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FRACTIONAL INTEGRAL FORMULAE CONCERNING CERTAIN SPECIAL FUNCTIONS

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Abstract. The main aim of the present paper is to derive some results related to fractional integral formulae on the product of multivariable H-function and two general class of multivariable polynomials. A large number of known and new result have also been obtained by proper choice of parameters.

1. Introduction

From the last four decades several mathematician (see, for example Ross [2], Kilbas and Saigo [1], Srivastava and Goyal [12], Srivastava and Hussain [6], Srivastava, Chandel and Vishwakarma [10], Manocha and Sharma [3], Saigo and Raina [17], Dhami and Gaira [15], Lin, Tu and Srivastava [19], Oldham and Spanier [13], Chaurasia and Godika [21], and Chaurasia and Gupta [20]) have made great and significant contribution in the field of fractional calculus, specially fractional derivatives and fractional integrals involving various functions.

The computation of fractional integrals (and fractional derivatives) of transcendental function of one and more variables are important from the point of view of the usefulness of these results in (for example) the evaluation of differential and integral equation. Motivated by these and other avenues of applications, a several mathematician and physician have made use of the fractional calculus operator in the theory of special functions of one and more variables.

In this paper, we are obtaining some results by using fractional integral operators on the product of general class of multivariable polymonials and multivariable H-function. Ross, B. [2] defined the fractional integral operator as the special case of the Riemann-Liouville fractional integral operator for $c = 0$ and represented it as ${}_cI_x^v\{f(x)\}$.

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The H-function of several complex variables [11] is defined in the following form:

$$H[x_1,\dots,x_r]$$
$$= H^{0,N:(M',N');\dots;(M^{(r)},N^{(r)})}_{P,Q:(P',Q');\dots;(P^{(r)},Q^{(r)})}\left[\begin{matrix}[(a):\theta',\dots,\theta^{(r)}]:[(b'):\phi'];\cdots;[(b^{(r)}):\phi^{(r)}];\\ [(c):\psi',\dots,\psi^{(r)}]:[(d'):\delta'];\cdots;[(d^{(r)}):\delta^{(r)}];\end{matrix} x_1,\dots,x_r\right]$$
$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r} m_1(\xi_1)\cdots m_r(\xi_r)\eta(\xi_1,\dots,\xi_r)x_1^{\xi_1}\cdots x_r^{\xi_r}d\xi_1\cdots d\xi_r, \tag{1.1}$$
where $\omega=\sqrt{-1}$. (1.2)

The convergence conditions and other details of the above function are given by Srivastava et al. [5, p.251, eq.(C.1)], see also equations (1.3) and (1.4) below

$$\Lambda_i=\sum_{j=N+1}^{P}\theta_j^{(i)}-\sum_{j=1}^{Q}\psi_j^{(i)}+\sum_{j=1}^{N^{(i)}}\phi_j^{(i)}-\sum_{j=N^{(i)}+1}^{P^{(i)}}\phi_j^{(i)}+\sum_{j=1}^{M^{(i)}}\delta_j^{(i)}-\sum_{j=M^{(i)}+1}^{Q^{(i)}}\delta_j^{(i)}>0 \tag{1.3}$$
$$\alpha_i=\min\{Re(d_j^{(i)}/\delta_j^{(i)}\},\quad j=1,\dots,M^{(i)},\quad \forall\ i\in(1,\dots,r) \tag{1.4}$$

We assume that the convergence and existence conditions of above function are satisfied by each of the various H-function involved throuthout the present work.

A general class of multivariable polynomials [8, p.686, eq.(1.4)], defined as follows

$$S_n^{m_1,\dots,m_s}[w_1\cdots w_s]=\sum_{k_1,\dots,k_s=0}^{m_1k_1+\cdots+m_sk_s\le n}(-n)_{m_1k_1+\cdots+m_sk_s}A(n;k_1,\dots,k_s)\frac{(w_1)^{k_1}}{k_1!}\cdots\frac{(w_s)^{k_s}}{k_s!}, \tag{1.5}$$

where $m_1,\dots,m_s$ are arbitrary positive integers and the coefficients $A(n;k_1\cdots k_s)$, $(n;k_{i'}\ge 0,\ i'=1,\dots,s)$ are arbitrary constants, real or complex.

