4\alpha\beta+4i\alpha t}{4\alpha}\left(\frac{z}{1-z^2}\right)\right] \tag{11}$$

for $0 < \alpha < \infty$, $0 \leq \beta < 1$ and t is real.

Theorem 1. *A function f in A is in $SP_s(\alpha,\beta)$ if and only if for all z in E for all*

$$H(z) \in \text{SP}'_S(\alpha,\beta) \text{ and } \frac{(f*H)(z)}{z} \neq 0.$$

Proof. Let us assume that for $f \in A$ and $\frac{(f*H)(z)}{z} \neq 0$, for all, $H(z) \in SP'_S(\alpha,\beta)$ and for $z \in E$. From the definition of $H(z)$ it follows that

$$\frac{(f*H)(z)}{z} = \frac{4\alpha}{4\alpha(1-\beta)-4\alpha i t - t^2}\left[zf'(z) - \frac{t^2+4\alpha\beta+4\alpha i t}{4\alpha}\left(\frac{f(z)-f(-z)}{2}\right)\right] \neq 0,$$

or equivalently

$$\frac{2zf'(z)}{f(z)-f(-z)} \neq \frac{t^2+4\alpha\beta+4\alpha i t}{4\alpha}, \text{ for } t \in R.$$

This means that $\frac{2zf'(z)}{f(z)-f(-z)}$ lies completely either inside Ω or complement of Ω for all z in E. At $z = 0$, $\frac{2zf'(z)}{f(z)-f(-z)} = 1 \in \Omega$. So that $\frac{2zf'(z)}{f(z)-f(-z)} \subset \Omega$, which shows that $f \in SP_s(\alpha,\beta)$.

Conversely let $f \in SP'_s(\alpha,\beta)$. Hence $\frac{2zf'(z)}{f(z)-f(-z)}$ lies in Ω which contains $w = 1$ and bounded by the parabola $y^2 = 4\alpha(x-\beta)$. Any point on the parabola can be taken as $\frac{t^2+4\alpha\beta}{4\alpha} + it$ for any $t \in R$. So $f \in SP_s(\alpha,\beta)$ if and only if $\frac{2zf'(z)}{f(z)-f(-z)} \neq \frac{t^2+4\alpha\beta+4\alpha i t}{4\alpha}$ or equivalently

$$f(z) * \left[\frac{z}{(1-z)^2} - \frac{t^2+4\alpha\beta+4\alpha i t}{4\alpha}\left(\frac{z}{1-z^2}\right)\right] \neq 0, \text{for } z \in E-\{0\}.$$

Normalizing the function with in the brackets we get $\frac{(f(z)*H)(z)}{z} \neq 0$, $z \in E$, where $H(z)$ is the function in Definition 3.

To establish the T - δ neighbourhoods of functions belonging to the class $SP_s(\alpha,\beta)$ we need the following lemmas.

Lemma 1. *Let* $H(z) = z + \sum_{k=2}^{\infty} c_k z^k \in SP'_s(\alpha,\beta)$ *then* $|c_k| \le \frac{k-\beta}{1-\beta}$, $\forall\ k \ge 2$.

Proof. Let $H(z) = z + \sum_{k=2}^{\infty} c_k z^k \in SP'_s(\alpha,\beta)$, then for any real t

$$\begin{aligned} H(z) &= \frac{4\alpha}{4\alpha(1-\beta)-4\alpha i t - t^2}\left[\frac{z}{(1-z)^2} - \frac{t^2+4\alpha\beta+4\alpha i t}{4\alpha}\left(\frac{z}{1-z^2}\right)\right] \\ &= \frac{4\alpha}{4\alpha(1-\beta)-4\alpha i t - t^2}\left[\left(z+2z^2+\cdots\right) - \frac{t^2+4\alpha\beta+4\alpha i t}{4\alpha}\left(z+z^3+\cdots\right)\right] \\ &= z + \sum_{k=2}^{\infty} c_k z^k \end{aligned}$$

Then comparing the coefficients on either side, we get

$$c_k = \begin{cases} \dfrac{4\alpha}{4\alpha(1-\beta)-4\alpha i t - t^2}, & \text{when } k \text{ is even,} \\ \dfrac{4\alpha(1-\beta)-t^2-4\alpha i t}{4\alpha(1-\beta)-t^2-4\alpha i t}, & \text{when } k \text{ is odd.} \end{cases}$$