For the sake of brevity, we use here the following notations

$$A(\phi)=\sum_{k_1,\dots,k_s=0}^{m_1k_1+\cdots+m_sk_s\le n}\ \sum_{k'_1,\dots,k'_t=0}^{m'_1k'_1+\cdots+m'_tk'_t\le n'}(-n)_{m_1k_1+\cdots+m_sk_s}(-n')_{m'_1k'_1+\cdots+m'_tk'_t}$$
$$\times A(n,k_1,\dots,k_s)A'(n',k'_1,\dots,k'_t)\frac{a_1^{k_1}}{k_1!}\cdots\frac{a_s^{k_s}}{k_s!}\cdot\frac{b_1^{k'_1}}{k'_1!}\cdots\frac{b_t^{k'_t}}{k'_t!} \tag{1.6}$$
$$B(\theta)=\alpha^{\sum_{i=1}^{s}v_ik_i+\sum_{j=1}^{t}v'_jk'_j-\ell}\beta^{\sum_{i=1}^{s}w_ik_i+\sum_{j=1}^{t}w'_jk'_j-\lambda} \tag{1.7}$$

$$I(\psi)=\sum_{k_1,\dots,k_s=0}^{m_1k_1+\cdots+m_sk_s\le n}(-n)_{m_1k_1+\cdots+m_sk_s}\frac{\prod_{i=1}^{E}(e_i)_{k_1y_i'+\cdots+k_sy_i^{(s)}}}{\prod_{i=1}^{G}(f_i)_{k_1\xi_i'+\cdots+k_s\xi_i^{(s)}}}$$

$$\times\frac{\prod_{i=1}^{U'}(T_i')_{k_1p_i'}\cdots\prod_{i=1}^{U^{(s)}}(T_i^s)_{k_sp_i^{(s)}}}{\prod_{i=1}^{V'}(\Omega_i')_{k_1q_i'}\cdots\prod_{i=1}^{V^{(s)}}(\Omega_i^{(s)})_{k_sq_i^{(s)}}}\cdot\frac{a_1^{k_1}}{k_1!}\cdots\frac{a_s^{k_s}}{k_s!} \tag{1.8}$$

$$I^*(\psi^*)=\sum_{k_1',\dots,k_t'=0}^{m_1k_1'+\cdots+m_tk_t'\le n'}(-n')_{m_1'k_1'+\cdots+m_t'k_t'}\frac{\prod_{j=1}^{E'}(e_j')_{k_1'y_j'+\cdots+k_t'y_j^{(t)}}}{\prod_{j=1}^{G'}(f_j')_{k_1'\xi_j'+\cdots+k_t'\xi_j^{(t)}}}$$

$$\times\frac{\prod_{i=1}^{(U^*)'}(T_j')_{k_1'p_j'}\cdots\prod_{i=1}^{(U^*)^t}(T_j^t)_{k_t'p_j^t}}{\prod_{i=1}^{(V^*)'}(\Omega_j')_{k_1'q_j'}\cdots\prod_{i=1}^{(V^*)^t}(\Omega_j^t)_{k_t'q_j^t}}\cdot\frac{b_1^{k_1'}}{k_1'!}\cdots\frac{b_t^{k_t'}}{k_t'!}. \tag{1.9}$$

2. Fractional integral formulae for the multivariable H-function

We shall prove the fractional integral formulae

(a)

$$\begin{aligned}
&{}_cI_x^v\{x^\rho(x+\alpha)^\sigma(x+\beta)^\mu S_n^{m_1,\dots,m_s}[a_1x^{\mu_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},\dots,a_sx^{u_s}(x+\alpha)^{v_s}(x+\beta)^{w_s}]\\
&\quad\times S_{n'}^{m_1',\dots,m_t'}[b_1x^{u_1'}(x+\alpha)^{v_1'}(x+\beta)^{w_1'},\dots,b_tx^{u_t'}(x+\alpha)^{v_t'}(x+\beta)^{w_t'}]\\
&\quad\times H[z_1x^{u_1''}(x+\alpha)^{v_1''}(x+\beta)^{w_1''},\dots,z_rx^{u_r''}(x+\alpha)^{v_r''}(x+\beta)^{w_r''}]\}\\
&=\alpha^\sigma\beta^\mu x^\rho\sum_{\eta,\ell,\lambda=0}^{\infty}A(\phi)B(\theta)\frac{(-1)^\eta(x-c)^{\eta+v}x^{\sum_{i=1}^{s}u_ik_i+\sum_{j=1}^{t}u_j'k_j'+\ell+\lambda-\eta}}{\ell!\lambda!\Gamma v(\eta+v)\eta!}\\
&\times H_{p+3,Q+3:[P',Q'];\cdots;[P^{(r)},Q^{(r)}]}^{0,N+3:[M',N'];\cdots;[M^{(r)},N^{(r)}]}\left[\begin{matrix}z_1\alpha^{v_1''}\beta^{w_1''}x_1^{u_1''}\\ \vdots\\ z_r\alpha^{v_r''}\beta^{w_r''}x_r^{u_r''}\end{matrix}\middle|\begin{matrix}(-\rho-\sum_{i=1}^{s}u_ik_i-\sum_{j=1}^{t}u_j'k_j'-\ell-\lambda:u_1'',\dots,u_r''),\\ \\ (\eta-\rho-\sum_{i=1}^{s}u_ik_i-\sum_{j=1}^{t}u_j'k_j'-\ell-\lambda:u_1'',\dots,u_r''),\end{matrix}\right.
\end{aligned}$$

$$(-\sigma-\sum_{i=1}^{s} v_i k_i-\sum_{j=1}^{t} v'_j k'_j{:}v''_1,\ldots,v''_r),\ (-\mu-\sum_{i=1}^{s} w_i k_i-\sum_{j=1}^{t} w'_j k'_j{:}w''_1,\ldots,w''_r),$$

$$(\ell-\sigma-\sum_{i=1}^{s} v_i k_i-\sum_{j=1}^{t} v'_j k'_j{:}v''_1,\ldots,v''_r),\ (\lambda-\mu-\sum_{i=1}^{s} w_i k_i-\sum_{j=1}^{t} w'_j k'_j{:}w''_1,\ldots,w''_r),$$

$$\left.\begin{matrix}[(a){:}\theta',\ldots,\theta^{(r)}]{:}[b'{:}\phi'];\ldots;[b^{(r)}{:}\phi^{(r)}]\\ [(c){:}\psi';\ldots;\psi^{(r)}]{:}[d'{:}\delta'];\ldots;[d^{(r)}{:}\delta^{(r)}]\end{matrix}\right]. \tag{2.1}$$

where

(i) $\Lambda_i > 0,\ \min\{u_i, v_i, w_i, u'_j, v'_j, w'_j, u''_{i'}, v''_{i'}, w''_{i'}\} > 0,\ i = 1,\ldots,s;\ j = 1,\ldots,t;$ $i' = 1,\ldots,r,$

(ii) $|\arg(z_i)| < \dfrac{\Lambda_i \pi}{2},$

(iii) $\mathrm{Re}(\rho) + \sum_{i=1}^{r} \alpha_i u''_i > -1$ and m_i and m'_i are arbitrary positive integers.