Hence when k is even, which yields on simplication $|c_k| \le \frac{k-\beta}{1-\beta}$ and when k is odd we have

$$\begin{aligned} |c_k|^2 &= \frac{\left(4\alpha(k-\beta)-t^2\right)^2+16\alpha^2t^2}{\left(4\alpha(1-\beta)-t^2\right)^2+16\alpha^2t^2} \le \frac{\left(4\alpha(k-\beta)-t^2\right)^2+16\alpha^2t^2}{\left(4\alpha(1-\beta)+t^2\right)^2} \quad if\ \alpha \ge 1-\beta \\ &= \frac{16\alpha^2(k-\beta)^2-8\alpha(k-\beta)t^2+t^4+16\alpha^2t^2}{16\alpha^2(1-\beta)^2-8\alpha(1-\beta)\ t^2+t^4} \\ &= \frac{16\alpha^2(k+1-2\beta)^2(k-1)-8\alpha t^2(k+1-2\alpha-2\beta)}{\left(4\alpha(1-\beta)+t^2\right)^2}+1 \\ &= \frac{8\alpha(k+1-2\beta-2\alpha)\left(2\alpha(k-1)-t^2\right)+32\alpha^3(k+1)}{\left(4\alpha(1-\beta)+t^2\right)^2}+1 \\ &\le \frac{16\alpha^2(k-1)\left(k+1-2(\alpha+\beta)\right)+32\alpha^3(k-1)}{16\alpha^2(1-\beta)^2}+1 \\ &= \frac{(k-1)\ \left(k+1-2(\alpha+\beta)\right)+(k-1)2\alpha}{(1-\beta)^2}+1 \\ &= \frac{(k-1)\ (k+1-2\beta)}{(1-\beta)^2}+1 = \frac{(k-\beta)^2}{(1-\beta)^2} \end{aligned}$$

Therefore

$$|c_k| \le \frac{k-\beta}{1-\beta}, \quad \forall\ k \ge 2.$$

□

Lemma 2. *For* $f \in A$ *and for every* $\varepsilon \in C$ *such that* $|\varepsilon| < \delta$ *if* $F_\varepsilon(z) = \left\{\frac{f(z)+\varepsilon z}{1+\varepsilon}\right\} \in SP_s(\alpha,\beta)$, *then for every* $H(z) \in SP'_s(\alpha,\beta)$ *implies* $\frac{(f*H)(z)}{z} \ne \delta$, $\forall z \in E$.

Proof. Let $F_{\varepsilon}(z) \in SP_s(\alpha,\beta)$, then by Theorem 1

$$\frac{(f*H)(z)}{z} \neq 0 \quad \forall\ H(z) \in SP'_s(\alpha,\beta)$$

and $z \in E$. Equivalently

$$\frac{(f*H)(z)+\varepsilon z}{(1+\varepsilon) z} \neq 0 \quad \text{or} \quad \frac{(f*H)(z)}{z} \neq -\varepsilon.$$

Hence

$$\left|\frac{(f*H)(z)}{z}\right| \geq \delta, \quad \forall\ z \in E.$$ □

Theorem 2. *For $f \in A$ and for every $\varepsilon \in C$ and for $|\varepsilon| < \delta < 1$, assume $F_{\varepsilon}(z) \in SP_s(\alpha,\beta)$. If further $\alpha \geq 1-\beta$ then $TN_{\delta}(f) \subset SP_s(\alpha,\beta)$ with $\{T_k\} = \left\{\frac{k-\beta}{1-\beta}\right\}$.*

Proof. Let $T_k = \frac{k-\beta}{1-\beta}$ and $g(z) = z + \sum_{k=2}^{\infty} b_k z^k$. Then for $\alpha \geq 1-\beta$, from Lemma 1 we have,

$$\begin{aligned}\left|\frac{1}{z}(g*H)(z)\right| &= \left|\frac{1}{z}((g*f)\,H)(z) + \frac{1}{z}(f*H)(z)\right| \\ &\geq \left|\frac{(f*H)(z)}{z}\right| - \left|\frac{((g-f)(z)*H)(z)}{z}\right| \\ &> \left|\frac{(f*H)(z)}{z}\right| - \sum_{k=2}^{\infty} |b_k - a_k|\left(\frac{k-\beta}{1-\beta}\right). \end{aligned} \tag{12}$$