Result (2.1) can also be written in the following form:

$$I_x^v\{x^\rho(x+\alpha)^\sigma(x+\beta)^\mu S_n^{m_1,\ldots,m_s}[a_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},\ldots,a_sx^{u_s}(x+\alpha)^{v_s}(x+\beta)^{w_s}]$$
$$\times S_{n'}^{m'_1,\ldots,m'_t}[b_1x^{u'_1}(x+\alpha)^{v'_1}(x+\beta)^{w'_1},\ldots,b_tx^{u'_t}(x+\alpha)^{v'_t}(x+\beta)^{w'_t}]$$
$$\times H[z_1x^{u''_1}(x+\alpha)^{v''_1}(x+\beta)^{w''_1},\ldots,z_rx^{u''_r}(x+\alpha)^{v''_r}(x+\beta)^{w''_r}]\}$$

$$= \alpha^\sigma\beta^\mu x^{\rho+v}\sum_{\ell,\lambda=0}^{\infty} A(\phi)B(\theta)\frac{x^{\sum_{i=1}^{s} u_ik_i+\sum_{j=1}^{t} u'_jk'_j+\ell+\lambda}}{\ell!\lambda!}$$

$$\times H^{0,N+3:[M',N'];\cdots;[M^{(r)},N^{(r)}]}_{p+3,Q+3:[P',Q'];\ldots;[P^{(r)},Q^{(r)}]}\left[\begin{matrix} z_1\alpha^{v''_1}\beta^{w''_1}x_1^{u''_1}\\ \vdots\\ z_r\alpha^{v''_r}\beta^{w''_r}x_r^{u''_r}\end{matrix}\left|\begin{matrix}(-\rho-\sum_{i=1}^{s} u_ik_i-\sum_{j=1}^{t} u'_jk'_j-\ell-\lambda{:}u''_1,\ldots,u''_r),\\ \\ (-v-\rho-\sum_{i=1}^{s} u_ik_i-\sum_{j=1}^{t} u'_jk'_j-\ell-\lambda{:}u''_1,\ldots,u''_r),\end{matrix}\right.\right.$$

$$(-\sigma-\sum_{i=1}^{s} v_i k_i-\sum_{j=1}^{t} v'_j k'_j{:}v''_1,\ldots,v''_r),\ (-\mu-\sum_{i=1}^{s} w_i k_i-\sum_{j=1}^{t} w'_j k'_j{:}w''_1,\ldots,w''_r),$$

$$(\ell-\sigma-\sum_{i=1}^{s} v_i k_i-\sum_{j=1}^{t} v'_j k'_j{:}v''_1,\ldots,v''_r),\ (\lambda-\mu-\sum_{i=1}^{s} w_i k_i-\sum_{j=1}^{t} w'_j k'_j{:}w''_1,\ldots,w''_r),$$

$$\left.\begin{matrix}[(a){:}\theta',\ldots,\theta^{(r)}]{:}[b'{:}\phi'];\ldots;[b^{(r)}{:}\phi^{(r)}]\\ [(c){:}\psi';\ldots;\psi^{(r)}]{:}[d'{:}\delta'];\ldots;[d^{(r)}{:}\delta^{(r)}]\end{matrix}\right]. \tag{2.2}$$

(b)

$$I_x^{\gamma,v}\{x^\rho(x+\alpha)^\sigma(x+\beta)^\mu S_n^{m_1,\ldots,m_s}[a_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},\ldots,a_sx^{u_s}(x+\alpha)^{v_s}(x+\beta)^{w_s}]$$
$$\times S_{n'}^{m'_1,\ldots,m'_t}[b_1x^{u'_1}(x+\alpha)^{v'_1}(x+\beta)^{w'_1},\ldots,b_tx^{u'_t}(x+\alpha)^{v'_t}(x+\beta)^{w'_t}]$$
$$\times H[z_1x^{u''_1}(x+\alpha)^{v''_1}(x+\beta)^{w''_1},\ldots,z_rx^{u''_r}(x+\alpha)^{v''_r}(x+\beta)^{w''_r}]\}$$

$$= \alpha^{\sigma}\beta^{\mu}x^{\rho}\sum_{\ell,\lambda=0}^{\infty}A(\phi)B(\theta)\frac{x^{\sum_{i=1}^{s}u_ik_i+\sum_{j=1}^{t}u'_jk'_j+\ell+\lambda}}{\ell!\lambda!}$$

$$\times H^{0,N+3:[M',N'];\cdots;[M^{(r)},N^{(r)}]}_{p+3,Q+3:[P',Q'];\ldots;[P^{(r)},Q^{(r)}]}\left[\begin{matrix} z_1\alpha^{v''_1}\beta^{w''_1}x_1^{u''_1} \\ \vdots \\ z_r\alpha^{v''_r}\beta^{w''_r}x_r^{u''_r}\end{matrix}\middle| \begin{matrix}(1-\gamma-\rho-\sum_{i=1}^{s}u_ik_i-\sum_{j=1}^{t}u'_jk'_j-\ell-\lambda:u''_1,\ldots,u''_r), \\ (-v+1-\gamma-\rho-\sum_{i=1}^{s}u_ik_i-\sum_{j=1}^{t}u'_tk'_t-\ell-\lambda:u''_1,\ldots,u''_r),\end{matrix}\right.$$

$$(-\sigma-\sum_{i=1}^{s}v_ik_i-\sum_{j=1}^{t}v'_jk'_j:v''_1,\ldots,v''_r),\ (-\mu-\sum_{i=1}^{s}w_ik_i-\sum_{j=1}^{t}w'_jk'_j:w''_1,\ldots,w''_r),$$

$$(\ell-\sigma-\sum_{i=1}^{s}v_ik_i-\sum_{j=1}^{t}v'_jk'_j:v''_1,\ldots,v''_r),\ (\lambda-\mu-\sum_{i=1}^{s}w_ik_i-\sum_{j=1}^{t}w'_jk'_j:w''_1,\ldots,w''_r),$$

$$\left.\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[b':\phi'];\ldots;[b^{(r)}:\phi^{(r)}] \\ [(c):\psi';\ldots;\psi^{(r)}]:[d':\delta'];\ldots;[d^{(r)}:\delta^{(r)}]\end{matrix}\right]. \tag{2.3}$$

where

(i) $\Lambda_i > 0$, $\min\{u_i, v_i, w_i, u'_j, v'_j, w'_j, u''_{i'}, v''_{i'}, w''_{i'}\} > 0$, $i = 1,\ldots,s$; $j = 1,\ldots,t$; $i' = 1,\ldots,r$

(ii) $|\arg(z_i)| < \dfrac{\Lambda_i\pi}{2}$,

(iii) $\mathrm{Re}(\rho) + \sum_{i=1}^{r}\alpha_iu''_i > -\eta$ and m_i and m'_i are arbitrary positive integers.

Proof. In order to prove (2.1), we first replace the multivariable H-function occurring on the left hand side by its Mellin-Barnes contour integral form given by (1.1) and express both general class of multivariable polynomials in series form given by (1.5). Now we collect the powers of x, $(x+\alpha)$ and $(x+\beta)$ and apply the binomial expansion

$$(x+\alpha)^{\sigma} = \alpha^{\sigma}\sum_{\ell=0}^{\infty}\binom{\sigma}{\ell}\left(\frac{x}{\alpha}\right)^{\ell} \qquad (|x/\alpha| < 1), \tag{2.4}$$

then we apply the fractional integral formula [2]

$${}_cI_x^v\{x^g\} = \sum_{\eta=0}^{\infty}\frac{(-1)^{\eta}(x-c)^{\eta+v}}{\Gamma v(\eta+v)\eta!}\cdot\frac{\Gamma g+1}{\Gamma g-\eta+1}x^{g-\eta} \quad [\mathrm{Re}(g) > -1], \tag{2.5}$$

and interpret the resulting Mellin-Barnes contour integral as an H-function of r-variables which is permissible under the stated conditions. We are thus led finally to the fractional integral formula (2.1).