Since

$$F_{\varepsilon}(z) = \frac{f(z)+\varepsilon z}{1+\varepsilon} \in SP_s(\alpha,\beta), \quad F_{\varepsilon}(z) * H(z) \neq 0, \quad \forall\ H(z) \in SP'_s(\alpha,\beta),\ z \in E$$

or

$$\frac{(f*H)(z)}{z} \neq -\varepsilon$$

which is equivalently

$$\left|\frac{(f*H)(z)}{z}\right| > \delta \quad \text{for} \quad |\varepsilon| < \delta.$$

Therefore $g \in TN_{\delta}(f)$ we get from (12) that $\left|\frac{(g*H)(z)}{z}\right| > 0$ and hence $\frac{(g*H)(z)}{z} \neq 0$ in E $\forall\ H(z) \in SP'_s(\alpha,\beta)$ and $\alpha \geq (1-\beta)$ there by showing $g \in SP_s(\alpha,\beta)$. This proves that $TN_{\delta}(f) \subset SP_s(\alpha,\beta)$. □

Next we will show that the class $SP_s(\alpha,\beta)$ is closed under convolution with functions f which are convex univalent in E.

Lemma 3. *If $g \in SP_s(\alpha,\beta)$ then $G(z) \in SP_s(\alpha,\beta) \subset ST$ where $G(z) = \frac{g(z)-g(-z)}{2}$.*

Proof. Since $g \in SP_s(\alpha,\beta)$, $\frac{2zg'(z)}{g(z)-g(-z)} \in \Omega$. Now

$$\frac{zG'(z)}{G(z)} = \frac{zg'(z)}{2G(z)} + \frac{(-z)g'(-z)}{2G(-z)} = \frac{\zeta_1}{2} + \frac{\zeta_2}{2} = \zeta_3 \text{ where } \zeta_1 \text{ and } \zeta_2 \in \Omega.$$

Since Ω is convex $\zeta_3 \in \Omega$ and hence $\frac{zG'(z)}{G(z)} \in \Omega$. It can be easily seen that $SP_s(\alpha,\beta) \subset ST$. Thus $G(z) \in SP_s(\alpha,\beta) \subset ST$. □

Theorem 4. *Let $f(z) \in CV$ the class of convex functions and $g(z) \in SP_s(\alpha,\beta)$. Then $(f*g)(z) \in SP_s(\alpha,\beta)$.*

Proof. Let $f(z) \in CV$ and $g(z) \in SP_s(\alpha,\beta)$, $G(z) = (g(z) - g(-z))/2$ and Ω is a convex domain. Since $g(z) \in SP_s(\alpha,\beta)$, $G(z) \in ST$ by Lemma 3. Hence by an application of **Lemma A** we get

$$\frac{z(f*g)'(z)}{(f*G)(z)} = \frac{(f*zg')(z)}{(f*G)(z)} = \frac{f*\frac{zg'(z)}{G(z)}G(z)}{(f*G)(z)} \subset \overline{C_0}\left(\frac{zg'(z)}{G(z)}\right) \subset \Omega.$$

Since Ω is convex and $g \in SP_s(\alpha,\beta)$. This proves that $(f*g)(z) \in SP_s(\alpha,\beta)$. □

Theorem 5. *If $f \in UCV_s(\alpha,\beta)$, then $\frac{f(z)+\varepsilon z}{1+\varepsilon} \in SP_s(\alpha,\beta)$ for $|\varepsilon| < \frac{1}{4}$.*

Proof. Let $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$. Then

$$\frac{f(z)+\varepsilon z}{1+\varepsilon} = \frac{z(1+\varepsilon) + \sum\limits_{k=2}^{\infty} a_k z^k}{1+\varepsilon} = \frac{f(z) * \left[z(1+\varepsilon) + \sum\limits_{k=2}^{\infty} z^k\right]}{1+\varepsilon}$$
$$= f(z) * \frac{\left(z - \frac{\varepsilon}{1+\varepsilon} z^2\right)}{(1-z)} = f(z) * h(z)$$

where $h(z) = \frac{\left[z - \frac{\varepsilon}{1+\varepsilon} z^2\right]}{(1-z)}$

$$\frac{zh'(z)}{h(z)} = \frac{\left[z - \frac{2\varepsilon}{1+\varepsilon} z^2\right]}{\left[z - \frac{\varepsilon}{1+\varepsilon} z^2\right]} + \frac{z}{1-z} = \frac{-\rho z}{1-\rho z} + \frac{1}{1-z}, \quad \text{where} \quad \rho = \frac{\varepsilon}{1+\varepsilon}.$$