By the application of the following formula

$$\sum_{\eta=0}^{\infty}\frac{(-1)^{\eta}}{\Gamma v(v+\eta)\eta!}H^{0,N:[M',N'];\ldots;[M^{(r)},N^{(r)}]}_{P,Q+1:[P',Q'];\ldots;[P^{(r)},Q^{(r)}]}\left[\begin{matrix}x_1\\ \vdots\\ x_r\end{matrix}\left|\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]: & [b':\phi'];\ldots;[b^r:\phi^r]\\ (\eta-k:u_1,\ldots,u_r),\ [(c):\psi',\ldots,\psi^{(r)}]:[d':\delta'];\ldots;[d^r:\delta^r]\end{matrix}\right.\right]$$

$$= H^{0,N:[M',N'];\ldots;[M^{(r)},N^{(r)}]}_{P,Q+1:[P',Q'];\ldots;[P^{(r)},Q^{(r)}]}\left[\begin{matrix}x_1\\ \vdots\\ x_r\end{matrix}\left|\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]: & [b':\phi'];\ldots;[b^r:\phi^r]\\ (v-k:u_1,\ldots,u_r),\ [(c):\psi',\ldots,\psi^{(r)}]:[d':\delta'];\ldots;[d^r:\delta^r]\end{matrix}\right.\right]. \tag{2.6}$$

We can evaluated result (2.2) which is an another form of result (2.1).

For result (2.3) we proceed on similar lines as adopted in result (2.1) and using

$$I_x^{\gamma,v}\{x^g\}=\frac{\Gamma g+\gamma}{\Gamma g+\gamma+v}x^g,\quad \mathrm{Re}(g)>-\gamma. \tag{2.7}$$

We can generate the result (2.3).

3. Special cases

Taking

$$A(n;k_1,\ldots,k_s)=\frac{\prod_{i=1}^{E}(e_i)_{k_1y_i'+\cdots+k_sy_i^{(s)}}}{\prod_{i=1}^{G}(f_i)_{k_1\xi_i'+\cdots+k_s\xi_i^{(s)}}}\cdot\frac{\prod_{i=1}^{U'}(T_i')_{k_1p_i'}\cdots\prod_{i=1}^{U^{(s)}}(T_i^{(s)})_{k_sp_i^{(s)}}}{\prod_{i=1}^{V'}(\Omega_i')_{k_1q_i'}\cdots\prod_{i=1}^{V^{(s)}}(\Omega_i^{(s)})_{k_sq_i^s}}, \tag{3.1}$$

and

$$A'(n';k_1',\ldots,k_t')=\frac{\prod_{j=1}^{E'}(e_j')_{k_1'y_j'+\cdots+k_t'y_j^{(t)}}}{\prod_{j=1}^{G'}(f_j')_{k_1'\xi_j'+\cdots+k_t'\xi_j^{(t)}}}\cdot\frac{\prod_{j=1}^{(U^*)'}(T_j')_{k_1'p_j'}\cdots\prod_{j=1}^{(U^*)^t}(T_j^t)_{k_t'p_j^t}}{\prod_{j=1}^{(V^*)'}(\Omega_j')_{k_1'q_j'}\cdots\prod_{j=1}^{(V^*)^t}(\Omega_j^t)_{k_t'q_j^t}}, \tag{3.2}$$

in (1.5), $S_n^{m_1,\ldots,m_s}[w_1,\ldots,w_s]$ and $S_n^{m_1',\ldots,m_t'}[w_1',\ldots,w_t']$ reduce to the generalized Lauricella function of Srivastava, H. M. and Daoust, M. C. [7. p.454] as follows

$$S_n^{m_1,\ldots,m_s}[w_1,\ldots,w_s]$$
$$=F^{1+E:U';\ldots;U^{(s)}}_{G:V';\ldots;V^{(s)}}\left[\begin{matrix}(-n:m_1,\ldots,m_s),\ ((e):y',\ldots,y^s):((T'):p');\ldots;((T^{(s)}):p^{(s)});\\ ((f):\xi',\ldots,\xi^s):((\Omega'):q');\ldots;((\Omega^s):q^s);\end{matrix}\ w_1,\ldots,w_s\right] \tag{3.3}$$

and

$$S_{n'}^{m'_1,\ldots,m'_t}[w_1,\ldots,w_t]$$
$$= F_{*G':V';\ldots;V^t}^{1+E':V';\ldots;V^t}\left[\begin{matrix}(-n':m'_1,\ldots,m'_t),\ ((e'):y',\ldots,y^t):((T'):p');\ldots;((T^{(t)}):p^{(t)});\\ ((f):\xi',\ldots,\xi^t):((\Omega^{(\prime)}):q^{(\prime)});\ldots;((\Omega^t):q^t);\end{matrix}\ w_1,\ldots,w_s\right] \tag{3.4}$$

and the result (2.2) and (2.3) readily reduce to the result involving generalized Lauricella function. For example

1.

$$I_x^v\{x^\rho(x+\alpha)^\sigma(x+\beta)^\mu F[a_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},\ldots,a_sx^{u_s}(x+\alpha)^{v_s}(x+\beta)^{w_s}]$$
$$\times F_*[b_1x^{u'_1}(x+\alpha)^{v'_1}(x+\beta)^{w'_1},\ldots,b_tx^{u'_t}(x+\alpha)^{v'_t}(x+\beta)^{w'_t}]$$
$$\times H[z_1x^{u''_1}(x+\alpha)^{v''_1}(x+\beta)^{w''_1},\ldots,z_rx^{u''_r}(x+\alpha)^{v''_r}(x+\beta)^{w''_r}]\}$$
$$=\alpha^\sigma\beta^\mu x^{\rho+v}\sum_{\ell,\lambda=0}^{\infty}I^*(\psi^*)B(\theta)\frac{x^{\sum_{i=1}^{r}u_ik_i+\sum_{j=1}^{t}u'_jk'_j+\ell+\lambda}}{\ell!\lambda!}$$
$$\times H_{P+3,Q+3:[P',Q'];\ldots;[P^{(r)},Q^{(r)}]}^{0,N+3:[M',N'];\cdots;[M^{(r)},N^{(r)}]}\left[\begin{matrix}z_1\alpha^{v''_1}\beta^{w''_1}x_1^{u''_1}\\ \vdots\\ z_r\alpha^{v''_r}\beta^{w''_r}x_r^{u''_r}\end{matrix}\left|\begin{matrix}(-\rho-\sum_{i=1}^{s}u_ik_i-\sum_{j=1}^{t}u'_jk'_j-\ell-\lambda:u''_1,\ldots,u''_r),\\ (-v-\rho-\sum_{i=1}^{s}u_ik_i-\sum_{j=1}^{t}u'_jk'_j-\ell-\lambda:u''_1,\ldots,u''_r),\end{matrix}\right.\right.$$
$$(-\sigma-\sum_{i=1}^{s}v_ik_i-\sum_{j=1}^{t}v'_jk'_j:v''_1,\ldots,v''_r),\ (-\mu-\sum_{i=1}^{s}w_ik_i-\sum_{j=1}^{t}w'_jk'_j:w''_1,\ldots,w''_r),$$
$$(\ell-\sigma-\sum_{i=1}^{s}v_ik_i-\sum_{j=1}^{t}v'_jk'_j:v''_1,\ldots,v''_r),\ (\lambda-\mu-\sum_{i=1}^{s}w_ik_i-\sum_{j=1}^{t}w'_jk'_j:w''_1,\ldots,w''_r),$$
$$\left.\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[b':\phi'];\ldots;[b^{(r)}:\phi^{(r)}]\\ [(c):\psi';\ldots;\psi^{(r)}]:[d':\delta'];\ldots;[d^{(r)}:\delta^{(r)}]\end{matrix}\right].$$