Hence $|\rho| < \frac{\varepsilon}{1-|\varepsilon|} < 1/3$ gives $|\varepsilon| < 1/4$. Thus

$$\operatorname{Re}\left\{\frac{zh'(z)}{h(z)}\right\} \geq \frac{1 - 2|\rho||z| - |\rho||z|^2}{(1-|\rho||z|)(1+|z|)} > 0,$$

if $|\rho|(|z|^2 + 2|z|) - 1 < 0$, which is true only when $|\rho| < 1/3$. Therefore h is starlike in E for $|\varepsilon| < 1/4$. Now $h(z) * \log\left(\frac{1}{1-z}\right) = \int_0^z \frac{h(t)}{t} dt$ is a convex function and

$$(f*h)(z) = (h*f)(z) = \left[h(z) * zf'(z) * \log\left(\frac{1}{1-z}\right)\right] = zf'(z) * \left[h(z) * \log\left(\frac{1}{1-z}\right)\right].$$

Hence $f(z) \in UCV_s(\alpha,\beta)$ implies $zf'(z) \in SP_s(\alpha,\beta)$ and $h(z) * \log\left(\frac{1}{1-z}\right) \in CV$.

Now by Theorem 4 we have

$$zf'(z) * \left[h(z) * \log\left(\frac{1}{1-z}\right)\right] \quad \text{is in } SP_s(\alpha,\beta).$$

Therefore

$$(f*h)(z) = \frac{f(z)+\varepsilon z}{1+\varepsilon} \in SP_s(\alpha,\beta) \text{ for } \quad |\varepsilon| < \frac{1}{4}.$$ □

Remark 1. By letting $\varepsilon = 0$ in Theorem 5 we get the following result.

Corollary 6. *If $f \in UCV_s(\alpha,\beta)$ then $f(z) \in SP_s(\alpha,\beta)$.*

Theorem 7. *Let $f \in UCV_s(\alpha,\beta)$ and $\alpha \geq 1-\beta$. Then $TN_{1/4}(f) \subset SP_s(\alpha,\beta)$.*

Proof. Let $f \in UCV_s(\alpha,\beta)$. Then from Theorem 5 we have $\frac{f(z)+\varepsilon z}{1+\varepsilon} \in SP_s(\alpha,\beta)$ for $|\varepsilon| < \frac{1}{4}$. Then an application of Theorem 2 gives when $\alpha \geq 1-\beta$, $TN_{1/4}(f) \subset SP_s(\alpha,\beta)$. □

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References

[1] A. W. Goodman, *On uniformly convex functions*, Ann. Polon. Math., **56**(1991), 87–92.

[2] A. W. Goodman, *On uniformly starlike functions*, J. Math. Anal. Appl., **155**(1991), 364–370.

[3] W. Ma and D. Minda, Uniformly convex functions, Ann. Polon, Math., **57** (1992), 165–175.

[4] K. S. Padmanabhan, *On uniformly convex functions in the unit disk*, J. Analysis, **2** (1994), 87–96.

[5] R. Parvatham and M. Premabai, *Neighbourhoods of uniformly starlike functions with respect to symmetric points*, Bull. Cal. Math. Soc., **88**(1996), 155–160.

[6] T. Ram Reddy and P. Thirupathi Reddy, *Neighbourhoods of a certain subclass of $SP(\beta)$*, J. Analysis, **15** (2007), 239-245.

[7] T. Ram Reddy and P. Thirupathi Reddy, *Neighbourhoods of a certain subclass of $SP(\alpha)$ with respect to symmetric points*, Bull. Cal. Math. Soc., **101**(2009), 359–366.

[8] F. Ronning, *On starlike functions associated with parabolic regions*, Ann. Univ. Mariae Curie-Sklodowska. Sect. A, **45** (1991), 117–122.

[9] F. Ronning, *Uniformly convex functions and a corresponding class of starlike functions*, Proc. Amer. Math. Soc. **118**(1993), 189–196.

[10] St. Ruscheweyh and T. Sheil-Small, *Hadamard products of schlicht functions and the Polya-Schoenberg Conjecture*, Comment. Math. Helv., **48** (1973), 119–135.

[11] St. Ruscheweyh, *Neighbourhoods of univalent functions*, Proc. Amer. Math. Soc., **81** (1981), 521–527.

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