2.

$$I_x^{\gamma,v}\{x^\rho(x+\alpha)^\sigma(x+\beta)^\mu F[a_1x^{u_1}(x+\alpha)^{v_1}(x+\beta)^{w_1},\ldots,a_sx^{u_s}(x+\alpha)^{v_s}(x+\beta)^{w_s}]$$
$$\times F_*[b_1x^{u'_1}(x+\alpha)^{v'_1}(x+\beta)^{w'_1},\ldots,b_tx^{u'_t}(x+\alpha)^{v'_t}(x+\beta)^{w'_t}]$$
$$\times H[z_1x^{u''_1}(x+\alpha)^{v''_1}(x+\beta)^{w''_1},\ldots,z_rx^{u''_r}(x+\alpha)^{v''_r}(x+\beta)^{w''_r}]\}$$
$$=\alpha^\sigma\beta^\mu x^{\rho}\sum_{\ell,\lambda=0}^{\infty}I(\psi)I^*(\psi^*)B(\theta)\frac{x^{\sum_{i=1}^{r}u_ik_i+\sum_{j=1}^{t}u'_jk'_j+\ell+\lambda}}{\ell!\lambda!}$$

$$\times H^{0,N+3:[M',N'];\cdots;[M^{(r)},N^{(r)}]}_{P+3,Q+3:[P',Q'];\ldots;[P^{(r)},Q^{(r)}]}\left[\begin{array}{c|l} z_1\alpha^{v_1''}\beta^{w_1''}x_1^{u_1''} & (1-\gamma-\rho-\sum_{i=1}^{s}u_ik_i-\sum_{j=1}^{t}u_j'k_j'-\ell-\lambda:u_1'',\ldots,u_r''), \\ \vdots & \\ z_r\alpha^{v_r''}\beta^{w_r''}x_r^{u_r''} & (-v+1-\gamma-\rho-\sum_{i=1}^{s}u_ik_i-\sum_{j=1}^{t}u_j'k_j'-\ell-\lambda:u_1'',\ldots,u_r''), \end{array}\right.$$

$$(-\sigma-\sum_{i=1}^{s}v_ik_i-\sum_{j=1}^{t}v_j'k_j':v_1'',\ldots,v_r''),\ (-\mu-\sum_{i=1}^{s}w_ik_i-\sum_{j=1}^{t}w_j'k_j':w_1'',\ldots,w_r''),$$

$$(\ell-\sigma-\sum_{i=1}^{s}v_ik_i-\sum_{j=1}^{t}v_j'k_j':v_1'',\ldots,v_r''),\ (\lambda-\mu-\sum_{i=1}^{s}w_ik_i-\sum_{j=1}^{t}w_j'k_j':w_1'',\ldots,w_r''),$$

$$\left.\begin{array}{l}[(a):\theta',\ldots,\theta^{(r)}]:[b':\phi'];\ldots;[b^{(r)}:\phi^{(r)}] \\ [(c):\psi';\ldots;\psi^{(r)}]:[d':\delta'];\ldots;[d^{(r)}:\delta^{(r)}]\end{array}\right].$$

3. By setting $s = t = 1$ in (2.1) through (2.3), we arrive at the results [15] after a little simplification.
4. By putting $s = t = 1$ and then $\mu = 0$, $w_i = 0$ $(i = 1, \ldots, r)$, $m_1', n' = 0$ in integral formula (2.3), we arrive at the results obtained by Gupta and Agrawal [14].
5. If we put $\mu = 0$, $n = n' = 0$ and $w_i = 0$ $(i = 1, \ldots, r)$ in (2.2), we get a result which is the same as obtained by Srivastava et al. [12].
6. Taking $m' = n' = k'$ $(i = 1, \ldots, t) = 0$ and $w_i = 0$ $(i = 1, \ldots, r)$ and $s = 1$ in equation (2.1), we get another result of Gupta et al. [14].

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